Unit IV

Derivation of variable sampling plan with single specification limit , known σ

- Let $F_x(x, \mu, \sigma)$ be the distribution function of x with mean μ and variance σ^2.
- When σ^2 is known, for a specific limit U, the proportion p_1 of defective items must be defined with reference to the distribution function as

$$p = 1- F_x(U, \mu, \sigma) = P(x > U/\mu)$$

Suppose that the following condition must satisfy

$$\alpha = P[Rejecting\ lots\ of\ good\ quality\ for\ which\ proportion\ nonconforming\ is\ p1\ (AQL)]$$

$$\beta = P\ [\ Accepting\ a\ lot\ poor\ quality\ level\ for\ which\ proportion\ nonconforming\ is\ p2\ (LTPD)]$$

Therefore, the OC curve of the sampling plan must pass two through the Points

P(Accepting the lot at AQL) ≥ 1 - α

P(Accepting the lot at LQL) ≤ β

AQL & LQL are defined by

AQL = p_1= 1-F_x(U; μ_1, σ)

LQL = $p2$= 1- F_x(U; μ_2, σ)

α and β are defined by

$$\alpha = 1 - F_{\bar{x}}(U; \mu_1, \frac{\sigma}{\sqrt{n}}) = P(\bar{x} + k\sigma > U/\mu = \mu_1)$$

$$1 - \beta = P(\bar{x} + k\sigma > U/\mu = \mu_2)$$

$$Where\ \mu_1 denote\ the\ process\ mean\ for\ which\ the\ proportion\ non - conforming\ is\ p1$$

$Z_1 = \frac{U-\mu_1}{\sigma}$ Where tail area outside Z_1 is p_1

p_1 = P(Z > Z_1)

Similarly, we have $Z_2 = \frac{U-\mu_2}{\sigma}$

μ_2 *is the process mean for which the proportion non conforming* $p_2 = P(Z > Z_2)$

Z_α and Z_β denote the upper tail of the standard normal distributions such that the tail areas are α & β respectively.

Under K method the lot is accepted otherwise reject the lot.

If $Z_u = \frac{U-\bar{x}}{\sigma} \geq K$

Adding and subtracting μ we get,

$$\frac{U-\mu+\mu-\bar{x}}{\sigma} \geq K$$

$$\frac{U-\mu}{\sigma} + \frac{\mu-\bar{x}}{\sigma} \geq K$$

$$\frac{U-\mu}{\sigma} - \frac{\bar{x}-\mu}{\sigma} \geq K$$

$$\frac{\bar{x}-\mu}{\sigma} \leq \frac{U-\mu}{\sigma} - K$$

Multiply $\sqrt{n}$ *by both sides*

$$\frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} \leq \left(\frac{U-\mu}{\sigma} - K\right)\sqrt{n} \text{-----------------------1}$$

Using the relationship $Z_1 = \frac{U-\mu_1}{\sigma}$

The requirement of accepting a lot of good quality (process mean at μ_1) with probability of 1-∝

$$P\left[\frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} \leq (Z_1 - K)\sqrt{n}\right] = 1-\alpha$$

$$P\left[\frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} > (Z_1 - K)\sqrt{n}\right] = \alpha \text{ ------------------------2}$$

$(Z_1 - K)\sqrt{n} = Z_\alpha$ ----------------------------3

Using the relationship $Z_2 = \frac{U-\mu_2}{\sigma}$

P$\left[\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \leq (Z_2 - \mathrm{K})\sqrt{n}\right]$ = β

P$\left[\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} > (Z_2 - \mathrm{K})\sqrt{n}\right]$ = 1-β

$(Z_2 - \mathrm{K})\sqrt{n}$ = $Z_{1-\beta}$ = -Z_β -------------------------4

From Equations 3 & 4, we get

$(Z_1 - \mathrm{K})\sqrt{n}$= Z_α (5)

$(Z_2 - \mathrm{K})\sqrt{n}$ = $-Z_\beta$ (6)

Subtracting equation 5-6

$\sqrt{n}\,(Z_1 - Z_2)$ = $Z_\alpha - (-Z_\beta)$

$$\sqrt{n} = \frac{Z_\alpha + Z_\beta}{Z_1 - Z_2}$$

$$n = \left(\frac{Z_\alpha + Z_\beta}{Z_1 - Z_2}\right)^2$$ --(7)

Hence the single sampling parameter n is determined.

Similarly, substituting (7) in (5) we get

$(Z_1 - K)\left(\frac{Z_\alpha + Z_\beta}{Z_1 - Z_2}\right)$= Z_α

$(z_1 - K)(Z_\alpha + Z_\beta)$ = $Z_\alpha(Z_1 - Z_2)$

$Z_1 Z_\alpha + Z_1 Z_\beta$ - k$(Z_\alpha + Z_\beta)$ = $Z_1 Z_\alpha - Z_2 Z_\alpha$

K $(Z_\alpha + Z_\beta) = Z_1 Z_\beta + Z_2 Z_\alpha$

K = $\frac{Z_1 Z_\beta + Z_2 Z_\alpha}{Z_\alpha + Z_\beta}$

Hence the variable factor of single sampling plan is determined.

Variable sampling plan when σ *is unkown*

When SD is unknown the following procedure may be used

Take a Random Sample $\bar{X}$ = $\sum x/n$

$s^2 = 1/n - 1 \ \sum(x - \bar{X})^2$

$Z_U = U - \bar{X} \,/\, S$

If $Z_U \geq K$ *accept the lot otherwsie rejected* $\bar{X} + ks \leq U$ accepted otherwise rejected.

To find n and k we should know the distribution of $\bar{X} \pm ks$

x~ N(μ, σ^2)

$\bar{X} \sim N(\mu, \sigma^2/n)$

s ~ N($\sigma, \sigma^2/2n$)

E($\bar{X} \pm ks) = E(\bar{X}) \pm k\, E(s)$

$\mu \ \pm k\sigma$

V($\bar{X} \pm ks$) = V(x) + k^2 v(s) ($\bar{X}\,'s$ are independent)

$\frac{\sigma^2}{n} + \ k^2 \ \left(\frac{\sigma^2}{2n}\right) \sigma^2/n$ (1+$k^2/2$)

therefore, $\bar{X} \pm ks$ *is approximately normally distributed with mean* $\mu \pm k\sigma$ accept the lot if

$\bar{X} \pm ks \ \leq U$

$\bar{X} \pm ks - (\,\mu + k\sigma) \,/ \frac{\sigma^2}{n}\,(\,1 + \frac{k^2}{2}) \leq U - (\,\mu + k\sigma) \,//\, \frac{\sigma^2}{n}\,(\,1 + \frac{k^2}{2})$

$\bar{X} \pm ks - (\,\mu + k\sigma) \,/\, \sigma\sqrt{1/n(1 + k^2/2)} \leq U - \mu - k\sigma \,/\, \sigma\sqrt{1/n(1 + k^2/2)}$

$P[\bar{X} - ks - (\mu_1 \ + \ k\sigma)/\,\sigma\sqrt{1/n(1 + k^2/2)} \leq U - \mu_1 /\sigma$ - K/ $\sigma\sqrt{1/n(1 + k^2/2)}]$ =

1-$\propto$

P[$\propto \leq z_1 - k/\sqrt{1/n(1+} k^2/2)$] = 1-$\propto$

P[Z $> z_1 - k/\sqrt{1/n(1+} k^2/2)$] = $\propto$

$z_1 - k/\sqrt{1/n(1+} k^2/2$ = $z_\propto$----------------------------*

$P[\bar{X} - ks - (\mu_2 + k\sigma)/\sigma\sqrt{1/n(1+} k^2/2$n $\leq U - \mu_2/\sigma$ - K/ $\sqrt{1/n(1+} k^2/2)$] = β

P[$\propto \leq z_2 - k/\sqrt{1/n(1+} k^2/2)$] = β

P[Z $\geq z_1 - k/\sqrt{1/n(1+} k^2/2)$] = 1 $- \beta$

$z_2 - k/\sqrt{1/n(1+} k^2/2)$] = $Z_{1-\beta}$ = -Zβ-------------------------------------**

$Solve * and **$

$z_1 - k/\sqrt{1/n(1+} k^2/2$ = $z_\propto$

$z_2 - k/\sqrt{1/n(1+} k^2/2)$] = -Z$\beta$ / 1/$\sqrt{1/n(1+} k^2/2)$] $(z_1 - z_2) = z_\propto + z_\beta$

$\sqrt{n}\,(z_1 - z_2)/\ \sqrt{(1+} k^2/2$ = $z_\propto + z_\beta$

$\sqrt{n} = (Z_\propto + Z_\beta\ /\ Z_1 + Z_2)\sqrt{(1+} k^2/2$---------------------------------------****

$sub *** and *$

$z_1 - k/\sqrt{1/n(1+} k^2/2$ = $z_\propto$

$(z_1 - k)\ (Z_\propto + Z_\beta\ /\ Z_1 - Z_2)/\ \sqrt{(1+} k^2/2$ / $\sqrt{(1+} k^2/2$= $Z_\propto$

$(z_1 - k)$ / $Z_\propto + Z_\beta$ = $Z_\propto\ (Z_1 - Z_2)$

$z_1 Z_\propto + z_1 Z_\beta -$ k$Z_\propto - kZ_\beta$ = $Z_\propto Z_1$ - $Z_\propto Z_2$

$z_1 Z_\beta$ - k $(Z_\propto + Z_\beta) = -Z_\propto Z_2$

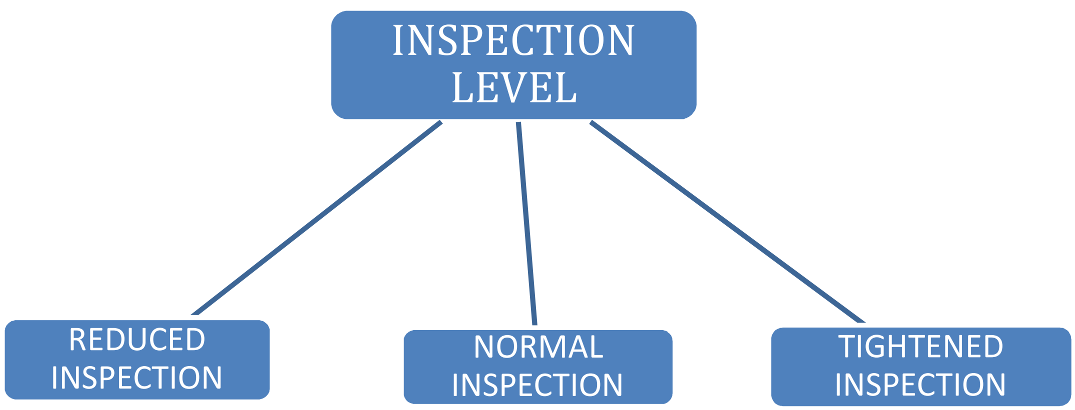
$z_1 Z_\beta + z_2 Z_\propto$ = k$(Z_\propto + Z_\beta)$

$k = z_1 Z_\beta + z_2 Z_\propto\ /\ Z_\propto + Z_\beta$

Military Standard 105D

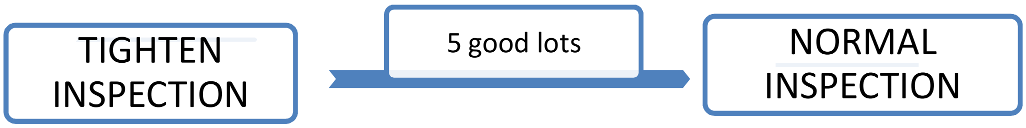
Objectives – MIL_STD_105D:

- The military standard 105D table provides plans that emphasize the protection of the producers against rejecting the lots.
- The MIL_STD_105D plans are based on the acceptable quality level (AQL).
- The AQL means it is the fraction defective from the production process agreed by the producer and consumer.
- The probability of rejecting the good lot with the process average AQL is called producer risk.
- P[rejecting the good lot/ AQL] = α
- MIL_STD_105D is developed by American military and it's also known as ABC_STD_105D.
- **Procedure**:
  - **Step 1**- Fix AQL from the production process p1
  - **Step 2**- Decide on the inspection level

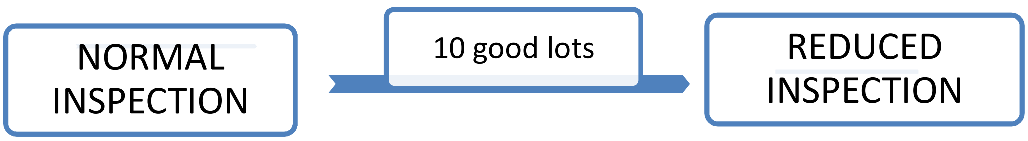


  - **Step 3**- Determine the lot size **N**.
  - **Step 4**- Refer the table to find sample size code letter.
  - **Step 5**- Decide on the parameter of the sampling plan (**n,c**).
  - **Step 6**- When 2 0ut of 5 consecutive lots are rejected under normal inspection then shift over to tightened inspection.

- **Step 7**- The tightened inspection will have lesser probability of acceptance than normal distribution.
- **Step8**- During tightened inspection is followed,when 10 consecutive lots are accepted then shift to reduced inspection.



- **Step 9**- During normal inspection, when 10 consecutive lots are accepted the shift to reduced inspection.

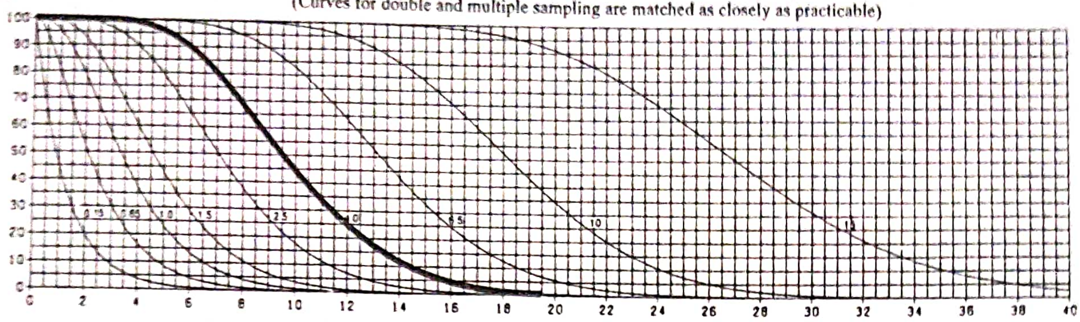


- **Step 10-** The Cumulative defective from 10 lot is less than or equal to the acceptance number **c**.

Table 4 Tables for sample size code letter J.

Percent of lots expected to be accepted (P_a)

Chart J—Operating characteristic curves for single sampling plans
(Curves for double and multiple sampling are matched as closely as practicable)



Quality of submitted product (p, in percent nonconforming for AQLs ≤ 10; in nonconformities per hundred units for AQLs > 10)
Note: Figures on curves are acceptance quality limits (AQLs) for normal inspection.

Tabulated values for operating characteristic curves for single sampling plans

P_a	Acceptance quality limits (normal inspection)																					
	0.15	0.65	1.0	1.5	2.5	4.0	X	6.5	X	10	0.15	0.65	1.0	1.5	2.5	4.0	X	6.5	X	10	X	15
	p (in percent nonconforming)										p (in nonconformities per hundred units)											
99.0	0.0126	0.187	0.550	1.04	2.28	3.73	4.51	6.17	7.93	9.76	0.0126	0.186	0.545	1.03	2.23	3.63	4.38	5.96	7.62	9.35	12.9	15.7
95.0	0.0641	0.446	1.03	1.73	3.32	5.07	6.00	7.91	9.89	11.9	0.064	0.444	1.02	1.71	3.27	4.98	5.87	7.71	9.61	11.6	15.6	18.6
90.0	0.132	0.667	1.39	2.20	3.99	5.91	6.90	8.95	11.0	13.2	0.132	0.665	1.38	2.18	3.94	5.82	6.79	8.78	10.8	12.9	17.1	20.3
75.0	0.359	1.201	2.16	3.18	5.30	7.50	8.61	10.9	13.2	15.5	0.360	1.20	2.16	3.17	5.27	7.45	8.55	10.8	13.0	15.3	19.9	23.4
50.0	0.863	2.09	3.33	4.57	7.06	9.55	10.8	13.3	15.8	18.3	0.866	2.10	3.34	4.59	7.09	9.59	10.8	13.3	15.8	18.3	23.3	27.1
25.0	1.72	3.33	4.84	6.30	9.14	11.9	13.3	16.0	18.6	21.3	1.73	3.37	4.90	6.39	9.28	12.1	13.5	16.3	19.0	21.7	27.2	31.2
10.0	2.84	4.78	6.52	8.16	11.3	14.3	15.7	18.6	21.4	24.2	2.88	4.86	6.65	8.35	11.6	14.7	16.2	19.3	22.2	25.2	30.9	35.2
5.0	3.68	5.79	7.66	9.41	12.7	15.8	17.3	20.3	23.2	26.0	3.74	5.93	7.87	9.69	13.1	16.4	18.0	21.2	24.3	27.4	33.4	37.8
1.0	5.59	8.01	10.1	12.0	15.6	18.9	20.5	23.6	26.6	29.5	5.76	8.30	10.5	12.6	16.4	20.0	21.8	25.2	28.5	31.8	38.2	42.9
	0.25	1.0	1.5	2.5	4.0	X	6.5	X	10	X	0.25	1.0	1.5	2.5	4.0	X	6.5	X	10	X	15	X
	Acceptance quality limits (tightened inspection)																					

Note: Binomial distribution used for percent nonconforming computations; Poisson for nonconformities per hundred units.

Adapted from *ANSI/ASQ Z1.4-2003 Sampling Procedures and Tables for Inspection by Attributes* (Milwaukee, WI: ASQ Quality Press), 47. Used with permission.

The actual operating characteristic can be found in Table 4. The subject standards have OC curves for each code letter. The operating characteristic data are presented both graphically and tabularly.

Continuous Sampling Plans

Sampling plans defined by MIL-STD-105 and MIL-STD-414 are applicable when the lot of units to be examined is in one large collection, or *lot,* of material. Many manufacturing processes do not accumulate the items to be inspected in a lot format but rather are continuous. When production is continuous and discrete lots are formed, there are a couple of disadvantages:

1. The rejection of the lot can lead to timely reinspection of the entire lot
2. Additional space is required for the accumulated lot

Continuous sampling plans (CSPs) allow the lot to be judged on a continuous basis with no surprises upon conclusion of the lot.

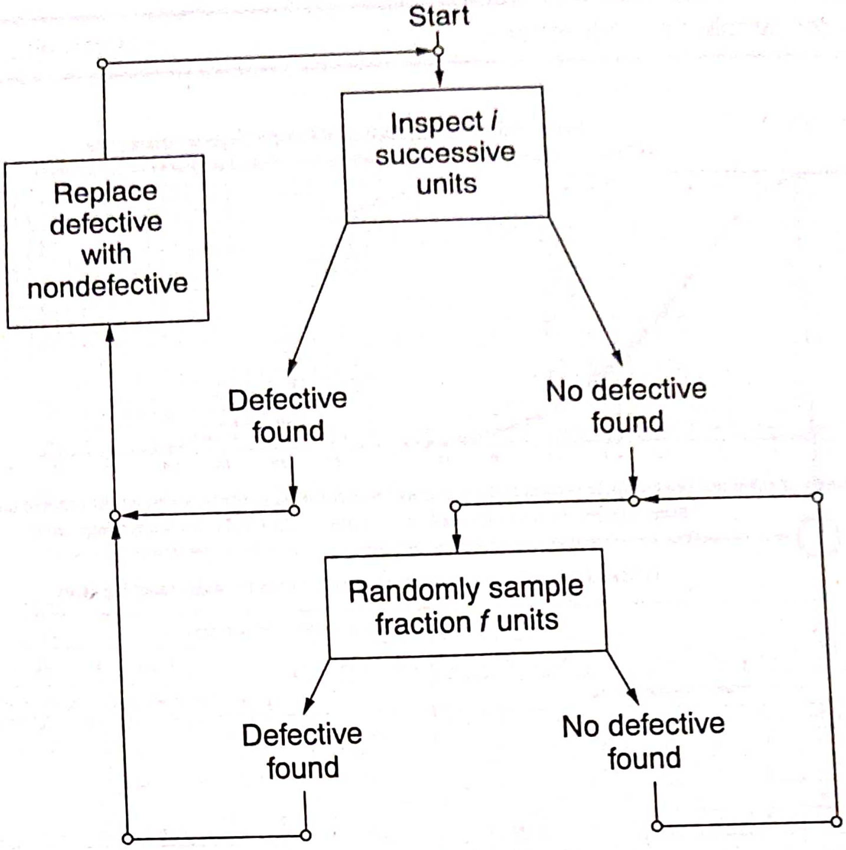


Figure 2 Procedure for CSP-1 plan.

CSP-1 Plan

The CSP-1 plan was first introduced by Harold F. Dodge in 1943 (Figure 2). Initially, 100 percent inspection is required until *i* consecutive units have been found to be satisfactory. Upon reaching the clearing number *i*, the 100 percent inspection decreases to a fraction *f* of the units. Discovery of a defective unit during the frequency inspection results in 100 percent inspection until *i* consecutive units are found, in which case interval inspection of every *f*th unit resumes. Table values for various combinations of *i* and *f* for designated average outgoing quality limits (AOQL) can be found in Table 5.

Application example:

1. An electronic assembly line produces 6500 units per day. What CSP-1 plan will yield a 0.33 percent AOQL?

Referring to the CSP-1 table, there are several choices. One plan would be to inspect 100 percent until 335 consecutive nondefective units are found, and then inspect every tenth unit. If any defective units are found during the initial 100 percent inspection, the defective unit would be replaced with a good unit, and the accumulation count would be restarted until 335 consecutive good units are found. After that, any tenth sample found defective will result in the reinstatement of the 100 percent inspection. The AOQL for this plan is 0.33 percent nonconforming.

The average number of units inspected in a 100 percent screening sequence following the occurrence of a defect is equal to

$$u = \frac{1-Q^i}{PQ^i},$$

where: P = proportion defective for process or lot
$Q = 1 - P$
i = initial clearing sample.

If the process proportion defective for the example case had been 0.80 percent, then the average number of units inspected would be

$$u = \frac{1-Q^i}{PQ^i}, \quad u = \frac{1-.992^{335}}{(.008)(.992^{335})}, \quad u = 1718.$$

The average number of units passed under CSP-1 plans before a defective unit v is found is given by

$$v = \frac{1}{fp} \quad f = 1/10 = 0.1 \text{ and } p = .008$$

$$v = \frac{1}{(0.1)(.008)} \quad v = 1250.$$

The average fraction of the total manufactured units inspected (AFI) in the long run is given by

$$\text{AFI} = \frac{u + fv}{u + v} \quad \begin{matrix} u = 1718 & f = 0.10 \\ v = 1250 & \end{matrix}$$

$$\text{AFI} = \frac{1718 + (0.1)(1250)}{1718 + 1250} \quad \text{AFI} = 0.621.$$

The average fraction of manufactured units passed under the CSP-1 plan is given by

$$Pa = \frac{v}{u + v} \quad Pa = 0.42 \text{ or } 42\%.$$

When Pa is plotted as a function of p, the OC curve for the subject plan is derived. In a traditional lot acceptance sampling plan, Pa equals the probability of accepting lots that are p proportional defective.

The OC curve for CSP-1 plans gives the probability or proportion of units passed under the subject plan as a function of process proportion defective.

Example of OC curve development:

Draw the OC curve for a CSP-1 plan where $i = 38$, $f = 1/10$, and the AOQL = 2.90.

The parameters u and v are calculated for several values of p, the process proportion defective. The proportion of units passing the sampling plan Pa is determined using the u and v values for each proportion defective p. A plot of Pa as a function of p defines the OC curve for the plan. The values for the required factors are listed in a tabular form.

P	Q	$u = \frac{1-Q^i}{PQ^i}$	$v = \frac{1}{fp}$	$Pa = \frac{v}{u+v}$
0.01	0.99	47	1000	0.9551

Calculation for $p = 0.01$:

$$Q = 1 - p \quad Q = 0.99$$

$$u = \frac{1-Q^i}{PQ^i} \quad u = \frac{1-.99^{38}}{(.01)(.99^{38})} \quad u = 47$$

$$v = \frac{1}{fp} \quad v = \frac{1}{(.10)(.01)} \quad v = 1000$$

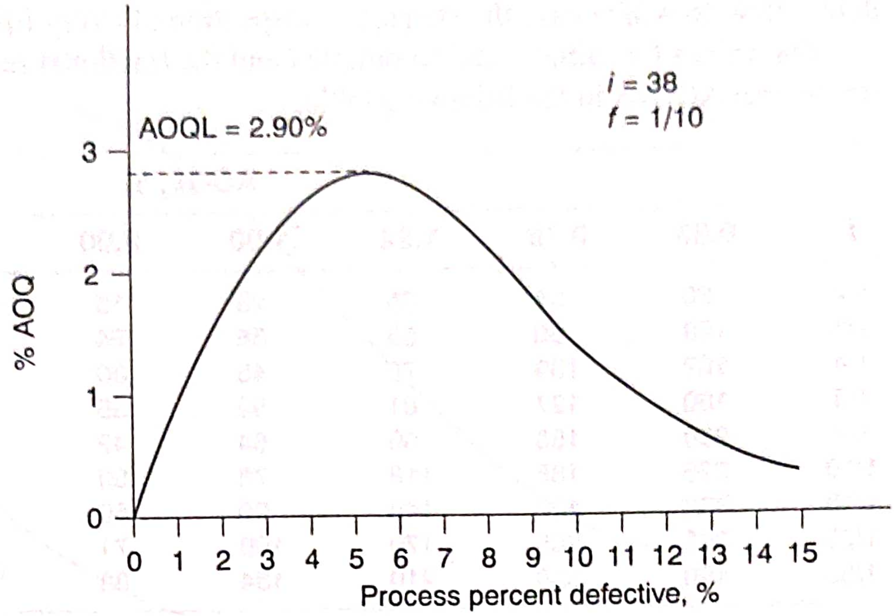
$$Pa = \frac{v}{u+v} \quad Pa = \frac{1000}{47+1000} \quad Pa = 0.9551$$

Continue the calculations of u, v, and Pa, increasing P in increments of 0.01.

P	Q	$u = \frac{1-Q^i}{PQ^i}$	$v = \frac{1}{fp}$	$Pa = \frac{v}{u+v}$
0.01	0.99	47	1000	0.9551
0.02	0.98	58	500	0.8961
0.03	0.97	73	333	0.8202
0.04	0.96	93	250	0.7289
0.05	0.95	120	200	0.6250
0.06	0.94	158	167	0.5138
0.07	0.93	211	143	0.4040
0.08	0.92	285	125	0.3049
0.09	0.91	389	111	0.2220
0.10	0.90	538	100	0.1567
0.11	0.89	753	91	0.1078
0.12	0.88	1064	83	0.0724
0.13	0.87	1521	77	0.0481
0.14	0.86	2195	71	0.0313
0.15	0.85	3200	67	0.0205

Calculating all the AOQs from $P = 1.0\%$ to 15%, we have

% *P*	% *AOQ*
1	0.86
2	1.61
3	2.22
4	2.62
5	2.81
6	2.77
7	2.54
8	2.20
9	1.80
10	1.41
11	1.07
12	0.78
13	0.56
14	0.40
15	0.28



CSP-2 Plan

As a modification to the original CSP-1 plan, the CSP-2 was developed by Dodge and Torrey (1951). Under the CSP-2 mode, 100 percent inspection is performed until *i* units are found to be defect free. If a defect is found, the inspection switches to a fraction *f* of the units. If a second defective is found during the sampling inspection of the *i* units, then 100 percent is resumed until *i*th consecutive units are found to be defect free. If no defectives are found during the sampling inspection, then the frequency inspection of a fraction *f* of the units is resumed.

The following table gives approximate values for *i* as a function of the AOQL.

	AOQL, %					
f	**0.5**	**1.0**	**2.0**	**3.0**	**4.0**	**5.0**
1/2	86	43	21	14	10	9
1/3	140	70	35	23	16	13
1/4	175	85	43	28	21	17
1/5	200	100	48	33	24	19
1/7	250	125	62	40	30	24
1/10	290	148	73	47	36	28
1/15	350	175	88	57	44	34
1/25	450	215	105	70	53	42
1/50	540	270	135	86	64	52

Example:

A 1.0 percent AOQL plan is wanted with a clearing cycle of $n = 100$. Because 100 units are inspected and found to be defect free, every fifth unit is now inspected. During this frequency inspection, a defective is found. If during the next 100 consecutive units a second defective is found, 100 percent inspection is required until 100 consecutive units are found defect free, in which case the frequency inspection of every fifth unit is reinstated.

The values for initial clearing sample i and the fractional sample frequency f are given for several AOQLs in the following table.

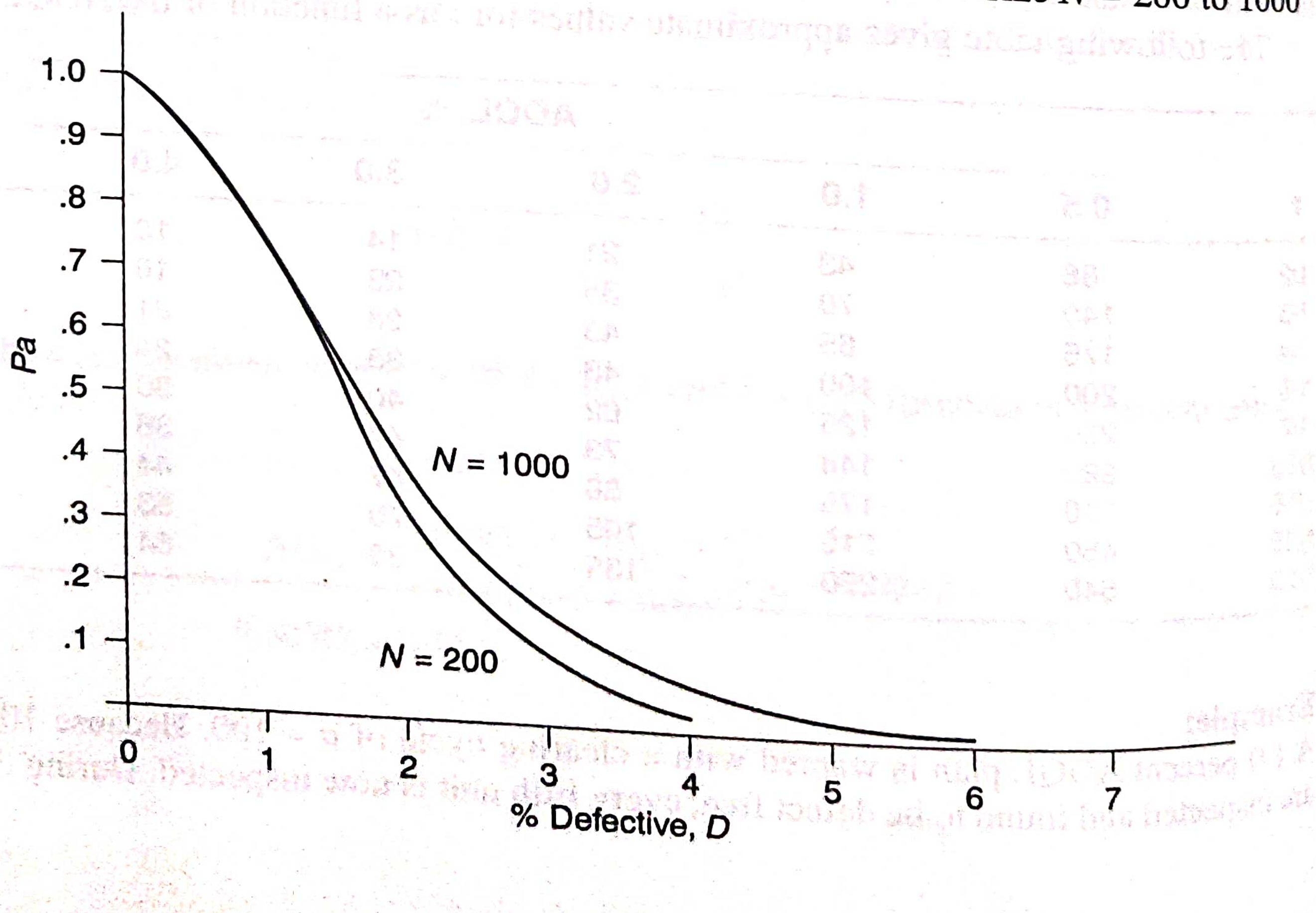
	AOQL, %							
f	0.53	0.79	1.22	1.90	2.90	4.94	7.12	11.46
1/2	80	54	35	23	15	9	7	4
1/3	128	86	55	36	24	14	10	7
1/4	162	109	70	45	30	18	12	8
1/5	190	127	81	52	35	20	14	9
1/7	230	155	99	64	42	25	17	11
1/10	275	185	118	76	50	29	20	13
1/15	330	220	140	90	59	35	24	15
1/25	395	265	170	109	71	42	29	18
1/50	490	330	210	134	88	52	36	22

Effects of Lot Size N, Sample Size n, and Acceptance Criteria C on the OC Curve

Traditional acceptance sampling plans such as ANSI Z1.4 and Z1.9 specify sampling plans as a function of the lot size. While lot size influences the overall OC curve, it is not the most contributing factor with respect to the shape of the OC curve. Other factors such as the sample size and the acceptance criteria C (the number of defective units, at which the lot will be accepted) are equally or more contributory.

Consider the following OC curves, where the lot size, sample size, and acceptance criteria are varied.

Case I: Constant $C = 1$ and constant $n = 100$ with variable lot size $N = 200$ to 1000



ation as related to the specification requirement. The following inspection plans are those covered by MIL-STD-414, sampling plans for variable data.

Single Specification Limit, Form I, Variability Unknown, Standard Deviation Method

Levels of inspection determine the sample size as a function of the submitted lot. Levels of inspection are directly related to the ratio of the acceptable quality limit (AQL) and the repeatable quality level (RQL). The closer the RQL is to the AQL, the more discriminating the sample plan. Levels available are

$$S3 \rightarrow S4 \rightarrow I \rightarrow II \rightarrow III$$

Increasing discrimination →.

For a given lot size and level of inspection, the sample size is determined and designated as a letter B through P.

Lot size		S3	S4	I	II	III	Letter	Sample size
2	to 8	B	B	B	B	C	B	3
9	to 15	B	B	B	B	D	C	4
16	to 25	B	B	B	C	E	D	5
26	to 50	B	B	C	D	F	E	7
51	to 90	B	B	D	E	G	F	10
91	to 150	B	C	E	F	H	G	15
151	to 280	B	D	F	G	I	H	20
281	to 400	C	E	G	H	J	I	25
401	to 500	C	E	G	I	J	J	35
501	to 1200	D	F	H	J	K	K	50
1201	to 3200	E	G	I	K	L	L	75
3201	to 10,000	F	H	J	L	M	M	100
10,001	to 35,000	G	I	K	M	N	N	150
35,001	to 150,000	H	J	L	N	P	P	200
150,001	to 500,000	H	K	M	P	P		
≥500,001		H	K	N	P	P		

Note: There is no letter "O."

Step 1. Select a level of inspection and sample size.

A lot size of 80 rods has been received. For a general level II, what will the sample size be?

Lot size		S3	S4	I	II	III	Letter	Sample size
2	to 8	B	B	B	B	C	B	3
9	to 15	B	B	B	B	D	C	4
16	to 25	B	B	B	C	E	D	5
26	to 50	B	B	C	D	F	*E*	7
51	~~to 90~~	~~B~~	~~B~~	~~D~~	*E*	G	F	10
91	to 150	B	C	E	F	H	G	15
151	to 280	B	D	F	G	I	H	20
281	to 400	C	E	G	H	J	I	25
401	to 500	C	E	G	I	J	J	35
501	to 1200	D	F	H	J	K	K	50
1201	to 3200	E	G	I	K	L	L	75
3201	to 10,000	F	H	J	L	M	M	100
10,001	to 35,000	G	I	K	M	N	N	150
35,001	to 150,000	H	J	L	N	P	P	200
150,001	to 500,000	H	K	M	P	P		
≥500,001		H	K	N	P	P		

Note: There is no letter "O."

Sample size E = 7

Step 2. Select the sample and calculate the average and sample standard deviation.

Seven samples are chosen and measured. The characteristic for this example is the diameter of the rods.

0.503
0.502
0.503
0.504
0.505
0.501
0.503

$\overline{X} = 0.503$

$S = 0.001$

Step 3. Select an AQL and look up the appropriate k value from Table 1.

The specification for this example is a minimum diameter of 0.500 or LSL of 0.500, and the selected AQL is 2.5 percent.

k for sample size letter E (sample size $n = 7$) and a 2.5 percent AQL is 1.33.

Step 4. Calculate the critical decision factor (CDF).

For lower specifications:
$CDF_L = \overline{X} - kS$

For upper specifications:
$CDF_U = \overline{X} + kS$

For this example, where there is a lower specification

$CDF_L = \overline{X} - kS$ $\quad CDF_L = 0.503 - (1.33)(0.001)$ $\quad CDF_L = 0.502$

Step 5. Confront the CDF with the acceptance criteria.

For lower specifications:

If CDF_L is less than the lower specification, reject the lot.
If CDF_L is greater than the lower specification, accept the lot.

For upper specifications:

If CDF_U is greater than the upper specification, reject the lot.
If CDF_U is less than the upper specification, accept the lot.

For this example, CDF_L is greater than the lower specification; therefore, the lot is accepted.

Problem:

A lot of 200 units has been received. The specification is 25.0 maximum. Perform a level II, AQL = 1.00 inspection using the single specification, form 1 (k method).

Randomly select the sample from the 200 values listed below.

21	20	22	23	17	20	22	21	21	14	19	22	19	19	20	25	18	19	25	19
19	23	12	23	20	25	21	17	22	18	23	19	21	22	20	18	23	17	19	20
16	22	20	22	23	21	17	14	18	20	24	21	18	20	24	14	17	19	20	22
18	19	23	22	21	21	20	16	24	20	18	15	20	21	22	18	18	19	21	18
20	22	21	20	17	18	21	15	19	19	17	22	19	19	22	21	16	18	18	24
16	18	20	20	23	21	27	15	19	20	22	23	22	21	26	23	19	21	19	23
20	24	16	21	22	23	19	22	15	14	21	19	18	20	20	23	23	17	17	20
18	20	20	22	19	18	19	21	20	17	24	17	19	23	20	20	17	15	19	16
21	15	17	22	22	19	21	21	16	23	24	20	22	14	18	20	19	21	20	21
20	23	20	22	20	24	14	17	18	19	21	23	19	22	19	20	20	20	22	19

Sampling Plans Based on Specified AQL, RQL, α Risk, and β Risk

The sample size n and the k factor can be estimated using the following relationships:

$$k = \frac{Z_{P_2} Z_\alpha + Z_{P_1} Z_\beta}{Z_\alpha - Z_\beta} \quad \text{and} \quad n = \left(\frac{Z_\alpha + Z_\beta}{Z_{P_1} - Z_{P_2}} \right)^2 \left(1 + \frac{k^2}{2} \right)$$