Unit V

We have seen that in Neyman - Pearson theory of testing of hypothesis, n, the sample size is regarded as a fixed constant and keeping α fixed, we minimise β. But in the sequential analysis theory propounded by A. Waldn, the sample size is not fixed but is regarded as a r.v whereas both α and β are fixed constants

Sequential Probability Ratio test:-

The best known procedure in sequential testing is the SPRT developed by A. Wald discussed below:-

Suppose we want to test $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$ for a distn with pdf $f(x, \theta)$. For any +ve integer m, the likelihood function of a sample $x_1, x_2, \ldots x_m$ from the popln with pdf (p.m.f) $f(x,\theta)$ is given by:-

$$L_{1m} = \prod_{i=1}^{m} f(x_i, \theta_1) \text{ when } H_1 \text{ is true}$$

$$L_{0m} = \prod_{i=1}^{m} f(x_i, \theta_0) \text{ when } H_0 \text{ is true}$$ and likelihood ratio

λ_m is given by

$$\lambda_m = \frac{L_{1m}}{L_{0m}} = \frac{\prod_{i=1}^{m} f(x_i, \theta_1)}{\prod_{i=1}^{m} f(x_i, \theta_0)} = \prod_{i=1}^{m} \frac{f(x_i, \theta_1)}{f(x_i, \theta_0)} \quad (m = 1, 2, 3 \ldots)$$

The SPRT for testing H_0 against H_1 is defined as follows:-

At each stage of the experiment (at the mth trial for any integral value m), the likelihood ratio λ_m $(m = 1, 2 \ldots)$ is computed.

i) If $\lambda_m \geq A$, we terminate the process with the rejection of H_0

ii) If $\lambda_m \leq B$, we terminate the process with the acceptance of H_0

iii) If $B < \lambda_m < A$, we continue sampling by taking an additional observation.

Here A and B ($B<A$) are constants which are determined by the relation

$$A = \frac{1-\beta}{\alpha} \qquad B = \frac{\beta}{1-\alpha}$$

where α and β are the prob. of type I error and prob. of type II error respectively.

From computational point of view, it is much convenient to deal with $\log \lambda_m$ rather than λ_m, since

$$\log \lambda_m = \sum_{i=1}^{m} \log \frac{f(x_i, \theta_1)}{f(x_i, \theta_0)} = \sum_{i=1}^{m} z_i$$

where $z_i = \log \frac{f(x_i, \theta_1)}{f(x_i, \theta_0)}$

In terms of z_i's SPRT is defined as follows

i) If $\Sigma z_i \geq \log A$, reject H_0

ii) If $\Sigma z_i \leq \log B$, reject H_1

iii) If $\log B < \Sigma z_i < \log A$, continue sampling by taking an additional observation

Operating characteristic function of SPRT

The OC function $L(\theta)$ is defined as

$L(\theta)$ = Prob. of accepting $H_0 : \theta = \theta_0$ when θ is the true value of the parameter

$P(\theta)$ = Prob. of rejecting H_0 where θ is the true value, we get

$$L(\theta) = 1 - P(\theta)$$

The O.C function of a SPRT for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$ in sampling from a popln with density function $f(x, \theta)$ is given by

$$L(\theta) = \frac{A^{h(\theta)} - 1}{A^{h(\theta)} - B^{h(\theta)}}$$

where for each value of θ, the value of $h(\theta) \neq 0$ is to be determined so that $$E\left[\frac{f(x, \theta_1)}{f(x, \theta_0)}\right]^{h(\theta)} = 1$$

Average sample Number: The sample size n in sequential testing is a r.v which can be determined in terms of the true density function $f(x, \theta)$. The A.S.N function for the SPRT for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$ is given by

$$E(n) = \frac{L(\theta) \log B + [1 - L(\theta)] \log A}{E(z)}$$

where $z = \log\left(\frac{f(x, \theta_1)}{f(x, \theta_0)}\right)$ $A = \frac{1-\beta}{\alpha}$ $B = \frac{\beta}{1-\alpha}$

Determine SPRT for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$ from a normal popln. Also obtain OC function and ASN function

solution:-

$$\lambda_m = \prod \frac{f(x_i, \theta_1)}{f(x_i, \theta_0)} \; ; \; f(x, \theta) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(x-\theta)^2} \quad -\infty < x < \infty$$

$$\lambda_m = \exp\left\{-\frac{1}{2\sigma^2}\Sigma\left[(x_i-\theta_1)^2 - (x_i-\theta_0)^2\right]\right\}$$

$$= \exp\left\{\frac{-1}{2\sigma^2}\Sigma\left[(x_i-\theta_1+x_i-\theta_0)(x_i-\theta_1-x_i+\theta_0)\right]\right\}$$

$$= \exp\left\{-\frac{1}{2\sigma^2}\Sigma\left[(2x_i-\theta_1-\theta_0)(\theta_0-\theta_1)\right]\right\}$$

$$= \exp\left\{-\frac{(\theta_0-\theta_1)}{\sigma^2}\left(\Sigma x_i - m\left(\frac{\theta_1+\theta_0}{2}\right)\right)\right\}$$

$$\log \lambda_m = \left(\frac{\theta_1-\theta_0}{\sigma^2}\right)\left[\Sigma x_i - m\left(\frac{\theta_1+\theta_0}{2}\right)\right]$$

hence SPRT for $H_0 : \theta = \theta_0$ against $\theta = \theta_1$ is given by

i) Reject H_0

$$\frac{\theta_1-\theta_0}{\sigma^2}\left[\Sigma x_i - m\left(\frac{\theta_0+\theta_1}{2}\right)\right] \geq \log A$$

$$\Sigma x_i - m\left(\frac{\theta_0+\theta_1}{2}\right) \geq \frac{\sigma^2}{\theta_1-\theta_0}\log A$$

$$\Sigma x_i \geq \frac{\sigma^2}{\theta_1-\theta_0}\log\frac{1-\beta}{\alpha} + \left(\frac{\theta_0+\theta_1}{2}\right)m$$

ii) Accept H_0,

$$\frac{\theta_1-\theta_0}{\sigma^2}\left[\Sigma x_i - m\left(\frac{\theta_0+\theta_1}{2}\right)\right] \leq \log B$$

$$\Sigma x_i \leq \frac{\sigma^2}{\theta_1-\theta_0}\log\frac{B}{1-\alpha} + m\left(\frac{\theta_0+\theta_1}{2}\right)$$

iii) Continue taking additional observations as long as

$$\log B \leq \frac{\theta_1-\theta_0}{\sigma^2}\left[\Sigma x_i - m\left(\frac{\theta_0+\theta_1}{2}\right)\right] < \log A$$

$$\frac{\sigma^2}{\theta_1-\theta_0}\log\frac{\beta}{1-\alpha} + m\left(\frac{\theta_0+\theta_1}{2}\right) < \Sigma x_i < \frac{\sigma^2}{\theta_1-\theta_0}\log\frac{1-\beta}{\alpha} + m\left(\frac{\theta_0+\theta_1}{2}\right)$$

$\theta_1 > \theta_0$

OC function for normal distn

First of all we shall determine $h = h(\theta) \neq 0$

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$$\int_{-\infty}^{\infty}\left[\frac{f(x,\theta_1)}{f(x,\theta_0)}\right]^h f(x,\theta)\,dx = 1$$

$$\frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\infty}\exp\left\{-\frac{1}{2}\left(\frac{x-\theta}{\sigma}\right)^2\right\}\exp\left\{-\frac{h}{2\sigma^2}\left[(\theta_1-\theta_0)(\theta_0+\theta_1-2x)\right]\right\}dx = 1$$

$$\frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\infty}\exp\left\{\frac{-1}{2\sigma^2}\left[x^2+\theta^2-2x\theta-2x\{h(\theta_1-\theta_0)\}+(\theta_1^2-\theta_0^2)h\right]\right\}dx = 1$$

$$\frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\infty}\exp\left\{-\frac{1}{2\sigma^2}\left[x^2+2x\left[(\theta_1-\theta_0)h+\theta\right]+\theta^2+(\theta_1^2-\theta_0^2)h\right]\right\}dx = 1$$

If we have $\lambda = (\theta_1 - \theta_0)h + \theta$

$$\lambda^2 = (\theta_1^2 - \theta_0^2)h + \theta^2$$

Then the LHS becomes $\frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left[-\frac{1}{2\sigma^2}(x-\lambda)^2\right] dx$

which being the total area under normal prob. curve with mean μ and variance σ^2 is always unity.

Thus $h = h(\theta)$

$$(\theta_1^2 - \theta_0^2)h + \theta^2 = \{(\theta_1 - \theta_0)h + \theta\}^2$$

$$\Rightarrow (\theta_1^2 - \theta_0^2)h = (\theta_1 - \theta_0)^2 h^2 + 2(\theta_1 - \theta_0)h\theta.$$

since $h = h(\theta) \neq 0$ and $\theta_1 \neq \theta_0$ dividing throughout $(\theta_1 - \theta_0)h$, we get

$$\frac{(\theta_1^2 - \theta_0^2)h(\theta_1 + \theta_0)}{(\theta_1 - \theta_0)h} = \frac{(\theta_1 - \theta_0)^2 h^2}{(\theta_1 - \theta_0)h} + 2\theta$$

$$\frac{(\theta_1 + \theta_0) - 2\theta}{\theta_1 - \theta_0} = h(\theta)$$

$$h(\theta) = \frac{A^{h(\theta)} - 1}{A^{h(\theta)} - B^{h(\theta)}}$$

ASN ... normal distn

$$Z = \log \frac{f(x,\theta_1)}{f(x,\theta_0)} = \frac{\theta_1-\theta_0}{\sigma^2}\left(x - \frac{\theta_0+\theta_1}{2}\right)$$

$$E(Z) = \frac{\theta_1-\theta_0}{2\sigma^2}\left[2E(x) - \theta_0 - \theta_1\right]$$

$$= \frac{\theta_1-\theta_0}{2\sigma^2}\left[2\theta - \theta_0 - \theta_1\right]$$

$$E(n) = \frac{L(\theta)\log B + [1-L(\theta)]\log A}{\frac{\theta_1-\theta_0}{2\sigma^2}\left[2\theta - \theta_0 - \theta_1\right]}$$

Binomial distn

Let x has the distn $f(x,\theta) = \theta^x(1-\theta)^{1-x}$; $x = 0,1$ $0<\theta<1$

For testing $H_0: \theta = \theta_0$ against $H_1: \theta = \theta_1$, construct SPRT and obtain its ASN and OC function

$$\lambda_m = \frac{L(x_1, x_2, \ldots x_m / H_1)}{L(x_1, x_2 \ldots x_m / H_0)}$$

$$= \left[\theta_1^{\Sigma x_i}(1-\theta_1)^{m-\Sigma x_i}\right] \Big/ \left[\theta_0^{\Sigma x_i}(1-\theta_0)^{m-\Sigma x_i}\right]$$

$$= \left(\frac{\theta_1}{\theta_0}\right)^{\Sigma x_i}\left(\frac{1-\theta_1}{1-\theta_0}\right)^{m-\Sigma x_i}$$

$$\log \lambda_m = \Sigma x_i \log(\theta_1/\theta_0) + (m - \Sigma x_i)\log\left(\frac{1-\theta_1}{1-\theta_0}\right)$$

$$= \Sigma x_i \log\left[\frac{\theta_1(1-\theta_0)}{\theta_0(1-\theta_1)}\right] + m\log\left(\frac{1-\theta_1}{1-\theta_0}\right)$$

Hence SPRT for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$

(i) Accept H_0 if $\log \lambda_m \le \log\left(\frac{\beta}{1-\alpha}\right) = b$,

(ie) if $\sum x_i \le \dfrac{b - m \log[(1-\theta_1)/(1-\theta_0)]}{\log[\theta_1(1-\theta_0)/\theta_0(1-\theta_1)]} = a_m$

(ii) Reject H_0 if $\log \lambda_m \ge \log\left(\frac{1-\beta}{\alpha}\right) = a$,

(ie) if $\sum x_i \ge \dfrac{a - m \log[(1-\theta_1)/(1-\theta_0)]}{\log[\theta_1(1-\theta_0)/\theta_0(1-\theta_1)]} = r_m$

(iii) Continue sampling if $b < \log \lambda_m < a \Rightarrow a_m < \sum x_i < r_m$

OC function

OC function is given by:

$$L(\theta) = \frac{A^{h(\theta)} - 1}{A^{h(\theta)} - B^{h(\theta)}}$$

where for each value of θ, $h(\theta) \neq 0$ is to be determined such that

$$E\left[\frac{f(x, \theta_1)}{f(x, \theta_0)}\right]^{h(\theta)} = 1$$

$$\sum_{x=0}^{1} \left[\frac{f(x, \theta_1)}{f(x, \theta_0)}\right]^{h(\theta)} f(x, \theta) = 1$$

$$\sum_{x=0}^{1}\left[\left(\frac{\theta_1}{\theta_0}\right)^{x}\left(\frac{1-\theta_1}{1-\theta_0}\right)^{1-x}\right]^{h(\theta)}\theta(1-\theta)^{1-x}=1$$

$$\left[\left(\frac{1-\theta_1}{1-\theta_0}\right)^{h(\theta)}\cdot(1-\theta)+\left(\frac{\theta_1}{\theta_0}\right)^{h(\theta)}\theta\right]=1$$

The soln of this eqn for $h=h(\theta)$ is very tedious. From practical point of view, instead of solving for h we regard h as a parameter and solve it for θ. Thus giving,

$$\theta\left[\left(\frac{\theta_1}{\theta_0}\right)^{h(\theta)}-\left(\frac{1-\theta_1}{1-\theta_0}\right)^{h(\theta)}\right]=1-\left(\frac{1-\theta_1}{1-\theta_0}\right)^{h(\theta)}$$

$$\Rightarrow \theta=\frac{1-[(1-\theta_1)/(1-\theta_0)]^{h(\theta)}}{(\theta_1/\theta_0)^{h(\theta)}-[(1-\theta_1)/(1-\theta_0)]^{h(\theta)}}=\theta(h)$$

$$\therefore L(\theta)=\frac{[(1-\beta)/\alpha]^{h}-1}{[(1-\beta)/\alpha]^{h}-[\beta/(1-\alpha)]^{h}}=L(\theta,h]$$

Various points on the OC curve are obtained by assigning arbitrary values to 'h' and computing the corresponding values of θ and $L(\theta)$ ~~f~~

ASN function

$$z=\log\left[\frac{f(x,\theta_1)}{f(x,\theta_0)}\right]\quad ; A=\frac{1-\beta}{\alpha}\qquad B=\frac{\beta}{1-\alpha}$$

$$E(z) = \sum_{x=0}^{1} \log\left[\frac{f(x,\theta_1)}{f(x,\theta_0)}\right] \cdot f(x,\theta)$$

$$= \sum_{x=0}^{1} \log\left[\left(\frac{\theta_1}{\theta_0}\right)^x \left(\frac{1-\theta_1}{1-\theta_0}\right)^{1-x}\right] \theta^x (1-\theta)^{1-x}$$

$$= (1-\theta)\log\left(\frac{1-\theta_1}{1-\theta_0}\right) + \theta \log\left(\frac{\theta_1}{\theta_0}\right)$$

$$E(z) = \theta \log\left[\frac{\theta_1(1-\theta_0)}{\theta_0(1-\theta_1)}\right] + \log\left(\frac{1-\theta_1}{1-\theta_0}\right)$$

ASN is given by

$$E(n) = \frac{L(\theta)\log B + [1-L(\theta)]\log A}{E(z)}$$

substituting the values of $E(z)$ and $L(\theta)$, we get ASN function

Poisson distribution

For testing $H_0: \lambda = \lambda_0$ Vs $H_1: \lambda = \lambda_1$ where λ is the parameter.

Derive SPRT

For an SPRT with strength (α, β) the decision at mth stage for testing $H_0: \theta = \theta_0$ Vs $H_1: \theta = \theta_1$ will be as follows. At mth stage if $\sum_{i=1}^{m} z_i \geq \log A$ reject H_0

if $\sum z_i \leq \log B$ ~~reject~~ accept H_0

if $\log A < \sum z_i < \log B$ continue sampling.

where $A = \frac{1-\beta}{\alpha}$ $B = \frac{\beta}{1-\alpha}$ $z_i = \log(f_1/f_0)$

f_1 is p.d.f under H_1 f_0 is p.d.f under H_0

$$f(x,\lambda) = \frac{e^{-\lambda}\lambda^x}{x!} \quad f(x,\lambda_1) = \frac{e^{-\lambda_1}\lambda_1^x}{x!} \quad f(x,\lambda_0) = \frac{e^{-\lambda_0}\lambda_0^x}{x!}$$

$$\log\left(\frac{f_1}{f_0}\right) = \log\left[\frac{e^{-\lambda_1}\lambda_1^x}{x!} \Big/ \frac{e^{-\lambda_0}\lambda_0^x}{x!}\right]$$

$$z = \log\left[e^{-(\lambda_1-\lambda_0)}(\lambda_1/\lambda_0)^x\right]$$

$$z = -(\lambda_1-\lambda_0) + x\log(\lambda_1/\lambda_0)$$

The SPRT with strength (α,β) for testing $H_0: \lambda = \lambda_0$ Vs $H_1: \lambda = \lambda_1$ is

$$\sum_{i=1}^{m}\left[-(\lambda_1-\lambda_0) + x_i\log(\lambda_1/\lambda_0)\right]$$

$$\sum_{i=1}^{m} z_i = -m(\lambda_1-\lambda_0) + \sum x_i\log(\lambda_1/\lambda_0)$$

Reject H_0 if $\sum z_i \geq \log A$

$$-m(\lambda_1-\lambda_0) + \sum x_i\log(\lambda_1/\lambda_0) \geq \log A$$

Accept H_0: if $\sum z_i \leq \log B$

$$-m(\lambda_1-\lambda_0) + \sum x_i\log(\lambda_1/\lambda_0) \leq \log B$$

Continue sampling if $\log B < z_i < \log A$

$$\log B < -m(\lambda_1-\lambda_0) + \sum x_i\log(\lambda_1/\lambda_0) < \log A$$

To draw the design lines:-

Rejection line R_m

$$-m(\lambda_1-\lambda_0)+\sum x_i \log(\lambda_1/\lambda_0) \geq \log A$$

$$\sum x_i \geq \frac{\log A + m(\lambda_1-\lambda_0)}{\log(\lambda_1/\lambda_0)}$$

$$\geq \frac{\log\left(\frac{1-\beta}{\alpha}\right)}{\log(\lambda_1/\lambda_0)} + \frac{m(\lambda_1-\lambda_0)}{\log(\lambda_1/\lambda_0)}$$

Acceptance line

$$-m(\lambda_1-\lambda_0)+\sum x_i \log(\lambda_1/\lambda_0) \leq \log B$$

$$\sum x_i \leq \frac{\log B + m(\lambda_1-\lambda_0)}{\log(\lambda_1/\lambda_0)}$$

$$\leq \frac{\log\left(\frac{\beta}{1-\alpha}\right)}{\log(\lambda_1/\lambda_0)} + \frac{m(\lambda_1-\lambda_0)}{\log(\lambda_1/\lambda_0)}$$

OC function

The OC function of the SPRT is $L(\theta) = \dfrac{A^{h(\theta)}-1}{A^{h(\theta)}-B^{h(\theta)}}$

where $A = \dfrac{1-\beta}{\alpha}$ $\quad B = \dfrac{\beta}{1-\alpha}$ $\quad h \in \theta \neq 0 \quad E\left(\dfrac{f_1}{f_0}\right)^{h(\theta)} = 1$

Exponential distribution

$H_0 : \theta = \theta_0 \qquad H_1 : \theta = \theta_1$

The given p.d.f is $f_0(x) = \frac{1}{\theta} e^{-x/\theta}$

Then the likelihood function for sample is

$$L = \prod_{i=1}^{n} f(x_i, \theta)$$

$$\lambda_m = \prod_{i=1}^{n} \left[\frac{f(x_1, \theta_1)}{f(x, \theta_0)} \right]$$

$$\lambda_m = \frac{(1/\theta_1)^n e^{-\frac{\Sigma x}{\theta_1}}}{\left(\frac{1}{\theta_0}\right)^n e^{-\frac{\Sigma x}{\theta_0}}}$$

$$\log \lambda_m = \log \left[\left(\frac{\theta_0}{\theta_1} \right)^n e^{-\frac{\Sigma x_i}{\theta_1} + \frac{\Sigma x}{\theta_0}} \right]$$

$$= \log \left(\frac{\theta_0}{\theta_1} \right)^n + \left[\frac{x}{\theta_0} - \frac{x}{\theta_1} \right]$$

$$= -\log \left(\frac{\theta_1}{\theta_0} \right)^n + \left[\frac{x\theta_1 - x\theta_0}{\theta_0 \theta_1} \right]$$

$$= -\log \left(\frac{\theta_1}{\theta_0} \right)^n + \left[\frac{\theta_1 - \theta_0}{\theta_0 \theta_1} \right] x$$

* The cummulative sum of likelihood function of all x is

$$\Sigma \log \lambda_m = -n \log \left[\frac{\theta_1}{\theta_0} \right] + \left[\frac{\theta_1 - \theta_0}{\theta_0 \theta_1} \right] \Sigma x_i$$

(i) Reject H_0: $-n \log \left(\frac{\theta_1}{\theta_0} \right) + \left(\frac{\theta_1 - \theta_0}{\theta_0 \theta_1} \right) \Sigma x_i \geq \log A$

(ii) Accept H_0 if $-n\log\left(\frac{\theta_1}{\theta_0}\right)+\left(\frac{\theta_1-\theta_0}{\theta_0\theta_1}\right)\Sigma x_i \le \log B$

(iii) $\log B < -n\log\left(\frac{\theta_1}{\theta_0}\right)+\left(\frac{\theta_1-\theta_0}{\theta_0\theta_1}\right)\Sigma x_i < \log A$

OC function

$$L(\theta) = \frac{A^h - 1}{A^h - B^h}$$ where $h(\theta) \neq 0$ such that

$$E[f_1/f_0] = 1$$

Consider SPRT for testing $H_0 : \theta = \theta_0$ Vs $H_1 : \theta = \theta_1$ given α and β will be as follows

Reject H_0 if $\frac{P_{1m}}{P_{0m}} \ge A$, accept H_0 if $\frac{P_{1m}}{P_{0m}} \le B$

Continue sampling if $B < \frac{P_{1m}}{P_{0m}} < A$. W.K.T $h(\theta) \neq 0$ subject such that $E[f_1/f_0] = 1$.

If $h(\theta) \neq 0$ it lie either $h(\theta) > 0$ (or) $h(\theta) < 0$. Let us assume $h(\theta) > 0$.

Further assume that X is continuous r.v

$$g(x,\theta) \cdot \left(\frac{f_1}{f_0}\right)^h f(x,\theta)$$

$$E[f_1/f_0]^h = \int_{-\infty}^{\infty} \left(\frac{f_1}{f_0}\right)^h f(x,\theta)\, dx = 1$$

Let us consider problem of testing, $H_1 : X \sim f(x,\theta)$

$H_0^* : X \sim g(x,\theta)$

The SPRT for testing H_0 Vs H^* will be

$$\frac{g(x:\theta_1)}{g(x;\theta_0)} \geq A^{h(\theta)}$$

$$\Rightarrow g(x;\theta_1) = \left(\frac{f(x;\theta_1)}{f(x;\theta_0)}\right)^h f(x;\theta)$$

$\cancel{f(x;\theta)}$ $F(x_i;\theta) = f(x_i,\theta)$

$$\Rightarrow \frac{g(x_i;\theta_1)}{F(x;\theta)} = \left[\frac{F(x;\theta_1)}{F(x;\theta_0)}\right]^h$$

The SPRT for testing $H: X \sim F(x:\theta)$ Vs $H^*: X \sim g(x:\theta)$

$$\prod\left[\frac{F(x_i;\theta_1)}{F(x_i;\theta_0)}\right]^h \geq A^{h(\theta)}, \text{ Reject } H_0$$

$$\prod\left[\frac{f(x_i;\theta_1)}{f(x_i;\theta_0)}\right]^h \leq B^{h(\theta)}, \text{ accept } H_0$$

$$B^{h(\theta)} \quad B^{(h)} \leq \prod\left[\frac{f(x_i;\theta_1)}{f(x_i;\theta_0)}\right]^h \leq A^{h(\theta)}, \text{ Continue Sampling}$$

We have $A^{h(\theta)} = \dfrac{1-\beta'}{\alpha'} \Rightarrow A^{h(\theta)}\alpha' = 1-\beta'$ —— ①

$B^{h(\theta)} = \dfrac{\cancel{1-}\beta'}{1-\alpha'} \Rightarrow B^{h(\theta)}(1-\alpha') = \beta'$ —— ②

① + ② $\Rightarrow$ $\alpha' A^h + B^h - \alpha' B^h = 1$

$$\alpha' = \frac{1-B^h}{A^h - B^h}$$

$$1 - \alpha' = L(\theta) = 1 - \frac{1 - B^h}{A^h - B^h}$$

$$L(\theta) = \frac{A^h - 1}{A^h - B^h}$$