Chapter 17

Simulation

17.1 Introduction

The representation of reality in *some physical form* or *in some form of* mathematical equations may be called **simulation.** *i.e.,* **simulation is an imitation of reality.**

Some examples of simulation models

(1) Testing an aircraft model in a wind tunnel.

(2) Children cycling park with various signals and crossing – to model a traffic system.

(3) Planetarium.

To determine the behaviour of a real system in true environment a number of experiments are performed on simulated models either in the laboratories or in a computer itself.

Some advantages of Simulation

(1) Less complicated mathematically.

(2) Flexible.

(3) Modified to suit the changing environments of the real situation.

(4) Can be used for training purposes.

(5) May be less expensive in quite a few real-world situations.

(6) May be less time consuming in quite a few real–world situations.

Some Limitations of Simulation

(1) Quantification of the variables may be difficult.

(2) Large number of variables makes simulation unwieldy and more difficult.

(3) Simulation may not yield optimum results.

(4) Simulation may not be cheap always.

(5) Simulation may not be less time consuming always.

(6) Cannot rely too much on the results obtained from simulation models sometimes.

When to use Simulation

Simulation may be especially used when

(1) The problem is susceptible to description by a mathematical model but the analysis of the model is beyond the level of mathematical sophistication of the analyst.

(2) The problem is not susceptible to description by a mathematical model.

(3) One is satisfied with sub optimal results for decision making.

17.2 Monte–Carlo Technique or Monte–Carlo simulation

This technique involves the selection of random observations with in the simulation model. It is constrained for application involving random numbers to solve deterministic and stochastic problems. The underlying principle of this technique is

(1) Replace the actual statistical universe by another universe described by some assumed probability distribution.

(2) Sample from this theoretical population by means of random numbers.

17.3 Generation of Random Numbers

Random number is a number (from a collection of numbers) whose probability of occurrence is the same as that of any other number in the collection.

(1) Random numbers may be drawn from a random number table. (Appendix Table 2 of this book)

(2) Random number may be obtained using electronic devices.

(3) One may use Pseudo random numbers in the place of random numbers.

Pseudo random numbers may be generated by some arithmetic operations. The most commonly used one is the ***congruence method*** or the ***residue method***, by using the formula.

$r_{i+1} = (ar_i + b)$ (modulo m)

where a, b are constants ; r_i, r_{i+1} are the i^{th} and $(i+1)^{th}$ pseudo random numbers. r_0 is called the seed which, if chosen properly, may, yield a large set of pseudo random numbers. Before using the pseudo random numbers, the validation of a pseudo random number is very important. A number of tests are available to test the randomness of the sequence.

Some tests to ensure the uniformity and independence of random numbers are

(1) Frequency test or uniformity test

(2) Chi-square test

(3) Independence test :

Even though there is a large number of tests for *randomness* a sequence of pseudo-random numbers that passes the *frequency* test and independence test will be sufficient *for most simulation* purposes. The multiplicative congruential generating

$x_{i+1} = x_i \cdot a \pmod{m}$ with the value of a, m and x_0 taken suitably is found to pass both these tests.

17.4 Steps in Simulation

(1) Identify the measure of effectiveness.

(2) Decide the variables which influence the measure of effectiveness – choose those variables which affects the measure of effectiveness significantly.

(3) Determine the probability distribution for each variable in step (2) and construct the cumulative probability distribution.

(4) Choose an appropriate set of random numbers.

(5) Consider each random number as decimal value of the cumulative probability distribution.

(6) Use the simulated values so generated into the formula derived from the measure of effectiveness.

(7) Repeat (5) & (6) until the sample is large enough to arrive at a satisfactory and reliable decision.

12.5 Uses of Simulation

Simulation is used for solving :

(1) Inventory problems

(2) Queueing problems

(3) Training programmes, *etc.*

17.6 SIMULATION APPLIED TO QUEUEING PROBLEMS

Example 1 : **Customers arrive at a milk booth for the required service. Assume that inter arrival and service time are constants and given by 1.5 and 4 minutes respectively. Simulate the system by hand computations for 14 minutes *(i)* What is the waiting time per customer ? *(ii)* What is the percentage idle time for the facility? (Assume that the system starts at $t = 0$)**

Solution : First customer starts getting the service. So its departure time becomes $t = 0 + 4 = 4$ minutes. Next event (arrival) occurs at $t = 0 + 1.5 = 1.5$ minutes, which is listed before d_1 at $t = 4$. The facility is still busy 2nd customer stands in the queue and the first one is to be considered in this queue. The 3rd arrival event a_4 (customer 4) at $t = 3.0 + 1.5 = 4.5$. This event succeeds d_1 at $t = 4$. At this moment first customer departs leaving the service facility free. Second customer who was the first to join the queue, now gets service. The waiting time is calculated as the time period from the moment he joined the queue until his service is started. This process is repeated until the simulated period is completed. The results of simulation are given in the following table.

Time	Event arrival/departure	Customer No	Waiting time
0 – 0	a_1	1	
1 – 5	a_2	2	
3 – 0	a_3	3	
4 – 0	d_1	1	4 – 1.5 = 2.5 (Customer 2)
4 – 5	a_4	4	
6 – 0	a_5	5	
7 – 5	a_6	6	
8 – 0	d_2	2	8 – 3 = 5 (Customer 3)
9 – 0	a_7	7	
10 – 5	a_8	8	
12 – 0	a_9, d_3	9,3	12 – 4.5 = 7.5 (Customer 4)
13 – 0	a_{10}	10	
14 – 0	*end*	–	

For customers who are yet to get the service after 14 minutes the waiting times are given by the following table :

Customer No	Waiting time in minutes
5	14 − 6 = 8
6	14 − 7.5 = 6.5
7	14 − 9.0 = 5.0
8	14 − 10.5 = 3.5
9	14 − 12.0 = 2
10	14 − 13.5 = 0.5

From this simulation table it is clear that

(i) Average waiting time for customer

$$= \frac{2.5 + 5 + 7.5 + 8 + 6.5 + 5.0 + 3.5 + 2 + 0.5}{10}$$

$$= \frac{40.5}{10} = 4.05 \text{ minutes.}$$

(ii) Average waiting per customer for those who must wait

$$= \frac{40.5}{9} = 4.5 \text{ minutes.}$$

(iii) Percentage idle time of the facility = 0%.
(since servicing facility is always busy)

Example 2 : A sample of 100 arrivals of customers at a retail sales depot is according to the following distribution.

Time between arrivals (min)	Frequency
0.5	2
1.0	6
1.5	10
2.0	25
2.5	20
3.0	14
3.5	10
4.0	7
4.5	4
5.0	2

A study of the time required to service customers by adding up the bills, receiving payments and placing packages yields the following distribution.

Time between service (min)	Frequency
0.5	12
1.0	21
1.5	36
2.0	19
2.5	7
3.0	5

Estimate the average percentage customer waiting time and average percentage idle time of the server by simulation for the next 10 arrivals.

Solution : Tag numbers are allocated to the events in the same proportions as indicated by the probabilities.

Arrivals (min)	Frequency	Probability	Cumulative Probability	Tag–numbers
0.5	2	0.02	0.02	00 – 01
1.0	6	0.06	0.08	02 – 07
1.5	10	0.10	0.18	08 – 17
2.0	25	0.25	0.43	18 – 42
2.5	20	0.20	0.63	43 – 62
3.0	14	0.14	0.77	63 – 76
3.5	10	0.10	0.87	77 – 86
4.0	7	0.07	0.94	87 – 93
4.5	4	0.04	0.98	94 – 97
5.0	2	0.02	1.00	98 – 99

Service time (min)	Frequency	Probability	Cumulative Probability	Tag–numbers
0.5	12	0.12	0.12	00 – 11
1.0	21	0.21	0.33	12 – 32
1.5	36	0.36	0.69	33 – 68
2.0	19	0.19	0.88	69 – 87
2.5	7	0.07	0.95	88 – 94
3.0	5	0.05	1.00	95 – 97

The random numbers are generated and linked to the appropriate events. The first 10 random numbers simulating arrival, the second 10 simulating service times. The results are incorporated into an appropriate table on the assumption that the system starts at 0.00 a.m.

Arrival No	Random No	Inter arrival time (min)	Arrival time (min)	Random No	Service Time (min)	Service Start	Service End	Waiting time of Customer	Waiting time of Server
1	93	4.0	4.0	78	2.0	4.0	6.0	–	4.0
2	22	2.0	6.0	76	2.0	6.0	8.0	–	–
3	53	2.5	8.5	58	1.5	8.5	10.0	–	0.5
4	64	3.0	11.5	54	1.5	11.5	13.0	–	1.5
5	39	2.0	13.5	74	2.0	13.5	15.5	–	0.5
6	07	1.0	14.5	92	2.5	15.5	18.0	1.0	–
7	10	1.5	16.0	38	1.5	18.0	19.5	2.0	–
8	63	3.0	19.0	70	2.0	19.5	21.5	0.5	–
9	76	3.0	22.0	96	3.0	22.0	25.0	–	0.5
10	35	2.0	24.0	92	2.5	25.0	27.5	1.0	–
							Total	4.5	7.0

(i) Average waiting time per customer is $= \frac{4.5}{10} = 0.45$ minutes

(ii) Average waiting time (or) idle time of the servers $= \frac{7.0}{10} = 0.7$ min

Example 3 : **An automobile production line turns out about 100 cars a day, but deviations occur owing to many causes. The production is more accurately described by the probability distribution given below :**

Production/day	Probability
95	0.03
96	0.05
97	0.07
98	0.10
99	0.15
100	0.20
101	0.15
102	0.10
103	0.07
104	0.05
105	0.03
	1.00

Finished cars are transported across the bay at the end of each day by ferry. If the ferry has space for only 101 cars, what will be the average number of cars waiting to be shipped and what will be the average number of empty spaces on the ship ?

Solution : The Tag – numbers are established as in the table below:

Production/day	Probability	Cumulative probability	Tag numbers
95	0.03	0.03	00 – 02
96	0.05	0.08	03 – 07
97	0.07	0.15	08 – 14
98	0.10	0.25	15 – 24
99	0.15	0.40	25 – 39
100	0.20	0.60	40 – 59
101	0.15	0.75	60 – 74
102	0.10	0.85	75 – 84
103	0.07	0.92	85 – 91
104	0.05	0.97	92 – 96
105	0.03	1.00	97 – 99

The simulated production of cars for the next 15 days is given in the following table :

Day	Random Number	Production per day	No. of cars waiting	No. of empty spaces in the ship
1	97	105	4	–
2	02	95	–	6
3	80	102	1	–
4	66	101	–	–
5	96	104	3	–
6	55	100	–	1
7	50	100	–	1
8	29	99	–	2
9	58	100	–	1
10	51	100	–	1
11	04	96	–	5
12	86	103	2	–
13	24	98	–	3
14	39	99	–	2
15	47	100	–	1
		Total	10	23

Average number of cars waiting to be shipped $= \frac{10}{15}$

$= 0.67$ per day

Average number of empty spaces on the ship $= \frac{23}{15}$

$= 1.53$ per day

17.7. Simulation applied to some other types of problems.

Example 4 : **Using simulation find the value of π.**

Solution : Taking the origin O as the centre draw an arc AB of unit circle cutting the coordinate axes OX, OY at A and B and complete the square OACB as shown in the figure.

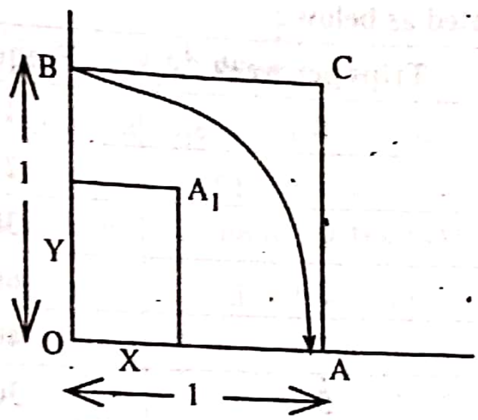

Fig. 12.1

Equation of the arc AB is $x^2 + y^2 = 1, 0 \le x \le 1, 0 \le y \le 1$ and $0 \le x \le 1, 0 \le y \le 1$ is nothing but all the points inside and on the square OACB.

Using the random number table, select any two random numbers, say 0.8262 and 0.9586 and let $x = 0.98262$ and $y = 0.9586$. The point (0.8262, 0.9586) will lie inside or on the arc AB if $x^2 + y^2 \le 1$ or will lie outside the arc and within the square if $x^2 + y^2 > 1$.

In this manner large number of pairs of random numbers are selected and whether the points representing the pairs lie in/on the arc or beyond the arc but inside the square is determined.

Let p be the total number of points considered (p pairs of random numbers are selected) and let q be those points

which lie inside or on the arc. Then, obviously

$$\frac{q}{p} = \frac{\text{area enclosed by the arc}}{\text{area of the square}} = \frac{\frac{\pi}{4} \times 1^2}{1} = \frac{\pi}{4}$$

$$\therefore \pi = \frac{4q}{p}$$

Larger the value of p, closer will be the value obtained for π.

Example 5 : A tourist car operator finds that during the past few months the car's use has varied so much that the cost of maintaining the car varied considerably. During the past 200 days the demand for the car fluctuated as below :

Trips per week day	Frequency
0	16
1	24
2	30
3	60
4	40
5	30

Using random numbers simulate the demand for a ten week period.

Solution : The Tag – numbers allotted for various demand levels is shown in the table below :

Trips/week (or) Demand/week	Fre quency	Proba bility	Cumulative Probability	Tag – Numbers
0	16	0.08	0.08	00 – 07
1	24	0.12	0.20	08 – 19
2	30	0.15	0.35	20 – 34
3	60	0.30	0.65	35 – 64
4	40	0.20	0.85	65 – 84
5	30	0.15	1.00	85 – 90

The simulated demand for the cars for the next 10 weeks period is given in the table below :

Week	Random Number	Demand
1	82	4
2	95	5
3	18	1
4	96	5
5	20	2
6	84	4
7	56	3
8	11	1
9	52	3
10	03	0

Total demand = 28 cars

$\therefore$ Average demand = $\frac{28}{10}$ = 2.8 cars per week.

Example 6 : Suppose that the demand for a particular item is normally distributed with a mean of 175 units and standard deviation of 25 units per day. Simulate the demand for the next 20 days.

Solution : Give that $\mu = 175$ and $\sigma = 25$. In normal distribution, 99.9% observations lie in $\mu \pm 3\sigma$ limits. Hence the demand can vary within $175 \pm 3 \times 25$ *i.e.*, from 100 to 250. The cumulate distribution of demand can be found as :

Demand X	$z = \frac{X-\mu}{\sigma}$	Cumulative Probability	Tag – Numbers
100	–3	0.00	00 – 00
125	–2	0.02	00 – 01
150	–1	0.16	02 – 15
175	0	0.50	16 – 49
200	1	0.84	50 – 83
225	2	0.98	84 – 97
250	3	1.00	98 – 99

The simulated demand for the next 20 days in given in the table below:

Day	Random Number	Demand	Day	Random Number	Demand
1	97	250	11	04	150
2	02	150	12	86	225
3	80	200	13	24	175
4	66	200	14	39	176
5	96	225	15	47	175
6	55	200	16	60	200
7	50	200	17	65	200
8	29	175	18	44	175
9	58	200	19	93	225
10	51	200	20	20	175

Example 7 : Suppose that the sales of a particular item per day is poisson with mean 5, then generate 20 days of sales by Monte–Carlo Method.

Solution : The cumulative distribution of sales is calculated on the information that sales have poisson distribution with mean $\lambda = 5$.

The probability for sales is given by

$$P(X = r) = \frac{e^{-\lambda}\lambda^r}{r!} = \frac{e^{-5}5^r}{r!} \qquad (\because \lambda = 5)$$

r	Cumulative Probability	Tag – Numbers
0	0.01	00 – 00
1	0.04	01 – 03
2	0.13	04 – 12
3	0.27	13 – 26
4	0.44	27 – 43
5	0.62	44 – 61
6	0.76	62 – 75
7	0.87	76 – 86
8	0.93	87 – 92
9	0.97	93 – 96
10	0.98	97 – 97
11	0.99	98 – 98
12	1.00	99 – 99

The simulated sales for the next 20 days is given in the table below :

Day	Random Number	Sales	Day	Random Number	Sales
1	49	05	11	99	12
2	58	05	12	89	08
3	89	08	13	10	02
4	15	03	14	27	04
5	12	02	15	50	05
6	94	09	16	93	09
7	85	07	17	92	08
8	34	04	18	57	05
9	07	02	19	50	05
10	53	05	20	78	07

Example 8 : **A manufacturing company keeps stock of a special product. Previous experience indicates the daily demand as given below :**

Daily demand :	5	10	15	20	25	30
Probability :	0.01	0.20	0.15	0.50	0.12	0.02

Simulate the demand for the next 10 days. Also find the daily average demand for that product on the basis of simulated data.

Solution :

Demand	Probability	Cumulative Probability	Tag – Numbers
5	0.01	0.01	00 – 00
10	0.20	0.21	01 – 20
15	0.15	0.36	21 – 35
20	0.50	0.86	36 – 85
25	0.12	0.98	86 – 97
30	0.02	1.00	97 – 98

The simulated demand for the special product for the next 10 days is given in the table below :

Day	Random Number	Demand
1	82	20
2	96	25
3	18	10
4	96	25
5	20	10
6	84	20
7	56	20
8	11	10
9	52	20
10	03	10
	Total	170

$\therefore$ Average demand $= \frac{170}{10} = 17$ units/day

EXERCISE

1. **Define Simulation. Why it is used ?**
   [BRU. B.E. Apr 97, Nov 97, MU. B.E. Oct 98]
2. **What is Simulation ?**
3. **Explain simulation ?**
4. **Explain Monte–Carlo method.**
5. **What are the advantages of simulation ?**
6. **What are the limitations of simulation ?**
7. **What are the uses of simulation ?**
8. **When simulation is preferable ?**
9. **What are the different steps involved while solving a problem by simulation ?**
10. **Explain the significance of simulation in model building.**
11. **Explain design of simulation experiments.**
12. **What are the advantages and disadvantages of simulation techniques ?** *[BNU. BE. Nov 98]*
13. **Explain Monte–Carlo method of simulation with suitable example.** *[BNU, BE. Nov 98]*
14. **Explain simulation and give its applications to queueing theory.** *[MU. M.B.A. Apr 97]*
15. **At a toll office, a sample of 100 arrivals of vehicles gives the following frequency distribution of the inter arrival and service time.**

Inter arrival time (min)	Frequency %	Service time	Frequency
1.0	2	1.5	10
1.5	5	–	–
2.0	9	2	22
2.5	25	–	–
3.0	22	2.5	40
3.5	11	–	–
4.0	10	3.0	20
4.5	6	–	–
5.0	3	3.5	8
5.5	2	–	–

There is a clerk at the office. Simulate the process for 20 arrivals and estimate the average percentage vehicle waiting time and average percent idle time of the clerk.

[*Ans :* Average waiting time of Customer = 0.18min,
Average Idle time of the server = 0.88 min]

16. A special purpose drill bores holes having a mean diameter of 1 cm. The process is normally distributed. Simulate a sequence of 10 diameters if the standard deviation of the process is 0.002 cm.

17. The following data is observed in a tea serving counter. The arrival is for one minute interval.

Number of persons arriving :	0	1	2	3	4	5
Probability :	0.05	0.15	0.40	0.20	0.15	0.05

The service is taken as 2 person for one minute interval. Using the following random numbers simulate for 15 minute period. 09, 54, 94, 01, 80, 73, 20, 26,90,79,25, 48, 99, 25, 89. Calculate also the average number of persons waiting in the queue per minute.

18. A company manufactures around 200 mopeds. Depending upon the availability of raw materials and other conditions, the daily production has been varying from 196 mopeds to 204 mopeds, whose probability distribution is as given below :

Producion/day	: 196	197	198	199	200
Probability	: 0.05	0.09	0.12	0.14	0.20
Production/day	: 201	202	203	204	
Probability	: 0.15	0.11	0.08	0.06	

The finished mopeds are transported in a specially designed three storeyed lorry that can accommodate 200 mopeds. Simulate the process to find out

(i) What will be the average number of mopeds waiting in the factory

(ii) What will be the average number of empty spaces on the lorry.

19. Customers arrive at a milk booth for the required service. Assume that inter arrival and service times are constants and given by 1.8 and 4 time units respectively. Simulate the system by hand computations for 14 time units. *(i)* What is the waiting time per customer? *(ii)* What is the percentage idle time of the facility?
(Assume that the system starts at $t = 0$).

20. Arrivals at a service station have been found to follow poisson process. The mean arrival rate is four units per hour. Simulate three hours of arrivals of the station.
[Hint : Similar to the Example 7, Page 17.12]
[MU. M.B.A. Nov 97]

21. Arrivals at a service station have been found to follow a poisson process. The mean arrival rate is 5 units per hour. Simulate three hours of arrivals at the station.
[MU. M.B.A. Apr 96]

22. Arrivals at a service station has been found to follow poisson process. The mean arrival rate is 6 units per hour. Simulate three hours of arrivals at the station. *[MU. M.B.A. Nov 96]*

23. Explain the simulation method for queueing models.
[MU. M.B.A. Apr 97]

ANSWERS

15. Average waiting time vehicles = 0.18 minutes
Average waiting time of the clerk = 0.88 minutes

17. 0.71 persons/minute

18. Average no of mopeds waiting = 0.67 per day
Average number of empty spaces in the lorry = 0.93 per day

19. *(i)* 3.57 time units. *(ii)* 4.08 time units.
(iii) 0% since service facility is always busy.

23

Simulation

"If man begins with certainties, he ends in doubts; but if he contents to begin with doubts, he shall end in certainties"

23 : 1. INTRODUCTION

Simulation is a numerical technique for conducting experiments that involve certain types of mathematical and logical relationships necessary to describe the behaviour and structure of a complex real world system over extended period of time. In other words, it is a quantitative technique that utilises a computerised mathematical model in order to represent actual decision-making under conditions of uncertainty for evaluating alternative courses of action based upon facts and assumptions.

A definition of simulation as given by Shannon :

"Simulation is the process of designing a model of a real system and conducting experiments with this model for the purpose of understanding the behaviour (within the limits imposed by a criterion or set of criteria) for the operation of the system."

23 : 2. WHY SIMULATION ?

Using *simulation*, an analyst can introduce the constants and variables related to the problem, set-up the possible courses of action and establish criteria which act as measures of effectiveness. The major reasons for applying simulation technique to O.R. problems may be listed as below :

1. It is an appropriate tool to use in solving a problem when *experimenting* on the *real system* (*a*) would be disruptive, (*b*) would be too expensive, (*c*) does not permit replication events, (*d*) does not permit control over key variables.

2. It is a desirable tool for solving a business problem when a mathematical model (*a*) is too complex to solve, (*b*) is beyond the capacity of available personnel, (*c*) is not detailed enough to provide information on all important decision variables.

3. The major reasons for adopting simulation in place of other mathematical techniques are :

(*i*) It may be the only method available, because it is difficult to observe the actual reality.

(*ii*) Without appropriate assumption, it is impossible to develop a mathematical solution.

(*iii*) It may be too expensive to actually observe the system.

(*iv*) There may not be sufficient time to allow the system to operate for a very long time.

4. It provides a trial-and-error movement towards the optical solution. The decision-maker selects an alternative, experiences the effect of the selection, and then improves the selection. In this way, the selection is adjusted until it approximates the optical solution.

23 : 3. METHODOLOGY OF SIMULATION

The *methodology* developed for simulation process consists of the following seven steps :

Step 1. Identify and clearly define the problem.

Step 2. List the statement of objectives of the problem.

Step 3. Formulate the variables that influence the situation and an exact or probabilistic description of their possible values or states.

Step 4. Obtain a consistent set of values (or states) for the variables, *i.e.*, a sample of what could happen. In the case of deterministic variables, this is simple and in the case of probabilistic variables, random sampling technique may be used.

Step 5. Use the sample obtained in *step 2* to calculate the value of the decision criterion, by actually following the relationships among the variables for each of the alternative decisions.

Step 6. Repeat *steps 2* and *3* until a sufficient number of samples are available.

Step 7. Tabulate the various values of the decision criterion and choose the best policy.

23 : 4. SIMULATION MODELS

The simulation models can be classified into the following four categories :

1. *Simulation of deterministic models.* In the case of these models, the input and output variables are not permitted to be random variables and models are described by exact functional relationships.

2. *Simulation of probabilistic models.* In such cases, method of random sampling is used. The technique used for solving these models is termed as 'Monte-Carlo Technique'.

3. *Simulation of static models.* These models do not take variable time into consideration.

4. *Simulation of dynamic models.* These models deal with time-varying interaction.

23 : 5. EVENT-TYPE SIMULATION

The concept of *event-type simulation* is best explained by the following illustration :

Consider a situation where customers arrive at a one-man barber shop for hair cutting. The problem is to analyse the system in order to evaluate the quality of service and the economic feasibility of offering the service. To measure the quality of service one has to make the assessment of the average waiting time per customer and the percentage of time the barber remains idle.

For the construction of a model of this system one notices that changes pertinent to the analysis of the system can occur only if a customer arrives for service or departs after completion of service. If a customer arrives at barber's shop, he will have to wait, if the server (barber) is busy. On the other hand, a departure of customer, after being served, indicates that the server is available to serve the waiting customers, if any. Thus, we conclude that there occur two events, namely an arrival and a departure. It indicates that as the simulator progresses on the time scale, one should pay attention to the system only when an event occurs.

Let E_a denote the arrival event; E_d, the departure event and T, the simulated period (time span). Then, the simulator starts at time $t=0$ and progresses up to $t=t_1$, the $t=t_2$ and so on until entire simulated period T is covered.

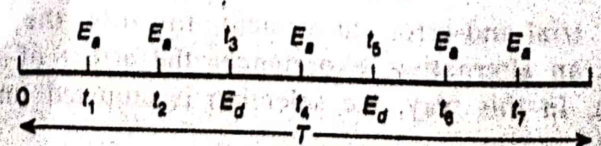

Fig. 23.1

Fig. 23.1 illustrates occurrences of E_a and E_d over T, where the simulation starts by generating E_a at t_1. Initially, as the facility is unoccupied, the customer starts service immediately. Then, the following two events must be generated :

(*a*) the next arrival may occur,

(*b*) the service of the customer may be completed.

The next arrival is determined from inter-arrival time. This determines E_a at t_2. Departure time of the customer in service is determined from service time and this generates E_d at t_3. Both E_d (at t_1) and E_d (at t_3) are now stored chronologically, so that the simulator recognizes that E_a occurs before E_d. The next event to be considered is E_a at t_2 and at this point E_a at t_1 is deleted from the stored list (because of past event).

The event E_a at t_2 generates E_a at t_4. Since the facility is busy, the arriving customer E_a (at t_2) joins a waiting line. Now, E_a at t_a is deleted from the list and E_d at t_3 is considered next. At this time a customer is taken from the waiting line and departure event E_d at t_5 is generated. The process is repeated until the entire simulated period T is covered.

SAMPLE PROBLEM

2301. *Customers arrive at a milk booth for the required service. Assume that inter-arrival and service times are constant and given by 1.8 and 4 time units, respectively. Simulate the system by hand computations for 14 time units. What is the average waiting time per customer? What is the percentage idle time of the facility? [Assume that the system starts at t = 0.]*

Solution. In the beginning, since the facility is free, first customer starts service. Its departure time is $t = 0 + 4 = 4$. Next event (arrival) occurs at $t = 0 + 1.8 = 1.8$, which is stored before E_d at $t = 4$. Now, since the facility is still busy, customer 2 is put in the queue and is first to be considered in this queue. A new arrival event (customer 3) occurs at $t = 1.8 + 1.8 = 3.6$ which precedes E_d at $t = 4$. Again, customer 3 is put in the queue and a new arrival event E_a (customer 4) occurs at $t = 3.6 + 1.8 = 5.4$. This event succeeds E_4 at $t = 4$. At this point, first customer departs which leaves the facility free. Customer 2, who was the first to join the queue, now gets service. The waiting time is computed as the time period from the instant he joined the queue until he commences service. The procedure is repeated until the simulated period is completed. The results of simulation are given in the following table :

Time	*Event*	*Customer*	*Waiting time*
0.0	E_a	1	
1.8	E_a	2	
3.6	E_a	3	
4.0	E_d	1	... 4 − 1.8 = 2.2 (customer 2)
5.4	E_a	4	
7.2	E_a	5	
8.0	E_d	2	... 8 − 3.6 = 4.4 (customer 3)
9.0	E_a	6	
10.8	E_a	7	
12.0	E_d	3	... 12 − 5.4 = 6.6 (customer 4)
13.6	E_a	8	
14.0	End	—	14 − 7.2 = 6.8 (customer 5) 14 − 9.0 = 5.0 (customer 6) 14 − 10.8 = 3.2 (customer 7) 14 − 13.6 = 0.4 (customer 8)

It is evident from this simulation that the average waiting time per customer is

$$(2.2 + 4.4 + 6.6 + 6.8 + 5.0 + 3.2 + 0.4)/8 = 3.57.$$

Average waiting time per customer for those who must wait is 28.6/7 (= 4.08) and percentage idle time of the facility = 0%.

PROBLEMS

2302. At a telephone booth, suppose that the customers arrive with an average time of 1.2 time units between one arrival and the next. Service times are assumed to be 2.8 time units. Simulate the system for 12 time units by assuming that the system starts at $t = 0$. What is the average waiting time per customer? *[Purvanchal M.C.A. 1996]*

2303. With the help of a single queueing model having inter-arrival and service time constantly 1.4 and 3 minutes respectively, explain discrete simulation technique taking 10 minutes as the simulation period. Find from this the average waiting time of a customer. (Assume that initially the system is empty and the first customer arrives at time $t = 0$.)

2304. Suppose there are two types of customers. Type *A* customers have priority for service so that those of type *B* cannot be serviced until all waiting type *A* customers have completed their service. The service times for type *A* and type *B* customers are 2 and 1, respectively. Further assume that the inter-arrival times for types *A* and *B* are 1.5 and 0.3 respectively. Simulate the system for 15 time units. Compute the average number of waiting customers for each type as well as the combined average. What is the average waiting time per each type customer? [Assume that the system starts at $t = 0$.]

23 : 6. GENERATION OF RANDOM NUMBERS

A simulation model need not be a deterministic one and may include some elements of uncertainty. For example, in the waiting line model of Sample Problem *2301*, inter-arrival and service times are usually probabilistic rather than deterministic. In inventory models, the variables include customer's demand and delivery times, which may also be probabilistic. The problem, in all such types of simulations, is based on the use of *random numbers*. These are the numbers which have equal probability of being generated. For example, when we are interested in one digit numbers 0, 1, 2, ..., 9, there are in all ten numbers and each of the numbers should have 1/10 probability of being generated.

There are several methods for the generation of random numbers, the most common among these are the "*Mid-square method*", "*Spinning arrow method*", "*Dice rolling method*" and the "*Spinning disc method*".

A convenient manual method for generation of random numbers may be summarised in the following seven steps :

Step 1. Collect the data related to the current problem.

Step 2. Construct a frequency distribution with these data.

Step 3. Construct the relative frequency distribution.

Step 4. Assign a coding system that relates the identified events to generated random numbers.

Step 5. Select a suitable method for obtaining the required random numbers.

Step 6. Match the random numbers to the assigned events and tabulate the results.

Step 7. Repeat Step *6* until the desired number of simulation runs has been generated.

Remarks 1. There are several published tables of random numbers (digits) which are very economical and convenient to use.

2. The numbers generated by the computer are always predictable and reproducible and hence cannot be treated as random. For this reason, they are sometimes given the name *pseudo-random numbers*. However, they satisfactorily play the role of random numbers in the simulation.

Illustration. *Kodak Photography Studios use an expensive grade of developing fluid when printing special colour portraits. Since the developing fluid cannot be stored for long periods, it is important to keep on hand only as much as is needed to fill anticipated demand. In the past few months, however, demand for the product has been fluctuating. The owner has decided to simulate the demand for this service.*

A study of Kodak Photography's appointment book resulted in the following frequency distribution :

Daily Demand :	*0*	*1*	2	3	4	5
Number of Days :	10	20	40	20	6	4

The data was taken for a 100-day period, during which no more than five special prints were requested on any given day. Using the data given above generate a ten-day sequence of demand values.

Solution.

Step 1. Collect the data relevant to the current problem.

Step 2. Using the data of *Step 1*, construct the frequency distribution.

Step 3. Construct the corresponding relative frequency distribution :

Daily demand	*Relative frequency*	*Probability*
0	10/100	0.10
1	20/100	0.20
2	40/100	0.40
3	20/100	0.20
4	6/100	0.06
5	4/100	0.04

Step 4. Given the relative frequency distribution of *Step 3*, assign a coding system that relates the identified events to generated random numbers. This coding system is the device whereby, given a random number, a particular event will be specified (in this case, a specific demand for special portraits).

The most practical coding system is one which assigns random numbers in proportion to the probability value. In this case, 10 per cent of the random numbers will be assigned to a daily demand of zero, 20 per cent to a daily demand of one, and 40 per cent to a daily demand of two. In a similar manner, random numbers will be assigned to the remaining daily demand figures. The results are summarized in Table 23.1 :

TABLE 23.1 : RANDOM NUMBER ASSIGNMENT

Daily demand	*Probability*	*Random number assignments*	*Numbers assigned*
0	0.10	00—09	10
1	0.20	10—29	20
2	0.40	30—69	40
3	0.20	70—89	20
4	0.06	90—95	6
5	0.04	96—99	4

Step 5. Select an appropriate method for generating the required random numbers. For the limited purposes of this example it is feasible to use a manual method such as the spinning arrow. To use this approach, we divide the face of a clock into 100 equal parts and number the parts from 00 to 99, inclusive, centre an arrow on the clock in such a way that it can spin freely. At each spin of the arrow, record the number to which it points when it stops. This is the generated random number. Each spin thus corresponds to one simulation run (or, as in the example, one day).

Step 6. Using the method selected in *Step 5*, generate the random numbers to be used in the simulation, match these to the assigned events, and summarise the results in an appropriate table. Since the owner wants a 10-day simulation, it is necessary to spin the arrow 10 times and record the random number generated each time. Once the random numbers have been generated, reference to the coded assignment of Table 23.2 gives the value of the generated demand. For example, if 35 is the first number generated, it falls in the range 30—69 and thus corresponds to the event of a daily demand for two portraits. Results for the full ten-day simulation are summarised below :

TABLE 23.2 : RANDOMLY GENERATED DATA

Day number	*Generated random demand*	*Generated demand*
1	35	2
2	92	4
3	68	2
4	03	0
5	51	2
6	05	0
7	72	3
8	84	3
9	98	5
10	34	2

Step 7. Repeat *Step 6* until the required number of simulation runs has been generated. Since the owner was only interested in one simulation run, this step is not required. *However, if there were to be more than one 10-day simulations, it would be necessary to repeat Step 6 as needed.*

Remarks. The demand data, that has been generated, can be used to estimate the expected daily demand. For the current example, total generated demand = 23 units, total number of days in the simulation = 10. Therefore the average daily demand is 2.3 units :

$$\text{Average daily demand} = \frac{\text{Total demand}}{\text{Number of days simulated}} = \frac{23}{10} = 2.3.$$

On the basis of this simulation, Kodak Studio can expect an average of 2.3 requests per day for the special colour portraits.

23 : 7. MONTE-CARLO SIMULATION

The *Monte-Carlo method* is a simulation technique in which statistical distribution functions are created by using a series of random numbers. This approach has the ability to develop many months or years of data in a matter of a few minutes on a digital computer. The method is generally used to solve problems which cannot be adequately represented by the mathematical models, or, where solution of the model is not possible by analytical method.

Monte-Carlo simulation yields a solution which should be very close to the optimal, but not necessarily the exact solution. However, it should be noted that this technique yields a solution that converges to the optimal or correct solution as the number of simulated trials lead to infinity.

The Monte-Carlo simulation procedure can be summarized in the following steps :

Step 1. Define the problem :

(*a*) Identify the objectives of the problem, and (*b*) Identify the main factors which have the greatest effect on the objectives of the problem.

Step 2. Construct an appropriate model :

(*a*) Specify the variables and parameters of the model.

(*b*) Formulate the appropriate decision rules, *i.e.*, state the conditions under which the experiment is to the performed.

(*c*) Identify the type of distribution that will be used—Models use either theoretical distributions or empirical distributions to state the patterns the occurrence associated with the variables.

(*d*) Specify the manner in which time will change.

(*e*) Define the relationship between the variables and parameters.

Step 3. Prepare the model for experimentation :

(*a*) Define the starting conditions for the simulation, and (*b*) Specify the number of runs of simulation to be made.

Step 4. Using Steps 1 to 3, experiment with the model :

(*a*) Define a coding system that will correlate the factors defined in *Step 1* with the random numbers to be generated for the simulation.

(*b*) Select a random number generator and create the random numbers to be used in the simulation.

(*c*) Associate the generated random numbers with the factors identified in *Step 1* and coded in *Step 4 (a)*.

Step 5. Summarize and examine the results obtained in *Step 4.*

Step 6. Evaluate the results of the simulation.

Step 7. Formulate proposals for advice to management on the course of action to be adopted and modify the model, if necessary.

SAMPLE PROBLEM

2305. *The occurrence of rain in a city on a day is dependent upon whether or not it rained on the previous day. If it rained on the previous day, the rain distribution is :*

Event	:	*No rain*	*1 cm. rain*	*2 cm. rain*	*3 cm. rain*	*4 cm. rain*	*5 cm. rain*
Probability	:	0.50	0.25	0.15	0.05	0.03	0.02

If it did not rain on the previous day, the rain distribution is :

Event	:	*No rain*	*1 cm. rain*	*2 cm. rain*	*3 cm. rain*
Probabiiity	:	0.75	0.15	0.06	0.04

Simulate the city's weather for 10 days and determine by simulation the total days without rain as well as the total rainfall during the period. Use the following random numbers :

67 63 39 55 29 78 70 06 78 76

for simulation. Assume that for the first day of the simulation it had not rained the day before.
[C.A. (Nov.) 1993]

Solution. We simulate the city's weather with and without rainfall in the following steps :

Step 1. Previous day *rain* distribution :

Event	*Probability*	*Cummulative probability*	*RN range*
No Rain	0.50	0.50	00—49
1 cm. rain	0.25	0.75	50—74
2 cm. rain	0.15	0.90	75—89
3 cm. rain	0.05	0.95	90—94
4 cm. rain	0.03	0.98	95—97
5 cm. rain	0.02	1.00	98—99

Step 2. Previous day *no rain* distribution :

Event	*Probability*	*Cummulative probability*	*RN range*
No Rain	0.75	0.75	00—74
1 cm. rain	0.15	0.90	75—89
2 cm. rain	0.06	0.96	90—95
3 cm. rain	0.04	1.00	96—99

Step 3. Simulate for 10 days using the given random numbers :

Day	*RN*	*Events*	*Cumulative rain*
1	67	No Rain	
2	63	No Rain	
3	39	No Rain	
4	55	No Rain	
5	29	No Rain	
6	78	1 cm. Rain	1 cm.
7	70	1 cm. Rain	2 cm.
8	06	No Rain	2 cm.
9	78	1 cm. Rain	3 cm.
10	76	2 cm. Rain	5 cm.

During the simulated period it did not rain on 6 out of 10 days. The total rainfall during the period is 5 cm.

23 : 8. SIMULATION OF INVENTORY PROBLEMS

Many of the inventory problems, especially storage problems, cannot be solved analytically because of the complex nature of the distribution followed by demand or supply. It is, however, possible to get the solution by using simulation techniques. The basic approach would be to determine the probability distribution of the input and output functions from the past data; and run the inventory system artificially by generating the future observations on the assumption of the same distributions. Subsequently, the decision-making regarding the optimization problems would be made by the *trial-and-error* method.

The artificial samples for future can be generated with the help of random numbers.

In inventory control, the reorder point is to be chosen with consideration for the demand during lead time to provide adequate service to customers. If both, the lead time and demand of inventory per unit of time, are random variables, then simulation technique can be used to investigate the effect of different inventory policies.

We illustrate the use of simulation in the study of inventory problems through some sample problems :

SAMPLE PROBLEMS

2306. *The automobile company manufactures around 150 scooters. The daily production varies from 146 to 154 depending upon the availability of raw materials and other working conditions :*

Production (per day) :	146	147	148	149	150	151	152	153	154
Probability :	0.04	0.09	0.12	0.14	0.11	0.10	0.20	0.12	0.08

The finished scooters are transported in a specially arranged lorry accommodating 150 scooters. Using following random numbers :

80, 81, 76, 75, 64, 43, 18, 26, 10, 12, 65, 68, 69, 61, 57.

Simulate the process to find out :

(i) What will be the average number of scooters waiting in the factory?

(ii) What will be the average number of empty space on the lorry?

[Delhi B.Sc. (Stat.) 2002; C.A. (May) 1998]

Solution. The random numbers are established as in table below :

TABLE 23.3 : RANDOM NUMBER CODING

Production per day	*Probability*	*Cumulative probability*	*Random number assigned*
146	0.04	0.04	00—03
147	0.09	0.13	04—12
148	0.12	0.25	13—24
149	0.14	0.39	25—38
150	0.11	0.50	39—49
151	0.10	0.60	50—59
152	0.20	0.80	60—79
153	0.12	0.92	80—91
154	0.08	1.00	92—99

Based on the 15 random numbers given, we simulate the production per day in the table below :

TABLE 23.4 : SIMULATION SHEET

S.No.	*Random number*	*Production per day*	*No. of scooters waiting*	*No. of empty spaces in the lorry*
1	80	153	3	
2	81	153	3	
3	76	152	2	
4	75	152	2	
5	64	152	2	
6	43	150	0	0
7	18	148		2
8	26	149		1
9	10	147		3
10	12	147		3
11	65	152	2	
12	68	152	2	
13	69	152	2	
14	61	152	2	
15	57	151	1	
Total			21	9

(*i*) Average number of scooters waiting = 21/15 = 1.4 per day
(*ii*) Average number of empty spaces = 9/15 = 0.6 per day.

2307. *Consider the setting up a 'Q' system of inventory control for a vital spare part. Though a demand pattern has not been established, yet the quantities demanded during the past 50 weeks are known. Similarly, the suppliers' delivery lead-time has been found to vary between 1 and 4 weeks with no established pattern. We know that the carrying cost is 30% per annum and the ordering cost is Rs. 60 per order. The stock-out cost in this case is around Rs. 75 per unit per week while the inventory carrying cost works out to Rs. 15 per unit per week. Simulate the demand for 20 weeks and obtain an optimal solution.*

Solution. Using the given information we obtain a table of frequency distribution and the corresponding percentage relative frequencies for both, the demand as well lead-time data. The cumulative frequencies are then computed and the random numbers assigned for both. This has been displayed in Table 23.5.

Let us now assume that an ordering lot size of 20 units at a reorder point of 7 units in stock would serve our purpose as a solution. By simulating the conditions of lead-time and demand, we will have to check the suitability of this solution.

From Table 23.5 we notice that the cumulative demand level frequencies of the demand are 4, 14, 44, 84, 94 and 100 for demand levels 0, 1, 2, 3, 4 and 5 respectively. If we consider 2-digit random numbers, we assign 00–03 to cumulative frequency 4, 04–13 to cumulative frequency 14, and so on. Thus, if the random number chosen is 02, then as it lies between 00 and 03, the demand is assumed to be 0. Similarly, if the next random number chosen is 63, the demand is 3 as it lies between 44 and 83.

TABLE 23.5 : DEMAND DATA

Quantities demanded (units/week)	0	1	12	3	4	5
Frequency of demand (number of weeks)	2	5	15	20	5	3
Relative frequency (%)	4	10	30	40	10	6
Cumulative frequency	4	14	44	84	94	100
Assigned random numbers	00—03	04—13	14—43	44—83	84—93	94—99

LEAD-TIME DATA

Lead-time (weeks)	1	2	3	4
Frequency of occurrence (number of times)	10	6	3	1
Relative frequency (%)	50	30	15	5
Cumulative frequency	50	80	95	100
Assigned random numbers	00—49	50—79	80—94	95—99

Let us begin with an initial stock of 13 units our strategy being to order 20 units whenever the stock on hand falls to 7 units. We shall determine the cost involved in adopting this policy by simulating the demand pattern and lead-time distribution in the next 20 weeks. In Table 23.6 (page 500), we see that for week *1*, the random number is 10, and so the demand is 1. The balance stock becomes 12. For week *2*, the random number is 91 and hence the demand is 4 thus bringing down the balance to 8. A demand of 3 in week *3*, brings down the balance to 5. Since it is less than 7, we place an order for 20 units and incur an ordering cost of Rs. 60. Moreover, we have been incurring the carrying cost for the quantities held at the end of the week. These have also been tabulated. We now choose a random number for the delivery lead-time and as it is 90, the delivery is available only after 3 weeks in the sixth week. In week *4*, the demand is for 4 units and in week *5* the demand is for 3 units which leads to a stock out of 2 units and a corresponding cost of Rs. 150. Therefore, when the delivery is obtained in week 6, this will have to be met. We proceed in this manner till week *20* and find that the total cost is Rs. 3,795. We now adopt a different strategy and repeat the process. Table 23.6 displays the simulation runs for two more strategies. One has to carry on this process and obtain the optimum solution :

TABLE 23.6 : WORKSHEET OF SIMULATION RUNS

Week No.	Demand simulation: Random number	Demand simulation: Demand (units)	Strategy One (R.O.L. = 7 units, Order quantity = 20 units): Receipt (units)	Balance (units)	Carrying cost (Rs.)	Order cost (Rs.)	Stock-out cost (Rs.)	Strategy Two (R.O.L. = 8 units, Order quantity = 15 units): Receipt (units)	Balance (units)	Carrying cost (Rs.)	Order cost (Rs.)	Strategy Three (R.O.L. = 8 units, Order quantity = 12 units): Receipt (units)	Balance (units)	Carrying cost (Rs.)	Order cost (Rs.)
0	—	—	—	13	—	—	—	—	13	—	—	—	13	—	—
1	9	1	—	12	180	—	—	—	12	180	—	—	12	180	—
2	92	4	—	8	120	—	—	—	8* (88)	120	60	—	8* (81)	120	60
3	70	3	—	5* (90)	75	60	—	—	5	75	—	—	5	75	—
4	88	4	1	—	15	—	—	—	1	15	—	—	1	15	—
5	45	3	—	−2	—	—	150	15	13	195	—	12	10	150	—
6	20	2	20	16	240	—	—	—	11	165	—	—	8* (63)	120	60
7	98	5	—	11	165	—	—	—	6* (52)	90	60	—	3	45	—
8	60	3	—	8	120	—	—	—	3	45	—	12	12	180	—
9	99	5	—	3* (77)	45	60	—	15	13	195	—	—	7* (20)	105	60
10	95	5	—	−2	—	—	150	—	8* (17)	120	60	12	14	210	—
11	48	3	20	15	225	—	—	15	20	300	—	—	11	165	—
12	16	2	—	13	195	—	—	—	18	270	—	—	9	135	—
13	82	3	—	10	150	—	—	—	15	225	—	—	6* (89)	90	60
14	65	3	—	7* (19)	105	60	—	—	12	180	—	—	3	45	—
15	74	3	20	24	360	—	—	—	9	135	—	—	0	—	—
16	84	4	—	20	300	—	—	—	5* (93)	75	60	12	8* (53)	120	60
17	01	0	—	20	300	—	—	—	5	75	—	—	8	120	—
18	07	1	—	19	285	—	—	—	4	60	—	12	19	285	—
19	72	3	—	16	240	—	—	15	16	240	—	—	16	240	—
20	83	3	—	13	195	—	—	—	13	195	—	—	13	195	—
			Totals :		3,315	180	300	Totals :		2,955	240	Totals :		2,595	300
				Total cost = Rs. 3,795					Total cost = Rs. 3,195				Total cost = Rs. 2,895		

The * in the above table indicates that the reorder point has been reached. The 2-digit number (within brackets) is the random number for obtaining lead-time. The entry in the receipt column, corresponds to this random number. There are no stock-out costs in strategies 2 and 3.

PROBLEMS

2308. Bright Bakery keeps stock of a popular brand of cake. Previous experience indicates the daily demand as given here :

Daily demand	:	0	10	20	30	40	50
Probability	:	.01	.20	.15	.50	.12	.02

Consider the following sequence of random numbers :

48, 78, 19, 51, 56, 77, 15, 14, 68, 09

Using this sequence, simulate the demand for the next 10 days. Find out the stock situation if the owner of the bakery decides to make 30 cakes every day. Also estimate the daily average demand for the cakes on the basis of simulated data. *[C.A. (Nov.) 1999]*

2309. A confectioner sells confectionery items. Past data of demand per week (in hundred kilograms) with frequency is given below :

Demand/week	:	0	5	10	15	20	25
Frequency	:	2	11	8	21	5	3

Using the following sequence of random numbers, generate the demand for the next 10 weeks. Also find the average demand per week :

35 52 90 13 23 73 34 57 35 83 94 56 67 66 60

2310. The manager of a book store has to decide the number of copies of a particular tax law book to order. A book costs Rs. 60 and is sold for Rs. 80. Since some of the tax laws change year after year, any copies unsold while the edition is current must be sold for Rs. 30. From past records, the distribution of demand for this book has been obtained as follows :

Demand (No. of copies)	:	15	16	17	18	19	20	21	22
Proportion	:	0.05	0.08	0.20	0.45	0.10	0.07	0.03	0.02

Using the following sequence of random numbers, generate data on demand for 20 time periods (years). Calculate the average profit obtainable under each of the courses of action open to the manager. What is the optimal policy?

14, 02, 93, 99, 18, 71, 37, 30, 12, 10,
88, 13, 00, 57, 69, 32, 18, 08, 92, 73.

2311. The management of *ABC* Company is considering the question of marketing a new product. The fixed cost required in the project is Rs. 4,000. Three factors are uncertain *viz.*, the selling price, variable cost and the annual sales volume. The product has a life of only one year. The management has the data on these three factors as under :

Selling price (Rs.)	*Probability*	*Variable cost (Rs.)*	*Probability*	*Sales volume (units)*	*Probability*
3	0.2	1	0.3	2,000	0.3
4	0.5	2	0.6	3,000	0.3
5	0.3	3	0.1	5,000	0.4

Consider the following sequence of thirty random numbers :

81, 32, 60, 04, 46, 31, 67, 25, 24, 10, 40, 02, 39, 68, 08,
59, 66, 90, 12, 64, 79, 31, 86, 68, 82, 89, 25, 11, 98, 16.

Using the sequence (First 3 random numbers for the first trial, etc.), simulate the average profit for the above project on the basis of 10 trials. *[C.A. (Nov.) 1994]*

2312. Suppose that the demand for a particular item is normally distributed with a mean of 200 units per week and a standard deviation of 50 units per week and the lead time is distributed exponentially with a mean of one week. The cost of making one order is Rs. 90 and the cost of holding one unit is 10 paise per week. Using Monte-Carlo simulation determine the *EOQ* and order point that results in minimum total system cost. *[Madras B.E. (Mech.) 1999]*

2313. The director of finance for a farm co-operative is concerned about the yields per acre she can expect from this year's corn crop. The probability distribution of the yields for the current weather conditions is given below :

Yield in kg. per acre	*Probability*
120	0.18
140	0.26
160	0.44
180	0.12

She would like to see a simulation of the yields she might expect 10 years for weather conditions similar to those she is now experiencing.

(*i*) Simulate the average yield she might expect per acre using the following random numbers :

20, 72, 34, 54, 30, 22, 48, 74, 76, 02.

She is also interested in the effect of market price fluctuations on the cooperative's farm revenue. She makes this estimate of per kg. prices for corn :

Price per kg. (Rs.)	*Probability*
2.00	0.05
2.10	0.15
2.20	0.30
2.30	0.25
2.40	0.15
2.50	0.10

(*ii*) Simulate the price she might expect to observe over the next 10 years using the following random numbers :

82, 95, 18, 96, 20, 84, 56, 11, 52, 03.

(*iii*) Assuming that prices are independent of yields, combine these two into the revenue per acre and also find out the average revenue per acre she might expect every year. ***[C.A. (May) 1991]***

2314 A book-store wishes to carry Systems Analysis and Design in stock. Demand is probabilistic and replenishment of stock takes 2 days (*i.e.*, if an order is placed on March 1, it will be delivered at the end of day on March 3). The probabilities of demand are given below :

Demand (daily)	:	*0*	*1*	*2*	*3*	*4*
Probability	:	0.05	0.10	0.30	0.45	0.10

Each time an order is placed, the store incurs an ordering cost of Rs. 10 per order. The store also incurs a carrying cost of Rs. 0.50 per book per day. The inventory carrying cost is calculated on the basis of stock at the end of each day. The manager of the book-store wishes to compare two options for his inventory decision :

A. Order 5 books, when the inventory at the beginning of the day plus orders outstanding is less than 8 books.

B. Order 8 books, when the inventory at the beginning of the day plus orders outstanding is less than 8 books.

Currently (beginning of the *1st day*) the store has stock of 8 books plus 6 books ordered 2 days ago and expected to arrive next day. Using Mante-Carlo simulation for 10 cycles, recommend which option the manager should choose?

The two-digit random numbers are given below :

89, 34, 78, 63, 61, 81, 39, 16, 13, 73.

[C.A. (Final) May 2000]

2315. A company has been having problems with stockouts for one of its components and is contemplating making alterations either to the reorder quantity or to the reorder level, or to both.

Before making any decision, the company wishes to explore whether any guidance can be obtained by simulating the operation of the system.

The pattern of weekly demand over the past few years has been as follows :

Weekly demand (units)	:	500	525	550	575	600	625	650	675	700	725
Frequency	:	10	15	30	50	55	60	40	20	10	5

Ordering costs are Rs. 20 per order and the carrying cost is Rs. 5 per unit. The estimated loss when an order cannot be met is Rs. 12 per unit. When stock reaches the pre-set order point a replenishment order is issued.

(*a*) You are required to describe, using a flow chart or other means, how a simulation model for this problem might work.

Using a re-order point of 2,500 and an order quantity of 2,000, a simulation of 20 weeks' operations has been run on a computer and the following summary produced :

Simulation summary

Number of Periods	20 weeks	Carrying Cost	Rs. 1,984.13
Average Inventory	1,031.8 units	Ordering Cost	Rs. 120.00
Orders	6	Stockout Cost	Rs. 20,580.00
Average Demand	606.3 units	Total Cost	Rs. 22,684.13
Lead Time	4 weeks		

(*b*) You are required to interpret the above summary and suggest what could be done next to make the simulation more realistic. *[CMA (May) 1992]*

2316. A retailer deals in a perishable commodity. The daily demand and supply are variables. The data for the past 500 days, show the following demand and supply :

Supply		*Demand*	
Availability (kg.)	*No. of days*	*Demand (kg.)*	*No. of days*
10	40	10	50
20	50	20	110
30	190	30	200
40	150	40	100
50	70	50	40

The retailer buys the commodity at Rs. 20 per kg. and sells it at Rs. 30 per kg. Any commodity remains at the end of the day, has no saleable value. Moreover, the loss (unearned profit) on any unsatisfied demand is Rs. 8 per kg. Given the following pair of random numbers, simulate 6 days sales, demand and profit.

(31, 18); (63, 84); (15, 79); (07, 32); (43, 75); (81, 27)

The first random number in the pair is for supply and the second random number is for demand, viz., in the first pair (31, 18), use 31 to simulate supply and 18 to simulate demand. *[C.A. (Nov.) 2000]*

23 : 9. SIMULATION OF A QUEUEING SYSTEM

Queueing theory provides techniques for determining measures of effectiveness, such as queue length, average waiting time, etc., when the distribution of inter-arrival times and service times are known. If costs be assigned to waiting time of customers and idle time of the service facility, the problem of establishing a proper balance between these costs can be determined.

However, many queueing problems cannot be solved explicitly by analytical methods. In such cases, the only possible method of solution is to simulate the experiment. We illustrate the use of simulation in the study of queues through some sample problems :

SAMPLE PROBLEMS

2317. *A company has a single service station which has the following characteristics : The mean arrival rate of customers and the mean service time are 6.2 minutes and 5.5 minutes respectively. The time between an arrival and its services varies from one minute to seven minutes. The arrival and service time distributions are given below :*

Time (minutes)	*Arrival (probability)*	*Service (probability)*
1—2	0.05	0.10
2—3	0.20	0.20
3—4	0.35	0.40
4—5	0.25	0.20
5—6	0.10	0.10
6—7	0.05	

The queueing process starts at 11 A.M. and closes at 12 P.M. An arrival moves immediately into the service facility if it is empty. On the other hand, if the service station is busy, the arrival will wait in the queue. Customers are served on the first come, first served basis.

If the clerk's wages are Rs. 6 per hour and the customer's waiting line costs Rs. 5 per hour, would it be economical for the manager to engage the second clerk? Use Monte-Carlo simulations technique. *[Madras M.C.A. 1999]*

Solution. From the given frequency distribution of arrivals and service times, the probabilities and cumulative probabilities are first worked out as shown in Table 23.7 and Fig. 23.2. These, then, become the basis for generating arrival and service times in conjunction with a table of random numbers :

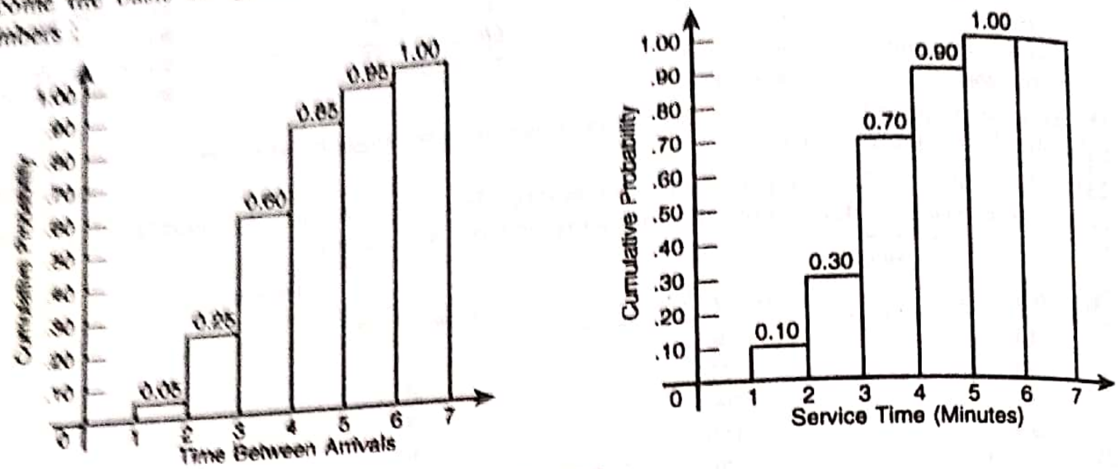

Fig. 23.2

TABLE 23.7 : CUMULATIVE PROBABILITIES

Time between arrivals (minutes)	*Cumulative probability*	*Service time (minutes)*	*Cumulative probability*
1—2	0.05	1—2	0.10
2—3	0.25	2—3	0.30
3—4	0.60	3—4	0.70
4—5	0.85	4—5	0.90
5—6	0.95	5—6	1.00
6—7	1.00	6—7	1.00

As we have to use the random number table, first of all we allot the random numbers to various intervals as shown in the table below :

TABLE 23.8 : RANDOM NUMBERS CODING

Inter-arrival time (minutes)	*Probability*	*RN allotted*	*Service time (minutes)*	*Probability*	*RN allotted*
1—2	0.05	00—04	1—2	0.10	00—09
2—3	0.20	05—24	2—3	0.20	10—29
3—4	0.35	25—59	3—4	0.40	30—69
4—5	0.10	60—84	4—5	0.20	70—89
5—6	0.10	85—94	5—6	0.10	90—99
6—7	0.05	95—99	6—7	0.00	—

A *simulation work sheet* is then developed in the following manner :

The random numbers developed above are related to the cumulative probability distributions of arrival and service times. The first random number of arrival time is 64. This number lies between 60 and 84 and indicates a simulated arrival time of 4 minutes. All simulated arrival and service times have been worked out in a similar fashion.

After generating the arrival and service times from a table of random numbers, the next step is to list the arrival time in the appropriate column. The first arrival comes in 4 minutes after the starting time. This means that the clerk waited for 4 minutes initially. It has been shown under the column—waiting time : clerk. The simulated service time for the first arrival is 3 minutes which results in the service being completed by 11.07 A.M. The next arrival comes at 11.08 A.M. which indicates that no one has waited in the queue.

The second arrival comes at 11.05 A.M. But the services will begin only at 11.07 A.M., since the service of first arrival ends at 11.07 A.M. This means that the second arrival has waited for 2 minutes before the start of its service. One customer waiting in the queue is shown in the last column of the simulation table. The procedure is followed throughout the preparation of the simulation work-sheet (See Table 23.9) :

TABLE 23.9 : SIMULATION WORKSHEET

RN	*Inter-arrival time*	*Arrival time (A.M.)*	*Service begins (A.M.)*	*RN*	*Service*		*Waiting time*		
					Time (min.)	*Ends (A.M.)*	*Clerk*	*Customer*	*Line length*
64	4	11.04	11.04	30	3	11.07	4	—	—
04	1	11.05	11.07	75	4	11.11	—	2	1
02	1	11.06	11.11	38	3	11.14	—	5	1
70	4	11.10	11.14	24	2	11.16	—	4	1
03	1	11.11	11.16	57	3	11.19	—	5	1
60	4	11.15	11.19	09	1	11.20	—	4	1
16	2	11.17	11.20	12	2	11.22	—	3	1
08	2	11.19	11.22	18	2	11.24	—	3	1
36	3	11.22	11.24	65	3	11.27	—	2	1
38	3	11.25	11.27	25	2	11.29	—	2	1
07	2	11.27	11.29	11	2	11.31	—	2	1
08	2	11.29	11.31	79	4	11.35	—	2	1
59	3	11.32	11.35	61	3	11.38	—	3	1
53	3	11.35	11.38	77	4	11.42	—	3	1
03	1	11.36	11.42	10	2	11.44	—	6	1
62	4	11.40	11.44	16	2	11.45	—	4	1
36	3	11.43	11.46	55	3	11.49	—	3	1
27	3	11.46	11.49	52	3	11.52	—	3	1
97	6	11.52	11.52	59	3	11.55	—	—	—
86	5	11.57	11.57	63	3	12.00	2	—	—
Total	57				54		6	56	17

The following information can be obtained from the simulation worksheet based on the period of one hour only :

(*a*) Average queue length

$$= \frac{\text{Number of customers in the waiting line}}{\text{Number}} = \frac{17}{20} = 0.85.$$

(*b*) Average waiting time of customer before service

$$= \frac{\text{Customer waiting time}}{\text{Number of arrivals}} = \frac{56}{20} = 2.80 \text{ minutes.}$$

(*c*) Average service time

$$= \frac{\text{Total service time}}{\text{Number of arrivals}} = \frac{54}{20} = 2.70 \text{ minutes.}$$

(*d*) Time a customer spends in the system

= Average service time + Average waiting time before service

= 2.70 + 2.80 = 5.50 minutes

Simulation worksheet developed in this problem also states that if one or more clerk is added, there is no need for a customer to wait in the queue. But before effecting any decision, the cost of having an additional clerk has to be compared with the cost due to customer waiting time. This can be worked out as follows :

One hour period	*Cost with one clerk*	*Cost with two clerks*
Customer waiting time (56 minutes × Rs. 5 per hour)	Rs. 4.50	Nil
Clerk's cost	Rs. 6.00	Rs. 12
Total cost of one hour period	Rs. 10.50	Rs. 12

If the above analysis based on simulation for a period of one hour only is representative of the actual situation, then it may be concluded that the cost with one clerk is lower than what it is with two clerks. Hence, it would not be an economical proposition to engage an additional clerk.

23.18. *Dr. Strong is a dentist who schedules all her patients for 30 minutes appointments. Some of the patients take more or less than 30 minutes depending on the type of dental work to be done. The following summary shows the various categories of work, their probabilities and the time actually to complete the work :*

Category	*Time required (minutes)*	*Probability of category*
Filling	45	0.40
Crown	60	0.15
Cleaning	15	0.15
Extraction	45	0.10
Check-up	15	0.20

Simulate the dentist's clinic for four hours and determine the average waiting time for the patients as well as the idleness of the doctor. Assume that all the patients show up at the clinic at exactly their scheduled arrival time starting at 8.00 A.M. Use the following random numbers in handling the above problem :

40 82 11 34 25 66 17 79

[C.A. (Nov.) 1990]

Solution. Allotment of random numbers for category of work for the given probabilities is shown below :

Category	*Probability*	*Cumulative probability*	*Random number*
Filling	0.40	0.40	00—39
Crown	0.15	0.55	40—54
Cleaning	0.15	0.70	55—69
Extraction	0.10	0.80	70—79
Check-up	0.20	1.00	80—99

TABLE 23.10 : FUTURE EVENTS

Patient No.	*Scheduled arrival*	*Random number*	*Category*	*Service time*
1	8.00	40	Crown	60 mts.
2	8.30	82	Check-up	15 "
3	9.00	11	Filling	45 "
4	9.30	34	Filling	45 "
5	10.00	25	Filling	45 "
6	10.30	66	Cleaning	15 "
7	11.00	17	Filling	45 "
8	11.30	79	Extraction	45 "

TABLE 23.11 : COMPUTATION OF ARRIVALS, DEPARTURES AND WAITING OF PATIENTS

Time	*Event (patient No.)*	*Status patient No. (time to go)*	*Waiting (patient No.)*
8.00	*1* arrives	*1* (60)	—
8.30	*2* arrives	*1* (30)	*2*
9.00	*1* departs; *3* arrives	*2* (15)	*3*
9.15	*2* departs	*3* (45)	—
9.30	*4* arrives	*3* (30)	*4*
10.00	*3* departs; *5* arrives	*4* (45)	*5*
10.30	*6* arrives	*4* (15)	*5, 6*
10.45	*4* departs	*5* (45)	*6*
11.00	*7* arrives	*5* (30)	*6, 7*
11.30	*5* departs; *8* arrives	*6* (15)	*7, 8*
11.45	*6* departs	*7* (45)	*8*
12.00	End	*7* (30)	*8*

The dentist was not idle during the entire simulated period. The waiting times for the patients were as follows :

Patient No.	*Arrival*	*Service starts*	*Waiting (minutes)*
1	8.00	8.00	0
2	8.30	9.00	30
3	9.00	9.15	15
4	9.30	10.00	30
5	10.00	10.45	45
6	10.30	11.30	60
7	11.00	11.45	45
8	11.30	12.30	60
		Total	285

The average waiting time was 285/8 = 35.625 minutes.

2319. ***Observations of past data show the following patterns in respect of inter-arrival duration and service duration in a single channel queueing system. Using the random number table below, simulate the queue behaviour for a period of 60 minutes and estimate the probability of the service being idle and then mean time spent by a customer waiting for service.***

Inter-arrival time		*Service time*	
Minutes	*Probability*	*Minutes*	*Probability*
2	0.15	1	0.10
4	0.23	3	0.22
6	0.35	5	0.35
8	0.17	7	0.23
10	0.10	9	0.10

Random Numbers (Start at NW Corner and proceed along the row)

9371	1463	7214	1053	2164
8142	8707	9054	3866	1053
2924	1725	1185	6885	9980
5119	4086	3083	5217	7105

Solution. The cumulative probability distributions and random number interval for inter-arrival time and service time are shown in Table 23.12.

TABLE 23.12 : RANDOM NUMBERS CODING

Arrival time		*Cumulative probability*	*RN interval*	*Service time*		*Cumulative probability*	*RN interval*
Minutes	*Probability*			*Minutes*	*Probability*		
2	0.15	0.15	00—14	1	0.10	0.10	00—09
4	0.23	0.38	15—37	3	0.25	0.35	10—31
6	0.35	0.73	38—72	5	0.32	0.67	32—66
8	0.17	0.90	73—89	7	0.23	0.90	67—89
10	0.10	1.00	90—99	9	0.01	1.00	90—99

The simulation worksheet developed for the given problem is shown in Table 23.13.

TABLE 23.13 : SIMULATION WORKSHEET

Random number (1)	*Inter-arrival time (min.)*	*Arrival time (min.)*	*Service starts (min.)*	*Random number (2)*	*Service time (min.)*	*Service ends (min.)*	*Waiting time: Attendant (min.)*	*Waiting time: Customer (min.)*	*Line length*
93	10	9.10	9.10	71	7	9.17	10	—	—
14	2	9.12	9.17	63	5	9.22	—	5	1
72	6	9.18	9.22	14	3	9.25	—	4	1
10	2	9.20	9.25	53	5	9.30	—	5	1
21	4	9.24	9.30	64	5	9.35	—	6	1
81	8	9.32	9.35	42	5	9.40	—	3	1
87	8	9.40	9.40	07	1	9.41	—	—	—
90	10	9.50	9.50	54	5	9.55	9	—	—
38	6	9.56	9.56	66	5	10.01	1	—	—
Total	56				41		20	23	5

(*i*) Average queue length = $5/9 = 0.56 \approx 1$ customer (approx.)
(*ii*) Average waiting time of customer before service = $23/9 = 2.56$ minutes.
(*iii*) Average service idle time = $20/9 = 2.22$ minutes.
(*iv*) Average service time = $41/9 = 4.56$ minutes
(*v*) Time a customer spends in the system = $(4.56 + 2.56) = 7.12$ minutes.
(*vi*) Percentage of service idle time = $20/(20 + 41) = 0.33$.

PROBLEMS

2320. The following table gives the arrival pattern at a coffee counter for one minute intervals The service is taken as 2 persons in one minute in one counter :

No. of persons arriving	:	0	1	2	3	4	5	6	7
Probability percentage	:	5	10	15	30	20	10	5	5

Using Monte-Carlo simulation technique and the following random numbers, generate the pattern of arrivals and the queue formed when the following 20 random numbers are given :

5,	25,	16,	80,	35,	48,	67,	79,	90,	92,
9,	14,	1,	55,	20,	71,	30,	42,	60,	85.

Find the queue length if two counters are used, *i.e.*, 4 persons in one minute.
[Madras B.Sc. (Appl. Sc.) 1999]

2321. The output of a production line is checked by an inspector for one or more of three different types of defects, called defects *A*, *B* and *C*. If defect *A* occurs, the item is scrapped. If defect *B* or *C* occurs, the item must be reworked. The time required to rework a *B* defect is 15 minutes and the time required to rework a *C* defect is 30 minutes. The probabilities of an *A*, *B* and *C* defects are 15, 20 and 10 respectively. For ten items coming off the assembly line, determine the number of items without any defects, the number scrapped and the total minutes of rework time. Use the following random numbers :

RN for defect *A* :	48	55	91	40	93	01	83	63	47	52
RN for defect *B* :	47	36	57	04	79	55	10	13	57	09
RN for defect *C* :	82	95	18	96	20	84	56	11	52	03

[C.A. (May) 1994]

2322. An airline has 20 fights leaving a base per day, each with a hostess. The airline keeps two hostesses in reserve so that they may be called in case the scheduled hostess for a flight is absent. The probability distribution for daily number of absenteeism by hostesses is as follows :

Number absent	:	0	1	2	3	4	5
Probability	:	0.30	0.35	0.20	0.10	0.03	0.02

Use Monte-Carlo method to estimate the utilisation of reserve hostesses and also the probability that at least one flight will be cancelled in a day because of non-availability of hostesses.
[Madurai M.B.A. 1995]

2323. A company manufactures around 200 mopeds. Depending upon the availability of raw materials and other conditions, the daily production has been varying from 196 mopeds to 204 mopeds, whose probability distribution is as given below :

Production per day	:	196	197	198	199	200	201	202	203	204
Probability	:	0.05	0.09	0.12	0.14	0.20	0.15	0.11	0.08	0.06

The finished mopeds are transported in a specially designed three-storeyed lorry that can accommodate only 200 mopeds. Using the given 15 random numbers, *viz.*, 82, 89, 78, 24, 53, 61, 18, 45, 04, 23, 50, 77, 27, 54, 10, simulate the process to find out (*i*) What will be the average number of mopeds waiting in the factory? and (*ii*) What will be the average number of empty spaces on the lorry?
[C.A. (May) 1999]

2324. (*a*) Patients arriving at a village dispensary are treated by a doctor on a first-come-first-served basis. The inter-arrival time of the patients is known to be uniformly distributed between 0 and 80 minutes, while their service time is known to be uniformly distributed between 15 and 40 minutes. It is desired to simulate the system and determine the average time a patient has to be in the queue for getting service and the proportion of time the doctor would be idle. Carry out the simulation using the following sequences of random numbers. The numbers have been selected between 00 and 80 to estimate inter-arrival times and between 15 and 40 to estimate the service times required by the patients.