Testing of hypothesis

Parameter and statistic statistical measure computed from the population are called parameter. calculation from the sample observation are called statistic

Eg:- population mean μ, population variance σ^2 - Parameter
Sample mean x, sample variance s^2 - statistic

statistical hypothesis: Making decisions about the population on the basis of sample information with decisions are called statistical decisions. For eg:- we may wish to decide on the basis of the sample data whether a new drug is really effective in curing a disease, whether one educational procedure is better than another or whether a given coin is biased.

In attempting to reach decisions it is useful to make assumption about the poplns involved such observations which he may or may not be true are called statistical hypothesis.

They

Explain the Simple and compositesis:-

If the statistical hypothesis specifies the population completely then it is termed as simple statistical hypothesis otherwise it is called as composite statistical hypothesis.

Eg:- In a bivariate normal distribution, with two means, two variables and one correlation coefficient if a hypothesis determines, only one, two three or four parameters it is called composite hypothesis, but if determines all the five parameters in addition to the normality of the distribution. It is called simple hypothesis.

Define Test of hypothesis A test of hypothesis procedure which specifies a set of rules for decision whether to accept or reject the hypothesis under the consideration.

That is, the testing of hypothesis is a procedure that helps us to as certain the likelihood of hypothesized population parameter being correct by making use of the sample statistic. statistical test of hypothesis play an important role in the biological, the agricultural, the medical science and also in the industry

Two types of hypothesis in a statistical test are:

Null hypothesis: A statistical hypothesis which is set up and whose validity is tested for possible rejection the basis of sample observations is called null hypothesis. That is, any statement or any assumption about the distribution on no difference basis. It is denoted as H_0

Alternative hypothesis:- Any hypothesis which is complementary to the null hypothesis is called alternative hypothesis. It is denoted as H_1.

$H_0: \mu_0 = 170$ $H_1: \mu \neq 170$ ($\mu > 170$ or $\mu < 170$)

One tailed and Two tailed tests:-

When the rejection region consist of two regions each associated with probability α, we call it a two tailed test. On the other hand, when the rejection region consist of only one region (either on the right or left associated with prob α) we call it as one tailed test.

One tailed Test:

H_0: The mean weight of a variety of wheat is 30 bushels per hectare, that is $\mu = 30$

H_1: The mean weight can be more than 30 bushels per hectare, $\mu > 30$ (Right tailed test)

H_1: The mean weight can be less than 30 bushels per hectare, $\mu < 30$

Two tailed test

H_0: The mean weight of a variety of wheat is 30 bushels per hectare $\mu = 30$

H_1: The mean weight is not 30 bushels per hectare, $\mu \neq 30$

Define type I and type II errors:

When a hypothesis H_0 is tested against an alternative hypothesis H_1 there arise one of the two types of errors. when a null hypothesis is rejected when it is true, it is known as type I error. If the null hypothesis is accepted when it is false it is type II error.

Type I error : Reject H_0, when H_0 is true

Type II error : Accept H_0, when H_0 is false

P[Type I error] = α P[type II error] = β

Symbolically, $P(X \in \omega / H_0) = \alpha$ where $X = (x_1, x_2 \ldots x_n)$

$\int_{\omega} L_0 \, dx = \alpha$, where L_0 is the likelihood function of the sample observations under H_0 and $\int dx$ represents the n fold integral $\int \ldots \int dx_1 \, \int dx_2, \ldots \int dx_n$.

Against, $P(x \in \omega / H_1) = \beta \Rightarrow \int_{\omega} L_1 \, dx = \beta$ where L_1 is the likelihood function of the sample observations under H_1

$$\int_{\bar{w}} L_1 \, dx + \int_{w} L_1 \, dx = 1$$

$$\int_{\bar{w}} L_1 \, dx = 1 - \int_{w} L_1 \, dx$$

$$\int_{\bar{w}} L_1 \, dx = 1 - \beta$$

Level of significance:- α, the prob of type I error is known as the level of significance of the test, it is also called the size of the critical region

Power of the set:- $1-\beta$ is called the power of the test and it gives the prob of making a correct decision

If w is the critical region and $\bar{w}$ is the acceptance region, then the power of the test is derived as

$$\beta = P(X \in w / H_1)$$

$$1-\beta = 1 - P(X \in w / H_1)$$

$$= P(X \in \bar{w} / H_1)$$

$$= P(\text{rejecting } H_0 / H_1 \text{ is true})$$

$$= P(\text{rejecting } H_0 / H_0 \text{ is false})$$

$$= P(\text{correct decision})$$

Critical Region:-

Let $x_1, x_2, \ldots x_n$ be the sample observations denoted by O, all the value of O will be aggregate of a sample and they constitute a space called sample space and is denoted by S. The basis of testing of hypothesis the division of the sample space into 2 exclusive regions w and $S - \bar{w}$ (or) w, the null hypothesis H_0 is rejected by the observations sample points fall in the w and if false in w we reject H_1, accept H_0. the region of rejecting H_0 when H_0 is true is that region of outcome set. when H_0 is rejected if the sample points falls in that region and is called as the region of reject or critical region.

Scanned with CamScanner

9/9/2020

Randomised test:-

A randomised test is one in which no test statistic is involved. The decision about i is taken on the basis of some predicted criteria. if It is decided that H_0 will be accepted of on tossing the coin falls with head and rejected if the coin fall with tail but randomised tests are seldom used.

Non-Random Test:-

A test 'T' of a hypothesis, it is said to be a non randomised test. If the decision about the rejection or acceptance of H_0 is based on a test statistic. It is rejected, if the test statistic in the critical region otherwise acceptance Region

Neyman - Pearson Generalised lemma.

Let $p_1, p_2, \dots p_{m+1}$ be integral functions and suppose that a for given set of constants $c_1, c_2 \dots c_m$, these exists a test function ϕ satisfying

$$E(\phi(x)) = \int \phi(x)\, p_i(x)\, dx = c_i \quad \text{--- (1)}$$

$$i = 1, 2, \dots m$$

Let C be the class of all test functions satisfying one which is assume to be non-empty. then

a) If a members of c satisfies the condition

$$\phi(x) = \begin{cases} 1 & \text{if } P_{m+1}(x) > k_1 P_1(x) + k_2 P_2(x) + \dots + k_m P_m(x) \\ 0 & \text{if } P_{m+1}(x) < k_1 P_1(x) + k_2 P_2(x) + \dots + k_m P_m(x) \end{cases} \quad \text{--- (2)}$$

Then it will maximize $\int \phi(x) P_{m+1}(x)\, dx$ — (3)

If a member of c satisfies (2) with $k_i > 0$ for $i = 1, 2, \dots m$ then it will maximize (3) subject to

$$E[\phi(x)] = \int \phi(x) P_i(x)\, dx \leq c_i \quad \text{for } i = 1, 2 \dots m$$

b) The set of $m = \{b_1 \phi(x), b_2 \phi(x) \dots b_m \phi(x)\}$ where $b_i \phi(x) = \int \phi(x) P_i(x)\, dx$; $i = 1, 2 \dots m$ is wave x and closed for some critical function ϕ. If $(c_1, c_2 \dots c_m)$ is an inner point of m then $\exists$ constants $k_1, k_2 \dots k_m$ and a test function ϕ satisfying (1) and (2), also (2) is necessary condition for member of c is maximize (3)

Power function

Let us denote the characteristics of a hypothesis test (then NH , the ANH and the decision rule) be the notation 'δ'

The power function of hypothesis test 'δ' is the prob. of rejecting H_0 given test the true value of the parameter is $\theta \in \Omega$

$P(\text{rejecting } H_0 / \theta \in \Omega) = P(X \in C / \theta)$ for $\theta \in \Omega$

Thus $\pi(\theta / \delta) = \alpha_\theta(\delta)$ if $\theta \in \Omega_0$

$1 - \pi(\theta/\delta) = \beta_\theta(\delta)$ if $\theta \in \Omega$

Defenition:-

(i) For $0 \le \alpha \le 1$, a test with power function $\beta(\theta)$ is a size α test if $\sup\limits_{\theta \in \Theta_0} \beta(\theta) = \alpha$

(ii) For $0 \le \alpha \le 1$, a test with power function $\beta(\theta)$ is a level α test if $\sup\limits_{\theta \in \Theta_0} \beta(\theta) \le \alpha$

A size α test and level α test are almost the same. The distinction is made because sometimes, we want a size α test and we can not construct a test with exact size α but we can construct one with a smaller error. That is a α-test, that is of level α. Hence the set of level α tests contains the set of size α tests.

In general :- Fix $\alpha \in [0,1]$. Now try to maximize $\beta(\theta)$ for $\theta \in \Theta_1$ subject to $\beta(\theta) \leq \alpha$ for $\theta \in \Theta_0$ with typical choice being $\alpha = 0.01, 0.05, 0.10$ That is the experimenter controls the type I error.

If this approach is taken, we should specify the NH, and ANH, so test is most important to control the type I error prob. with maximizing the power of rejecting H_0 when H_1 is true.

Unit - II

Uniformly most powerful test :-

Let us take the most powerful test $H_0 : \mu = \mu_0$ against $H_1 : \mu = \mu_1$ (or) $(\mu > \mu_0)$ using a random sample of n-observations from the distn $N(\mu, \sigma^2)$, σ^2 known, at level α is given by

$$\phi(x) = \begin{cases} 1 ; \text{ if } \bar{x} \geq \mu_0 + z_\alpha \sigma/\sqrt{n} \\ 0 ; \text{ otherwise} \end{cases}$$

and if $\mu_1 < \mu_0$. the MPT of level α is given by

$$\phi(x) = \begin{cases} 1, \text{ if } \bar{x} \leq \mu_0 - z_\alpha \frac{\sigma}{\sqrt{n}} \\ 0, \text{ otherwise} \end{cases}$$