Engineering. The creation and development of a product is basically engineering; the development of quality evaluation through improved inspection procedures is also engineering ; again, the knowledge of causes of defects and sub-standard products and their rectification is engineering.

Statistical. The concept of the behaviour of a process, which has brought in the idea of 'prevention' and 'control', is statistical ; building an information system to satisfy the concept of 'prevention' and 'control' and improving upon product quality, requires statistical thinking.

Managerial. The competent use of the engineering and statistical technology is managerial ; the creation of a climate for quality consciousness in the organisation depends upon the policies and practices of the management ; the effective coordination of the quality control function with those of others is managerial.

1·2. BASIS OF STATISTICAL QUALITY CONTROL

The basis of statistical quality control is the degree of '*variability*' in the size or the magnitude of a given characteristic of the product. Variation in the quality of manufactured product in the repetitive process in industry is inherent and inevitable. These variations are broadly classified as being due to two causes, *viz*., (*i*) *chance causes,* and (*ii*) *assignable causes.*

Chance Causes. *Some "stable pattern of variation"* or *"a constant cause system"* is inherent in any particular scheme of production and inspection. This pattern results from many minor causes that behave in a random manner. The variation due to these causes is beyond the control of human hand and cannot be prevented or eliminated under any circumstances. One has got to allow for variation within this stable pattern, usually termed as *allowable variation*. The range of such variation is known as 'natural tolerance of the process'.

Assignable Causes. The second type of variation attributed to any production process is due to non-random or the so-called assignable causes and is termed as *preventable variation.* The assignable causes may creep in at any stage of the process, right from the arrival of the raw materials to the final delivery of goods. Some of the important factors of assignable causes of variation are sub-standard or defective raw materials, new techniques or operations, negligence of the operators, wrong or improper handling of machines, faulty equipment, unskilled or inexperienced technical staff, and so on. These causes can be identified and eliminated and are to be discovered in a production process before it goes wrong, *i.e.,* before the production becomes defective.

Chance causes of variation	*Assignable causes of variation*
(*i*) *Consist* of many individual causes.	*Consist* of just a few individual causes.
(*ii*) Any one chance cause results in only a small amount of variation.	Any one assignable cause can result in a large amount of variation.
(*iii*) Chance variation cannot economically be eliminated from a process.	The presence of assignable variation can be detected, and action to eliminate the causes is usually economically justified.
(*iv*) Some typical chance causes of variation are : — Slight vibration of a machine. — Lack of human perfection in reading instruments and setting controls. — Voltage fluctuations and variation in temperatures.	Some typical assignable causes of variation are : —Negligence of operators. —Defective raw material. —Faulty equipment. —Improper handling of machines.

S.Q.C. means planned collection and effective use of data for studying causes of variations in quality either as between processes, procedures, materials, machines, etc., or over periods of time. This cause-effect analysis is then fed back into the system with a view to continuous action on the processes of handling, manufacturing, packaging, transporting and delivery at end-use.

The main purpose of *Statistical Quality Control (S.Q.C.)* is to devise statistical techniques which would help us in separating the assignable causes from the chance causes, thus enabling us to take immediate remedial action whenever assignable causes are present. The elimination of assignable causes of erratic fluctuations is described as bringing a process under control.

> *A production process is said to be in a state of statistical control, if it is governed by chance causes alone, in the absence of assignable causes of variation.*

S.Q.C. is a productivity enhancing and regulatory technique (PERT) with three factors—Management, Methods and Mathematics. More particularly, if we want to properly design a self-regulating system for quality, we must look to the field of cybernetics for design information. There are six elements to a cybernetic or self-regulating system :

1. Management
2. Standard/Specification
3. Measurement/Comparison
4. Action on process/product/system
5. Information (Feedback System/Quality Information Service)
6. R & D with two objectives :
   (*a*) better quality standard or more economical quality standard.
   (*b*) better means for achieving the standard.

It is important to note that if any element of the system is missing or mismatched, it will not function. Further, control is two-fold—controlling the process (process control) and controlling the finished products (product control). It is expected that items from a controlled process should have a higher degree of conformity to specifications.

1·3. STATISTICAL QUALITY CONTROL (DEFINITION)

A few definitions are being reproduced below to make the term understandable :

> ❶ *"S.Q.C. may be broadly defined as that industrial management technique by means of which product of uniform acceptable quality are manufactured. It is mainly concerned with setting things right rather than discovering and rejecting those made wrong."*—**Duncan**
>
> ❷ *"S.Q.C. refers to the systematic control of those variables encountered in a manufacturing process which affect the excellence of the end product. Such variables are from the application of materials, men, machines and manufacturing conditions."*
>
> —**Bethel, Atwater and Stackman**
>
> ❸ *"Statistical Quality Control is simply a statistical method for determining the extent to which quality goals are being met without necessarily checking every item produced and for indicating whether or not the variations which occur are exceeding normal expectations. S.Q.C. also enables us to decide whether to reject or accept a particular product."* —**Grant**

1.4. BENEFITS OF STATISTICAL QUALITY CONTROL

The following are some of the benefits that result when a manufacturing process is operating in a state of statistical control :

❶ An obvious advantage of S.Q.C. is the control, maintenance and improvement in the quality standards.

❷ The act of getting a process in statistical quality control involves the identification and elimination of assignable causes of variation and possibly the inclusion of good ones, *viz.,* new material or methods. This (*a*) helps in the detection and correction of many production troubles, and (*b*) brings about a substantial improvement in the product quality and reduction of spoilage and rework.

❸ It tells us when to leave a process alone and when to take action to correct troubles, thus preventing frequent and unwarranted adjustments.

❹ If a process in control (which is doing about all we can expect of it) is not good enough, we shall have to make more or less a radical (fundamental) change in the process—just meddling (tampering) with it won't help.

❺ A process in control is predictable—we know what it is going to do and thus we can more safely guarantee the product. In the presence of good statistical control by the supplier, the previous lots supply evidence on the present lots, which is not usually the case if the process is not in control.

❻ If testing is destructive (*e.g.,* testing the breaking strength of chalk ; proofing of ammunition, explosives. crackers, etc.), a process in control gives confidence in the quality of untested product which is not the case otherwise.

❼ It provides better quality assurance at lower inspection cost.

❽ Quality control finds its applications not only in the sphere of production, but also in other areas like packaging, scrap and spoilage, recoveries, advertising, etc. Foreign trade items of developing countries like India are particularly appropriate for every type of quality control in every possible area.

❾ The very presence of a quality control scheme in a plant improves and alerts the personnel. Such a scheme is likely to breed 'quality consciousness' throughout the organisation which is of immense long-run value.

❿ S.Q.C reduce waste of time and material to the absolute minimum by giving an early warning about the occurrence of defects. Savings in terms of the factors stated above mean less cost of production and hence may ultimately lead to more profits.

Remarks **1.** An SQC department is, thus, an essential part of a modern plant, and its important functions are as follows :

(*i*) Evaluation of quality standards of incoming materials, products in process and of finished goods.

(*ii*) Judging the conformity of the process to established standards and taking suitable action when deviations are noted.

(*iii*) Evaluation of optimum quality obtainable under given conditions.

(*iv*) Improvement of quality and productivity by process control and experimentation.

2. The following diagram [Fig. 1·1] gives a summary of the advantages of quality control in industry :

A substantial increase in productivity, basic savings in costs, and improvement in quality of products are attainable in any industry by means of the application of quality control techniques.

The basic pre-requisites for successful quality control applications are :

(a) an alert and progressive management,

(b) competent technical staff, searching for new methods and new economies,

(c) competitive or social pressures for technical advancement, and

(d) the training of top management staff in new statistical methods.

QUALITY CONTROL APPROACH AND BENEFITS

Planned collection of data, analysis and interpretation

↓

Improvement in product quality and design
Reduction in operating costs and losses
Reduction in scrap
Saving in excess use of materials
Removing production bottlenecks
Reduction in inspection
Evaluation of scientific tolerances
Improvement of employee morale
Maintenance of operating efficiency quality consciousness
Greater customer satisfaction

↓

ENHANCED PRODUCTIVITY

Fig. 1·1.

3. The increasing role of computers in process control and acceptance decisions not only reduces time lags, but additionally makes possible certain cost reductions. The reduced calculations that were formerly required to manually prepare control charts and make acceptance plan decisions have freed up quality control personnel to do other duties. In some cases the number of these personnel has actually been reduced. Inventory levels may be reduced because fewer lots are in quality control holding areas waiting for disposition. Reduced inventories can reduce holding costs, free up capital funds, and promote increased operating efficiency through reduced congestion.

The growth of computers in quality control departments has been spearheaded by a growing list of computer companies and computer software companies which provide standard quality control computer programs. These special programs are usually an integral part of larger management information systems and thus involve little out-of-pocket cost to users.

1·5. PROCESS CONTROL AND PRODUCT CONTROL

As already stated the main objective in any production process is to control and maintain a satisfactory quality level of the manufactured product so that it conforms to specified quality standards. In other words, we want to ensure that the proportion of defective items in the manufactured product is not too large. This is termed as '*process control*' and is achieved through the technique of '*Control Charts*' pioneered by W.A. Shewhart in 1924.

On the other hand, by *product control* we mean controlling the quality of the product by critical examination at strategic points and this is achieved through '*Sampling Inspection Plans*' pioneered by H.F. Dodge and H.C. Romig. Product control aims at guaranteeing a certain quality level to the consumer regardless of what quality level is being maintained by the producer. In other words, it attempts to ensure that the product marketed by sale department does not contain a large number of defective (unsatisfactory) items. Thus, product control is concerned with classification of raw materials, semi-finished goods or finished goods into acceptable or rejectable items.

The following diagram [Fig. 1·2] summarizes the techniques of S.Q.C. :

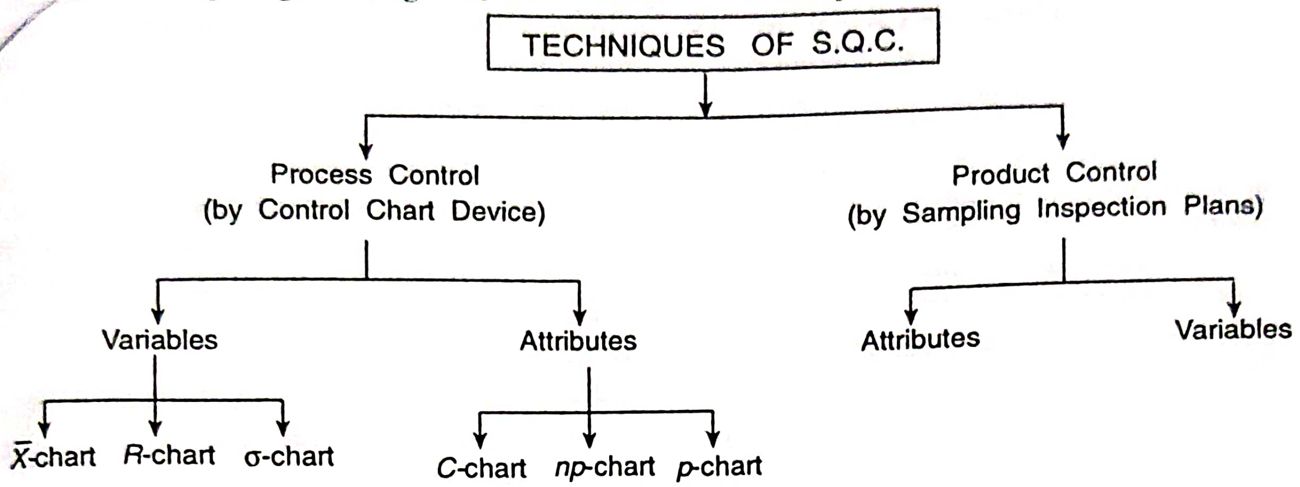


Fig. 1·2

1·5·1. Control Limits, Specification Limits and Tolerance Limits

1. *Control Limits.* These are limits of sampling variation of a statistical measure (*e.g.*, mean, range, or fraction-defective) such that if the production process is under control, the values of the measure calculated from different rational sub-groups will lie within these limits. Points falling outside control limits indicate that the process is not operating under a system of chance causes, *i.e.*, assignable causes of variation are present, which must be eliminated. Control limits are used in 'Control Charts'.

2. *Specification Limits.* When an article is proposed to be manufactured, the manufacturers have to decide upon the maximum and the minimum allowable dimensions of some quality characteristics so that the product can be gainfully utilised for which it is intended. If the dimensions are beyond these limits, the product is treated as defective and cannot be used. These maximum and minimum limits of variation of individual items, as mentioned in the product design, are known as '*specification limits*'.

3. *Tolerance Limits.* These are limits of variation of a quality measure of the product between which at least a specified proportion of the product is expected to lie (with a given probability), provided the process is in a state of statistical quality control. For example, we may claim with a probability of 0·99 that at least 90% of the products will have dimensions between some stated limits. These limits are also known as '*statistical tolerance limits*'.

Remark. The three wings of 'specification', 'production' and 'inspection' often display a very poor appreciation of one another's problems. Engineer, who prepare the specifications, often complain of poor quality ; the production wing grows dissatisfied with stringency of specifications and unnecessary rejections by the inspection wing and the inspection personnel complain not only about the poor quality of the manufactured products but also about the unreasonableness of the specified tolerances. Quality control techniques can be said to provide the necessary data and thereby a basis on which these three wings can discuss the common problems and reach agreements based on mutual understanding.

1·6. CONTROL CHARTS

The epoch-making discovery and development of control charts was made by a young physicist, Dr. Walter A. Shewart of Bell Telephone Laboratories, in 1924 and the following years. Based on the theory of probability and sampling, Shewhart's control charts provide a powerful tool of discovering and correcting the assignable causes of variation outside the 'stable pattern' of chance causes, thus enabling us to stabilize and control our processes at desired performances and thus bring the process under statistical control.

In industry one is faced with two kinds of problems : (*i*) to check whether the process is conforming to standards laid down, and (*ii*) to improve the level of standard and reduce variability consistent with cost considerations. Shewhart's control charts provide an answer to both. Control chart, as conceived and devised by Shewhart, is a simple pictorial device for detecting unnatural patterns of variations in data resulting from repetitive processes *i.e.*, control charts provide criteria for detecting lack of statistical control. Control charts are simple to construct and easy to interpret and tell us at a glance whether the sample point falls within 3-σ control limits (discussed below) or not. Any sample point going outside the 3-σ control limits is an indication of the lack of statistical control, *i.e.*, presence of some assignable causes of variation which must be traced, identified and eliminated.

A typical control chart consists of the following three horizontal lines :

(*i*) A Central Line (C.L.), indicating the desired standard or the level of the process.

(*ii*) Upper Control Limit (U.C.L.), indicating the upper limit of tolerance,

(*iii*) Lower Control Limit (L.C.L.), indicating the lower limit of tolerance

The control line as well as the upper and lower limits are established by computations based on the past records or current production records.

Major Parts of a Control Chart. A control chart generally includes the following four major parts :

1. *Quality Scale.* This is a vertical scale. The scale is marked according to the quality characteristics (either in variables or in attributes) of each sample.

2. *Plotted Samples.* The qualities of individual items of a sample are not shown on a control chart. Only the quality of the entire sample represented by a single value (a statistic) is plotted. The single value plotted on the chart is in the form of a dot (sometimes a small circle or a cross).

3. *Sample (or Sub-group) Numbers.* The samples plotted on a control chart are numbered individually and consecutively on a horizontal line. The line is usually placed at the bottom of the chart. The samples are also referred to as sub-groups in statistical quality control. Generally 25 sub-groups are used in constructing a control chart.

4. *The Horizontal Lines.* The central line represents the average quality of the samples plotted on the chart. The line above the central line shows the upper control limit (*UCL*) which is commonly obtained by adding 3 sigma's to the average, *i.e.*, Mean + 3(S.D.). The line below the central line is the lower control limit (L.C.L.) which is obtained by subtracting 3 sigmas from the average, *i.e.*, Mean – 3(S.D.). The upper and lower control limits are usually drawn as dotted lines, and the central line is plotted as a bold (dark) line.

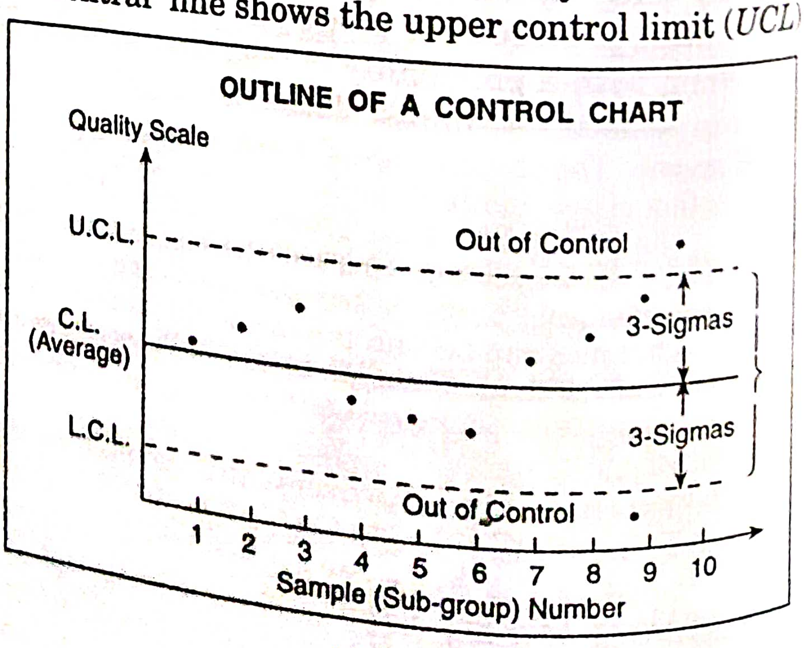


Fig. 1·3

The adjoining diagram (*Fig.* 1·3) depicts the principle of Shewhart's control chart.

In the control chart, Upper Control Limit (U.C.L.) and Lower Control Limit

(L.C.L.) are usually plotted as dotted lines and central line (C.L.) is plotted as a bold (dark) line. If t is the underlying statistic then these values depend on the sampling distribution of t and are given by :

$$\text{U.C.L.} = E(t) + \text{S.E. }(t)$$
$$\text{L.C.L.} = E(t) - 3\text{ S.E.}(t)$$
$$\text{C.L.} = E(t)$$

1·6·1. 3σ Control Limits. 3-σ limits were proposed by **Dr. Shewhart** for his control charts from various considerations, the main being probabilistic considerations. Consider the statistic $t = t(x_1, x_2, ..., x_n)$, a function of the sample observations $x_1, x_2, ..., x_n$. Let

$$E(t) = \mu_t \quad \text{and} \quad \text{Var}(t) = \sigma_t^2$$

If the statistic t is normally distributed, then from the fundamental area property of the normal distribution, we have

$$P[\mu_t - 3\sigma_t < t < \mu_t + 3\sigma_t] = 0{\cdot}9973 \Rightarrow P[|t - \mu_t| < 3\sigma_t] = 0{\cdot}9973 \text{ i.e., } P[|t - \mu_t| > 3\sigma_t] = 0{\cdot}0027$$

In other words, the probability that a random value of t goes outside the 3-σ limits, *viz.*, $\mu_t \pm 3\sigma_t$ is 0·0027, which is very small. Hence, if t is normally distributed, the limits of variation should be between $\mu_t + 3\sigma_t$ and $\mu_t - 3\sigma_t$ which are termed respectively the *Upper Control Limit* (U.C.L.) and *Lower Control Limit* (L.C.L.). If, for the ith sample, the observed t_i lies between the upper and lower control limits, there is nothing to worry as in such a case variation between samples is attributed to chance *i.e.*, in this case the process is in statistical control. It is only when any observed t_i falls outside the control limits, it is considered to be a danger signal indicating that some assignable cause has crept in which must be identified and eliminated.

***Remarks* 1.** If the assumption regarding normality of the statistic t does not hold, then the above argument does not remain strictly valid. In practice, the quality characteristic can seldom be supposed to be exactly normal. For non-normal population, (*i.e.*, if the sampling distribution of statistic t is not normal) we apply *Chebychev's Inequality* in probability theory which states that for any constant $k > 0$,

$$P[|t - E(t)| < k] \geq 1 - \frac{\text{Var }(t)}{k^2} \quad \Rightarrow \quad P[|t - \mu_t| < 3\sigma_t] \geq 1 - \frac{\sigma_t^2}{9\sigma_t^2} = \frac{8}{9} \simeq 0{\cdot}9,$$

which is also fairly high for practical purposes and the above argument holds, more or less. However, in practice, σ_t is not known and is estimated from the sample data and consequently Chebychev's inequality does not hold if σ_t is not known.

Moreover, according to the central limit theorem in probability, the statistics of observations drawn from non-normal populations will exhibit nearly normal behaviour. Of course, such behaviour will be more closely normal, the closer the underlying population is to a normal distribution, but the principle applies in any case.

Hence, even for non-normal population, 3-σ limits are almost universally used, as they have been found to be most suitable empirically in the sense that the 3-σ control charts have been found to give excellent protection against both types of wrong actions we can take, *viz.*, looking for trouble when there is none and not looking for trouble when there really is one.

2. If none of the sample points falls outside the control limits and if there is no evidence of non-random variation within the limits, it does not imply the absence of assignable causes altogether. All we can infer is that the hypothesis of random variations alone is reasonable one and from management point of view, looking for special assignable causes at this stage is unlikely to be profitable.

3. It has been emphasised strongly by Dr. Shewhart that a production process should not be adjudged in statistical control unless the random variation pattern persists for quite some time and for a sizable volume of output. More specifically he states :

"This potential state of economic control can be approached only as a statistical limit even after the assignable causes of variability have been detected and removed. Control of this kind cannot be reached

in a day. It cannot be reached in the production of a product in which only a few pieces are manufactured. It can, however, be approached scientifically in a continuing mass production."

Usually, a process should be considered in statistical control if the pattern of random variation is exhibited by a sequence of not less than twenty-five samples, each of size four.

1·7. TOOLS FOR S.Q.C.

The following four, separate but related techniques, are the most important statistical tools for data analysis in quality control of the manufactured products :

1. *Shewhart's Control Chart for Variables i.e.,* for a characteristic which can be measured quantitatively. Many quality characteristics of a product are measurable and can be expressed in specific units of measurements such as diameter of a screw, tensile strength of steel pipe, specific resistance of a wire, life of an electric bulb, etc. Such variables are of continuous type and are regarded to follow normal probability law. For quality control of such data, two types of control charts are used and technically these charts are known as :

   (*a*) *Charts for* $\overline{X}$ *(mean) and R (Range)*, and

   (*b*) *Charts for* $\overline{X}$ *(Mean) and* σ *(standard deviation).*

2. *Shewhart's Control Chart for fraction Defective or p-Chart.* This chart is used if we are dealing with attributes in which case the quality characteristics of the product are not amenable to measurement but can be identified by their absence or presence from the product or by classifying the product as defective or non-defective.

3. *Shewhart's Control Chart for the 'Number of Defects' per unit or c-Chart.* This is usually used with advantage when the characteristic representing the quality of a product is a discrete variable, *e.g.*, (*i*) the number of defective rivets in an aircraft wing, and (*ii*) the number of surface defects observed in a roll of coated paper or a sheet of photographic film.

4. The portion of the sampling theory which deals with the quality protection given by any specified sampling acceptance procedure.

1·8. CONTROL CHARTS FOR VARIABLES

These charts may be applied to any quality characteristic that is measurable. In order to control a measurable characteristic we have to exercise control on the measure of location as well as the measure of dispersion. Usually $\overline{X}$ and R charts are employed to control the mean (location) and standard deviation (dispersion) respectively of the characteristic.

1·8·1. $\overline{X}$ and R Charts. No production process is perfect enough to produce all the items exactly alike. Some amount of variation, in the produced items, is inherent in any production scheme. This variation is the totality of numerous characteristics of the production process *viz.*, raw material, machine setting and handling, operators, etc. As pointed out earlier, this variation is the result of (*i*) chance causes, and (*ii*) assignable causes. The control limits in the $\overline{X}$ and R charts are so placed that they reveal the presence or absence of assignable causes of variation in the

(*a*) average—mostly related to machine setting, and

(*b*) range—mostly related to negligence on the part of the operator

Steps for $\overline{X}$ and R Charts

1. *Measurement.* Actually the work of a control chart starts first with measurements. Any method of measurement has its own inherent variability. Errors in measurement can enter

X ~ N(μ, σ²)

X ~ Non normal

into the data by :

(*i*) the use of faulty instruments,

(*ii*) lack of clear-cut definitions of quality characteristics and the method of taking measurements, and

(*iii*) lack of experience in the handling or use of the instrument, etc.

Since the conclusions drawn from control chart are broadly based on the variability in the measurements as well as the variability in the quality being measured, it is important that the mistakes in reading measurement instruments or errors in recording data should be minimised so as to draw valid conclusions from control charts.

2. *Selection of Samples or Sub-groups.* In order to make the control chart analysis effective, it is essential to pay due regard to the rational selection of the samples or sub-groups. The choice of the sample size n and the frequency of sampling, *i.e.*, the time between the selection of two groups, depend upon the process and no hard and fast rules can be laid down for this purpose. Usually n is taken to be 4 or 5 while the frequency of sampling depends on the state of the control exercised. Initially more frequent samples will be required (15 to 30 minutes) and once a state of control is maintained, the frequency may be relaxed. Normally 25 samples of size 4 each or 20 samples of size 5 each under control will give good estimate of the process average and dispersion.

Remark. While collecting data it may not be necessary to go exactly at the specified time, in fact this should not be practised. This is to avoid (*i*) the operative being careful at the time of sampling, or (*ii*) any periodicities of the process to coincide with sampling.

3. *Calculation of $\overline{X}$ and R for each Sub-group.* Let X_{ij}, $j = 1, 2, \ldots, n$ be the measurements on the ith sample ($i = 1, 2, \ldots, k$). The mean $\overline{X}_i$, the range R_i and the standard deviation s_i for the ith sample are given by :

$$\overline{X}_i = \frac{1}{n}\sum_j X_{ij}\ ,\quad R_i = \max_j X_{ij} - \min_j X_{ij}\ ,\quad s_i^2 = \frac{1}{n}\sum_j (X_{ij} - \overline{X}_i)^2 \quad (i = 1, 2, \ldots, k) \qquad \ldots(1{\cdot}1)$$

Next we find $\overline{\overline{X}}$, $\overline{R}$ and $\overline{s}$, the averages of sample means, sample ranges and sample standard deviations, respectively, as, follows :

$$\overline{\overline{X}} = \frac{1}{k}\sum_i \overline{X}_i\ ,\qquad \overline{R} = \frac{1}{k}\sum_i R_i\ ,\qquad \overline{s} = \frac{1}{k}\sum_i s_i \qquad \ldots(1{\cdot}2)$$

4. *Setting of Control Limits.* It is well known that if σ is the process standard deviation (standard deviation of the universe from which samples are taken), then the standard error of sample mean is $\sigma/\sqrt{n}$, where n is the sample size, *i.e., S.E.* $(\overline{X}_i) = \sigma/\sqrt{n}$, $(1, 2, \ldots, k)$.

Also from the sampling distribution of range, we know that

$$E(R) = d_2 . \sigma,$$

where d_2 is a constant depending on the sample size. Thus an estimate of σ can be obtained from $\overline{R}$ by the relation :

$$\overline{R} = d_2 . \sigma \quad \Rightarrow \quad \hat{\sigma} = \overline{R}/d_2 \qquad \ldots(1{\cdot}3)$$

Also $\overline{\overline{X}}$ gives an unbiased estimate of the population mean μ, since

$$E\left(\overline{\overline{X}}\right) = \frac{1}{k}\sum_{i=1}^{k} E(\overline{X}_i) = \frac{1}{k}\sum_{i=1}^{k} \mu = \mu$$

Control Limits for $\overline{X}$-chart :

Case 1. *When standards are given, i.e.,* both μ and σ are known. The 3-σ control limits for $\overline{X}$ chart are given by : $\quad E(\overline{X}) \pm 3\ S.E.\ (\overline{X}) = \mu \pm 3\sigma/\sqrt{n} = \mu \pm A\sigma$, $(A = 3/\sqrt{n})$.

If μ' and σ' are known or specified values of μ and σ respectively, then

$$UCL_{\overline{X}} = \mu' + A\sigma' \quad \text{and} \quad LCL_{\overline{X}} = \mu' - A\sigma' \qquad \ldots(1\cdot4)$$

where $A\ (= 3/\sqrt{n})$ is a constant depending on n and its values are tabulated for different values of n from 2 to 25 in Table VIII in the Appendix.

Case 2. *Standards not given.* If both μ and σ are unknown, then using their estimates $\overline{\overline{X}}$ and $\hat{\sigma}$ given in Eqns. (1·2) and (1·3) respectively, we get the 3-σ control limits for the $\overline{X}$-chart as :

$$\overline{\overline{X}} \pm 3\,\frac{\overline{R}}{d_2}\cdot\frac{1}{\sqrt{n}} = \overline{\overline{X}} \pm \left(\frac{3}{d_2\sqrt{n}}\right)\overline{R} = \overline{\overline{X}} \pm A_2\overline{R}, \left(A_2 = \frac{3}{d_2\sqrt{n}}\right)$$

$$\therefore \quad UCL_{\overline{X}} = \overline{\overline{X}} + A_2\overline{R} \quad \text{and} \quad LCL_{\overline{X}} = \overline{\overline{X}} - A_2\overline{R} \qquad \ldots(1\cdot4a)$$

Since d_2 is a constant depending on n, $A_2 = 3/(d_2\sqrt{n})$ also depends only on n and its values have been computed and tabulated for different values of n from 2 to 25 and are given in the Table at the end of the chapter.

If, on the other hand, the control limits are to be obtained in terms of $\overline{s}$ rather than $\overline{R}$, then an estimate of σ can be obtained from the relation. [See Remarks 1 and 2, § 1·8·4, on page 1·17] : $\qquad \ldots(1\cdot4b)$

$$E(s) = C_2\,\sigma \quad \Rightarrow \quad \overline{s} = C_2\,\sigma \quad i.e., \quad \hat{\sigma} = \overline{s}/C_2$$

where
$$C_2 = \sqrt{\frac{2}{n}}\cdot\frac{\left(\frac{n-2}{2}\right)!}{\left(\frac{n-3}{2}\right)!}, \text{ is a constant depending on } n.$$

$$\therefore \quad UCL_{\overline{X}} = \overline{\overline{X}} + \left(\frac{3}{\sqrt{n}\,C_2}\right)\overline{s} = \overline{\overline{X}} + A_1\overline{s} \quad \text{and} \quad LCL_{\overline{X}} = \overline{\overline{X}} - \left(\frac{3}{\sqrt{n}\,C_2}\right)\overline{s} = \overline{\overline{X}} - A_1\overline{s}$$

The factor $A_1 = 3/\left(\sqrt{n}\ C_2\right)$ has been tabulated for different values of n from 2 to 25 in Table at the end of the chapter. $\qquad \ldots(1\cdot4c)$

Control Limits for R-chart. R-Chart is constructed for controlling the variation in the dispersion (variability) of the product. The procedure of constructing R-chart is similar to that for the $\overline{X}$-chart and involves the following steps :

1. Compute the range $R_i = \max_j X_{ij} - \min_j X_{ij}$, $(i = 1, 2, \ldots, n)$ for each sample .

2. Compute the mean of the sample ranges :

$$\overline{R} = \frac{1}{k}\sum_{i=1}^{k} R_i = \frac{1}{k}\left(R_1 + R_2 + \ldots + R_k\right).$$

3. ***Computation of Control Limits.*** The 3-σ control limits for R-chart are : $E(R) \pm 3\sigma_R$. $E(R)$ is estimated by $\overline{R}$ and σ_R is estimated from the relation :

$$\sigma_R = d_3 \hat{\sigma} = d_3 . \frac{\overline{R}}{d_2}, \quad \text{[From (1·3)]} \qquad ...(1·5)$$

where d_2 and d_3 are constants depending on n.

$$\therefore \quad UCL_R = E(R) + 3\sigma_R = \overline{R} + \frac{3d_3}{d_2}\overline{R} \quad \text{[From (1·5)]}$$

$$\Rightarrow \quad UCL_R = \left(1 + \frac{3d_3}{d_2}\right)\overline{R} = D_4 \overline{R} \qquad ...(1·5a)$$

Similarly

$$LCL_R = \left(1 - \frac{3d_3}{d_2}\right)\overline{R} = D_3 \overline{R} \qquad ...(1·5b)$$

The values of D_4 and D_3 depend only on n and have been computed and tabulated for different values of n from 2 to 25 in Table given at the end of the chapter.

However, if *σ is known*, then

$$UCL_R = E(R) + 3\sigma_R = d_2 \sigma + 3d_3 \sigma = (d_2 + 3d_3)\sigma = D_2 \sigma \qquad ...(1·5c)$$

$$LCL_R = E(R) - 3\sigma_R = d_2 \sigma - 3d_3 \sigma = (d_2 - 3d_3)\sigma = D_1 \sigma \qquad ...(1·5d)$$

In each case, (σ known or unknown), the central line is given by :

$$CL_R = E(R) = \overline{R} \qquad ...(1·5e)$$

Since range can never be negative, LCL_R must be greater than or equal to 0. In case it comes out to be negative, it is taken as zero.

Remark. It should be noted carefully that the control limits for $\overline{X}$ and R-charts are based on the assumption that different samples or sub-groups are of constant size n.

4. ***Construction of Control Charts for $\overline{X}$ and R,*** *i.e., plotting of Central Line and the Control Limits.* Control charts are plotted on a rectangular co-ordinate axis—vertical scale (ordinate) representing the statistical measures $\overline{X}$ and R, and horizontal scale (abscissa) representing the sample number. Hours, dates or lot numbers may also be represented on the horizontal scale. Sample points (mean or range) are indicated on the chart by points, which may or may not be joined.

For $\overline{X}$-chart, the central line is drawn as a *solid* horizontal line at $\overline{\overline{X}}$ and $UCL_{\overline{X}}$ and $LCL_{\overline{X}}$ are drawn at the computed values as *dotted* horizontal lines.

For $\overline{R}$-chart, the central line is drawn as a solid horizontal line at $\overline{R}$ and UCL_R is drawn at the computed value as a dotted horizontal line. If the sample size is seven or more ($n \geq 7$), LCL_R is drawn as dotted horizontal line at the computed value, otherwise ($n < 7$) LCL_R is taken as zero.

Remarks on $\overline{X}$ and R-Charts. We give below some of the very important remarks which should be clearly understood by the reader.

1. The values of constants A, A_1, A_2, D_1, D_2, D_3, and D_4, for different values of n are given in the Table at the end of the chapter.

2. $\overline{X}$-chart reveals undesirable variations between samples as far as their averages are concerned while the R-chart reveals any undesirable variation within samples.

3. For a process to be working under statistical control, points both in the $\overline{X}$ and R-charts should lie between the control limits. A process which is not in statistical quality control suggests the presence

1·9. CONTROL CHART FOR ATTRIBUTES

In spite of wide applications of $\overline{X}$ and R-(or σ) charts as a powerful tool of diagnosis of sources of trouble in a production process, their use is restricted because of the following imitations :

1. They are charts for variables only, *i.e.*, for quality characteristics which can be measured and expressed in numbers.

2. In certain situations they are impracticable and un-economical, *e.g.*, if the number of measurable characteristics, each of which could be a possible candidate for $\overline{X}$ and R charts, is too large, say 30,000 or so then obviously there can't be 30,000 control charts.

As an alternative to $\overline{X}$ and R-charts, we have the control chart for attributes which can be used for quality characteristics :

(*i*) which can be observed only as attributes by classifying an item as defective or non-defective *i.e.*, conforming to specifications or not, and

(*ii*) which are actually observed as attributes even though they could be measured as variables, *e.g.*, go and no-go gauge test results.

There are two control charts for attributes :

(*a*) Control chart for fraction defective (p-chart) or the number of defectives (np or d chart).

(*b*) Control chart for the number of defects per unit (c-chart).

1·9·1. Control Chart for Fraction Defective (p-chart). While dealing with attributes, a process will be adjudged in statistical control if all the samples or sub-groups are ascertained to have the same population proportion P.

If 'd' is the number of defectives in a sample of size n, then the sample proportion defective is $p = d/n$. Hence, d is a binomial variate with parameters n and P.

$$\therefore \quad E(d) = nP \quad \text{and} \quad \text{Var}(d) = nPQ, \quad Q = 1 - P$$

Thus $E(p) = E(d/n) = \frac{1}{n} E(d) = P$ and $\text{Var}(p) = \text{Var}(d/n) = \frac{1}{n^2}\text{Var}(d) = \frac{PQ}{n}$ ...(1·7)

Thus, the 3-σ control limits for p-chart are given by :

$$E(p) \pm 3\ \text{S.E.}(p) = P \pm 3\sqrt{PQ/n} = P \pm A\sqrt{PQ} \qquad \text{...(1·8)}$$

where $A = 3/\sqrt{n}$ has been tabulated for different values of n.

Case (*i*) *Standards specified*. If P' is the given or known value of P, then

$$UCL_p = P' + A\sqrt{P'(1-P')} \quad ; \quad LCL_p = P' - A\sqrt{P'(1-P')} \quad ; \quad CL_p = P' \quad \ldots(1{\cdot}8a)$$

Case (*ii*) *Standards not specified*. Let d_i be the number of defectives and p_i the fraction defective for the ith sample (i = 1, 2, ..., k) of size n_i. Then the population proportion P is estimated by the statistic $\bar{p}$ given by :

$$\bar{p} = \frac{\Sigma d_i}{\Sigma n_i} = \frac{\sum_i n_i p_i}{\sum_i n_i} \quad \ldots(1{\cdot}8b)$$

It may be remarked here that $\bar{p}$ is an unbiased estimate of P, since

$$E(\bar{p}) = \sum_i E(d_i)/\Sigma n_i = \left[\Sigma(n_i P)/\Sigma n_i\right] = P$$

In this case

$$UCL_p = \bar{p} + A\sqrt{\bar{p}(1-\bar{p})} \quad ; \quad LCL_p = \bar{p} - A\sqrt{\bar{p}(1-\bar{p})} \quad ; \quad CL_p = \bar{p} \quad \ldots(1{\cdot}8c)$$

1·9·2. Control Chart for Number of Defectives (*d*-chart). If instead of p, the sample proportion defective, we use d, the number of defectives in the sample, then the 3-σ control limits for d-chart are given by :

$$E(d) \pm 3\ S.E.(d) = nP \pm 3\sqrt{nP(1-P)} \quad \ldots(1{\cdot}9)$$

Case (*i*) *Standards specified*. If P' is the given value of P then

$$UCL_d = nP' + 3\sqrt{nP'(1-P')} \quad ; \quad LCL_d = nP' - 3\sqrt{nP'(1-P')} \quad ; \quad CL_d = nP' \quad \ldots(1{\cdot}9a)$$

Case (*ii*) *Standards not specified*. Using $\bar{p}$ as an estimate of P as in (1·8*b*), we get

$$UCL_d = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})} \quad ; \quad LCL_d = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})} \quad ; \quad CL_d = n\bar{p} \quad \ldots(1{\cdot}9b)$$

Since p cannot be negative, if LCL as given by above formulae comes out to be negative, then it is taken to be zero.

***Remarks* 1. *p* and *d*-charts for Fixed Sample Size.** If the sample size remains constant for each sample *i.e.*, if $n_1 = n_2 = \ldots = n_k = n$, (say), then using (1·8*b*) an estimate of the population proportion p is given by :

$$\hat{p} = \bar{p} = \frac{\sum_{i=1}^{k} d_i}{\sum_i n_i} = \frac{\sum_i d_i}{nk} = \frac{n\sum_{i=1}^{k} p_i}{nk} = \frac{1}{k}\sum_{i=1}^{k} p_i \quad \ldots(1{\cdot}9c)$$

In this case, the same set of control limits can be used for all the samples inspected and it is immaterial if one uses p-chart or d-chart.

2. *p* and *d*-charts for Variable Sample Size. *Method I.* If the number of items inspected (n) in each sample varies, for p-chart separate control limits have to be computed for each sample while the central line is invariant whereas for d-chart control limits as well as the central line has to be computed for each sample. This type of limits are known as *variable control limits*. In such a situation p-chart is relatively simple and is preferred to d-chart which becomes very confusing.

Method 2. As pointed out in *Remark 2*, if n varies, separate control limits are calculated for each sample. Since S.E. $(p) = \sqrt{PQ/n}$, it should be noted that smaller the sample size wider the control band and *vice versa*. If the sample size does not vary appreciably then a single set of control limits based on the average sample size $\left(\sum_{i=1}^{k} n_i / k\right)$ can be used. For practical purposes, this holds good for situations in which the largest sample size does not exceed the smallest sample size by more than 20% of the smallest sample size.

Alternatively, for all sample sizes two sets of limits, one based on the largest sample size and the other based on the smallest sample size can be used. The largest sample size gives the smallest control band which is called *inner band* and the smallest sample size gives the largest control band which is called *outer band*. Points falling within the inner band indicate the process in control while points lying outside the outer band are indicative of the presence of assignable causes of variation which must be searched and rectified. For other points, action should be based on the exact control limits.

***Method* 3.** Another procedure is to standardise the variate, *i.e.*, instead of plotting p or d on the control chart, we plot the corresponding standardised values, *viz.*,

$$Z = \frac{p - p'}{\sqrt{P'Q'/n}} \quad \text{or} \quad \frac{p - \bar{p}}{\sqrt{\bar{p}\,(1 - \bar{p})/n}} \qquad \ldots(1\cdot10)$$

according as P is given or not, the symbols having their usual meanings. This stabilises our variable and the resulting chart is called *stabilised* p-chart or d-chart. In this case the control limits as well as the central line for p and d-charts are invariant with n (*i.e.*, they are constants independent of n) being given by :

$$UCL = 3, \qquad CL = 0, \qquad LCL = -3 \qquad \ldots(1\cdot10a)$$

Hence, the problem of variable control limits can be solved with a little more computational work discussed above.

Interpretations of p-chart. **1.** From the p-chart a process is judged to be in statistical control in the same way as is done for $\bar{X}$ and R charts. If all the sample points fall within the control limits without exhibiting any specific pattern, the process is said to be in control. In such a case, the observed variations in the fraction defective are attributed to the stable pattern of chance causes and the average fraction defective $\bar{p}$ is taken as the standard fraction defective P.

2. Points outside the UCL are termed as *high spots*. These suggest deterioration in the quality and should be regularly reported to the production engineers. The reasons for such deterioration could possibly be known and removed if the details of conditions under which data were collected, were known. Of particular interest and importance is, if there was any change of inspection or inspection standards.

3. Points below LCL are called *low spots*. Such points represent a situation showing improvement in the product quality. However, before taking this improvement for guaranteed, it should be investigated if there was any slackness in inspection or not.

4. When a number of points fall outside the control limits, a revised estimate of P should be obtained by eliminating all the points that fall above UCL (it is assumed that points that fall below LCL are not due to faulty inspection). The standard fraction defective P should be revised periodically in this way.

Remark. The interpretation for the control chart for number of defects (d-chart) is same as that for p-chart.

Example 1·10. *The following are the figures of defectives in 22 lots each containing 2,000 rubber belts :*

425, 430, 216, 341, 225, 322, 280, 306, 337, 305, 356,
402, 216, 264, 126, 409, 193, 326, 280, 389, 451, 420

Draw control chart for fraction defective and comment on the state of control of the process.

Solution. Here we have a fixed sample size $n = 2,000$ for each lot. If d_i and p_i are respectively the number of defectives and the sample fraction defective for the ith lot, then

$$p_i = \frac{d_i}{2,000}, \ (i = 1, 2, \ldots, 22)$$

which are given in Table 1·2.

TABLE 1·2 : COMPUTATIONS FOR C.C. FOR FRACTION DEFECTIVE

S. No.	*d*	*p = (d/2000)*	*S. No.*	*d*	*p = (d/2000)*
1	425	0·2125	12	402	0·2010
2	430	0·2150	13	216	0·1080
3	216	0·1080	14	264	0·1320
4	341	0·1705	15	126	0·0630
5	225	0·1125	16	409	0·2045
6	322	0·1610	17	193	0·0965
7	280	0·1400	18	326	0·1630
8	306	0·1530	19	280	0·1400
9	337	0·1685	20	389	0·1945
10	305	0·1525	21	451	0·2255
11	356	0·1780	22	420	0·2100
Total	3,543	1·7715		3,476	1·7380

In the usual notations, we have

$$\bar{p} = \frac{\Sigma p_i}{k} = \frac{1{\cdot}7715 + 1{\cdot}7380}{22} = \frac{3{\cdot}5095}{22} = 0{\cdot}1595 \quad \Rightarrow \quad \bar{q} = 1 - \bar{p} = 0{\cdot}8405$$

$$\left[\text{Or} \qquad \bar{p} = \frac{\Sigma d_i}{nk} = \frac{3543 + 3476}{2000 \times 22} = \frac{7019}{44000} = 0{\cdot}1595\right]$$

3-σ control limits for p-chart are given by :

$$\bar{p} \pm 3\sqrt{\bar{p}\,\bar{q}/n} = 0{\cdot}1595 \pm 3\sqrt{0{\cdot}1595 \times 0{\cdot}8405/2000}$$

$$= 0{\cdot}1595 \pm 3\sqrt{0{\cdot}000067} = 0{\cdot}1595 \pm 0{\cdot}0246$$

$$\therefore \quad UCL_p = 0{\cdot}1595 + 0{\cdot}0246 = 0{\cdot}1841; \ LCL_p = 0{\cdot}1595 - 0{\cdot}0246 = 0{\cdot}1349; \ CL_p = \bar{p} = 0{\cdot}1595$$

The control chart for fraction defective (*p*-chart) is drawn in *Fig. 1·9.*

From the *p*-chart, we find that the sample points (fraction defectives) corresponding to the sample numbers 1, 2, 3, 5, 12, 13, 14, 15, 16, 17, 20, 21 and 22, fall outside the control limits. Hence, the process cannot be regarded in statistical control.

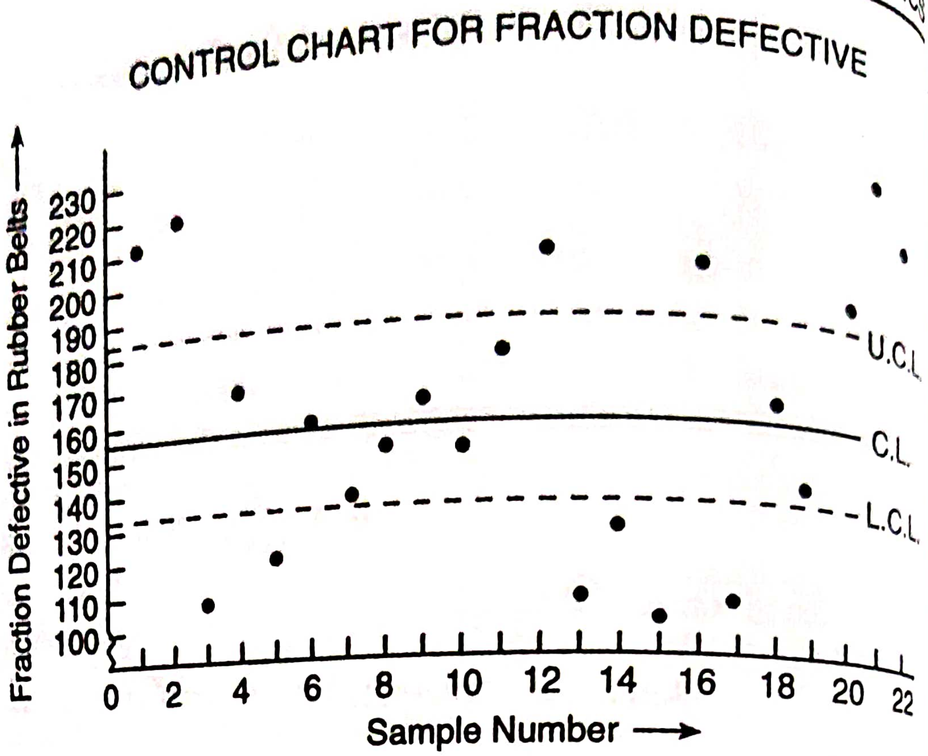


Fig. 1·9

Example 1·11. *From the following inspection results, construct 3-σ control limits for p chart :*

Date Sept.	*No. of Defectives*	*Date Sept.*	*No. of Defectives*	*Date Sept.*	*No. of Defectives*
1	22	11	70	21	66
2	40	12	80	22	50
3	36	13	44	23	46
4	32	14	22	24	32
5	42	15	32	25	42
6	40	16	42	26	46
7	30	17	20	27	30
8	44	18	46	28	38
9	42	19	28	29	40
10	38	20	36	30	24

The sub-groups, from which the defectives were taken out, were of the same size, i.e., 1,000 items each.

Without constructing the control chart, comment on the state of control of the process. If the process is out of control, then suggest the revised control limits for future use.

Solution. Here we have a fixed sample size for each lot. If d_i and p_i are respectively the number of defectives and the sample fraction defective for the ith lot then

$$p_i = \frac{d_i}{1{,}000}, \quad (i = 1, 2, \ldots, 30)$$

which are given in *Table 1·3* :

case of surface defects, area of the surface is the sample size ; in case of casting defects, a single part (such as base plate, side cover) is the sample size. However, defined sample size should be constant in the sense that different samples have essentially equal opportunity for the occurrence of defects.

Control Limits for *c*-chart. In many manufacturing or inspection situations, the sample size n *i.e.*, the area of opportunity is very large (since the opportunities for defects to occur are numerous) and the probability p of the occurrence of a defect in any one spot is very small such that np is finite. In such situations from statistical theory we know that the pattern of variations in data can be represented by Poisson distribution, and consequently 3-σ control limits based on Poisson distribution are used. Since for a Poisson distribution, mean and variance are equal, if we assume that c is Poisson variate with parameter, λ, we get

$$E(c) = \lambda \quad \text{and} \quad \text{Var}(c) = \lambda$$

Thus 3-σ control limits for c-chart are given by :

$$\left.\begin{aligned} UCL_c &= E(c) + 3\sqrt{\text{Var}(c)} = \lambda + 3\sqrt{\lambda} \\ LCL_c &= E(c) - 3\sqrt{\text{Var}(c)} = \lambda - 3\sqrt{\lambda} \\ CL_c &= \lambda \end{aligned}\right\} \qquad \ldots(1\cdot11)$$

Case (*i*) *Standards specified.* If λ' is the specified value of λ, then

$$UCL_c = \lambda' + 3\sqrt{\lambda'} \quad ; \quad LCL_c = \lambda' - 3\sqrt{\lambda'} \quad ; \quad CL_c = \lambda' \qquad \ldots(1\cdot12)$$

Case (*ii*) *Standards not Specified.* If the value of λ is not known, it is estimated by the mean number of defects per unit. Thus, if c_i is the number of defects observed on the ith $(i = 1, 2, \ldots, k)$ inspected unit, then an estimate of λ is given by :

$$\hat{\lambda} = \bar{c} = \sum_{i=1}^{k} c_i / k \qquad \ldots(1\cdot12a)$$

It can be easily seen that $\bar{c}$ is an unbiased estimate of λ. The control limits, in this case, are given by :

$$UCL_c = \bar{c} + 3\sqrt{\bar{c}} \quad ; \quad LCL_c = \bar{c} - 3\sqrt{\bar{c}} \quad ; \quad CL_c = \bar{c} \qquad \ldots(1\cdot12b)$$

Since c can't be negative, if LCL given by above formulae comes out to be negative, it is regarded as zero.

The central line is drawn at $\bar{c}$, and UCL and LCL are drawn at the values given by (1·12*a*). The observed number of defects on the inspected units are then plotted on the control chart. The interpretations for c-chart are similar to those of p-chart.

Remark. Usually k, the number of samples (inspected units), is taken from 20 to 25. Normal approximation to Poisson distribution may be used provided $\bar{c} < 5$.

1·9·4. *c*- Chart for Variable Sample Size or *u*-Chart. In this case instead of plotting c, the statistic $u = c/n$ is plotted, n being the sample size which is varying. If n_i is the sample size and c_i the total number of defects observed in the ith sample, then

$$u_i = c_i / n_i, \; (i = 1, 2, \ldots, k), \qquad \ldots(1\cdot13)$$

gives the average number of defects per unit for the ith sample.

In this case an estimate of λ, the mean number of defects per unit in the lot, based on all the k-samples is given by :

$$\hat{\lambda} = \bar{u} = \frac{1}{k} \sum_{i=1}^{k} u_i \qquad \ldots(1\cdot13a)$$

We know that if $\overline{X}$ is the mean of a random sample of size n then S.E. $(\overline{X}) = \sigma/\sqrt{n}$. Hence, the standard error of the average number of defects per unit is given by :

$$S.E.\ (u) = \sqrt{\lambda / n} = \sqrt{\overline{u} / n} \quad ; \quad \text{[On using (1·13}a\text{)]} \qquad \ldots(1\cdot13b)$$

Hence, 3-σ countrol limits for u-chart (or c - Chart for variable sample size) are given by :

$$UCL_u = \overline{u} + 3\sqrt{\overline{u} / n} \quad ; \quad LCL_u = \overline{u} - 3\sqrt{\overline{u} / n} \quad ; \quad CL_u = \overline{u} \qquad \ldots(1\cdot13c)$$

As is obvious, control limits will vary for each sample. The central line, however, will be same. The interpretation of these charts is similar to the p-chart or d-chart.

Applications of c-chart

The universal nature of Poisson distribution as the law of small numbers makes the c-chart technique quite useful. In spite of the limited field of application of c-chart (as compared to $\overline{X}$, R, p-charts), there do exist situations in industry where c-chart is definitely needed. Some of the representative types of defects to which c-chart can be applied with advantage are :

1. c is number of imperfections observed in a bale of cloth.

2. c is the number of surface defects observed in (i) roll of coated paper or a sheet of photographic film, and (ii) a galvanised sheet or a painted, plated or enamelled surface of given area.

3. c is the number of defects of all types observed in aircraft sub-assemblies or final assembly.

4. c is the number of breakdowns at weak spots in insulation in a given length of insulated wire subject to a specified test voltage.

5. c is the number of defects observed in stains or blemishes on a surface.

6. c is the number of soiled packages in a given consignment.

7. c-chart has been applied to sampling acceptance procedures based on number of defects per unit, *e.g.*, in case of inspection of fairly complex assembled units such as T.V. sets, aircraft engines, tanks, machine-guns, etc., in which there are very many opportunities for the occurrence of defects of various types and the total number of defects of all types found by inspection is recorded for each unit.

8. c-chart technique can be used with advantage in various fields other than industrial quality control, *e.g.*, it has been applied (i) to accident statistics (both of industrial accidents and highway accidents), (ii) in chemical laboratories, and (iii) in epidemiology.

Example 1·14. *In welding of seams, defects included pinholes, cracks, cold taps, etc. A record was made of the number of defects found in one seam each hour and is given below.*

1·12·2005	8 A.M.	2	2·12·2005	8 A.M.	5	3·12·2005	8 A.M.	6
	9 A.M.	4		9 A.M.	3		9 A.M.	4
	10 A.M.	7		10 A.M.	7		10 A.M.	3
	11 A.M.	3		11 A.M.	11		11 A.M.	9
	12 A.M.	1		12 A.M.	6		12 A.M.	7
	1 P.M.	4		1 P.M.	4		1 P.M.	4
	2 P.M.	8		2 P.M.	9		2 P.M.	7
	3 P.M.	9		3 P.M.	9		3 P.M.	12

Draw the control chart for number of defects and give your comments.

Cumulative Sum Chart

Construct CUSUM chart and derive its limits using V-mask procedure?

- The CUSUM chart deals with **Cumulative Sum** of the deviation of the sample values from the target value.
- CUSUM chart is developed by **a British Statistician** E.S page in the year 1954.
- CUSUM chart is primarily used to maintain current control of a process.
- The Advantage of this chart is less expensive.
- CUSUM chart will detect sudden and persistent change in the process.
- Let $\overline{X}_i$ be the average of i^{th} sample. Let $\mu=\mu_0$ be the target of the process.
- Cumulative Sum is defined us

$$\mathbf{c(k)=\sum(\overline{X}_i-T)}$$

$$\mathbf{c(k)=\sum(\overline{X}_i-\mu_0)}$$

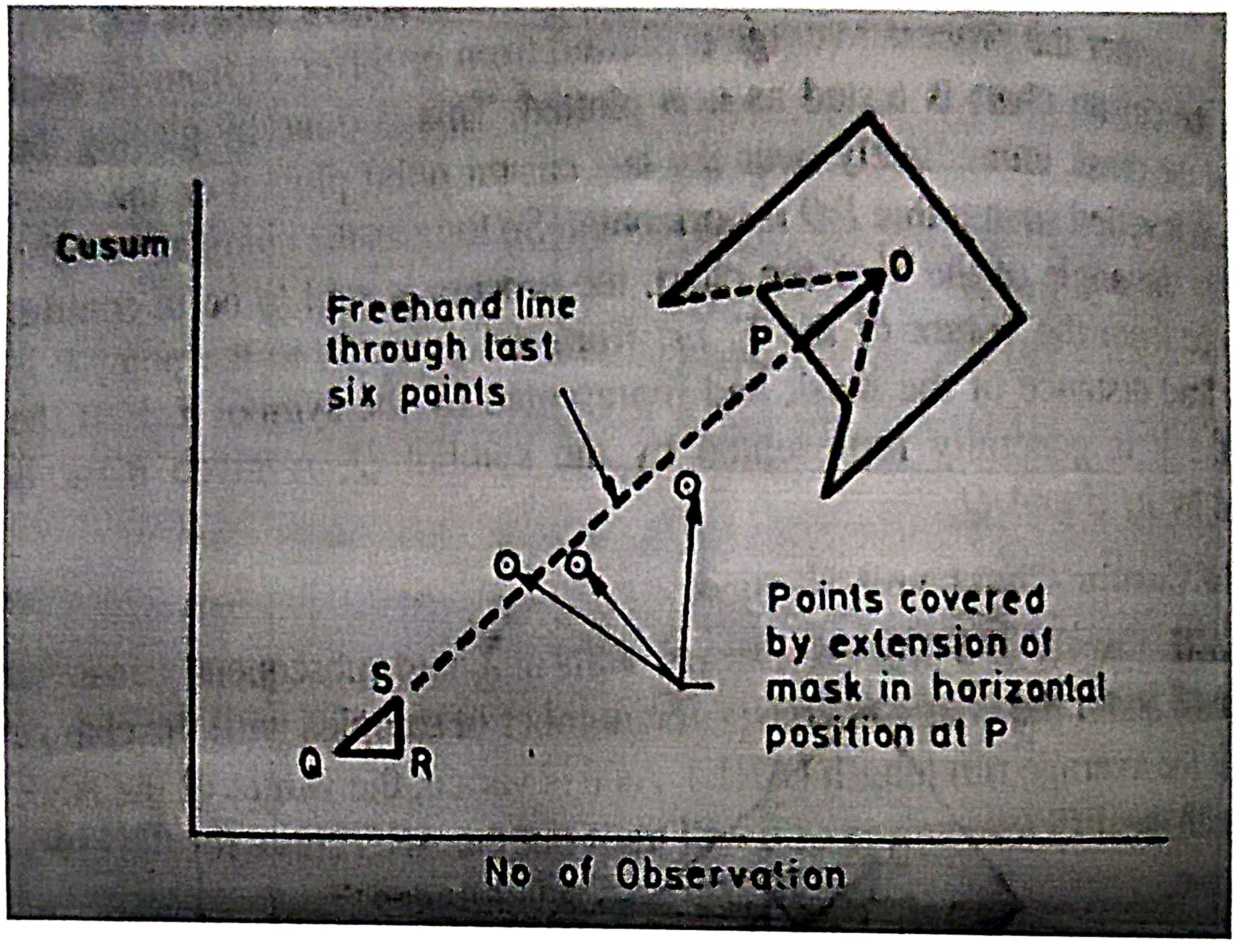
- During Normal process operation, c(k) fluctuates around zero.
- If a process change causes a small shift in $\overline{X}$, c(k) will drift either upward or downward.
- The CUSUM control chart was developed using a graphical approach based on V-mask.

V-mask Two sided decision procedure:-

- Let a sample of n-items be taken periodically from a process and let the mean $\bar{X}$ be computed.
- Let μ_0 be the target for the process and let the data plotted on the cumulative sum chart be the cumulative sums of $\bar{X}$-μ_0

$$C_k=\sum(\bar{x}_i-\mu_0)$$

- The process average shifted from or is different from μ_0, each point on the CUSUM chart is tested as it is plotted.
- This is done by placing the point P of the mask immediately over the last CUSUM point plotted on the chart with the mask leveled so that the line PO is horizontal.
- If any of the previously plotted points is covered by the mask it is an indication that the process had changed.
- As long as the cumulative sums stay within the angle of the V of the chart, the process in being in control.
- The distance from the vertex of the V, (ie) from the point O, to the point P is called the lead distance of the mask and it is denoted by d.
- Half the angle of the V is generally represented by the symbol θ.
- A given mask is thus determined by its d and θ.



❖ It is possible to run concurrently two sided scheme with upper and lower reference values k_1 and k_2 are known. Let w is the horizontal distance between successive points on the chart measured terms of unit distance on the vertical scale.

❖ The Two One sided schemes and the V-mask scheme will be equivalent if,

$$k_1-\mu_0=\mu_0-k_2=w\tan\theta$$

and

$$h=d\tan\theta$$

Substituting eqn 1 & 2, we get

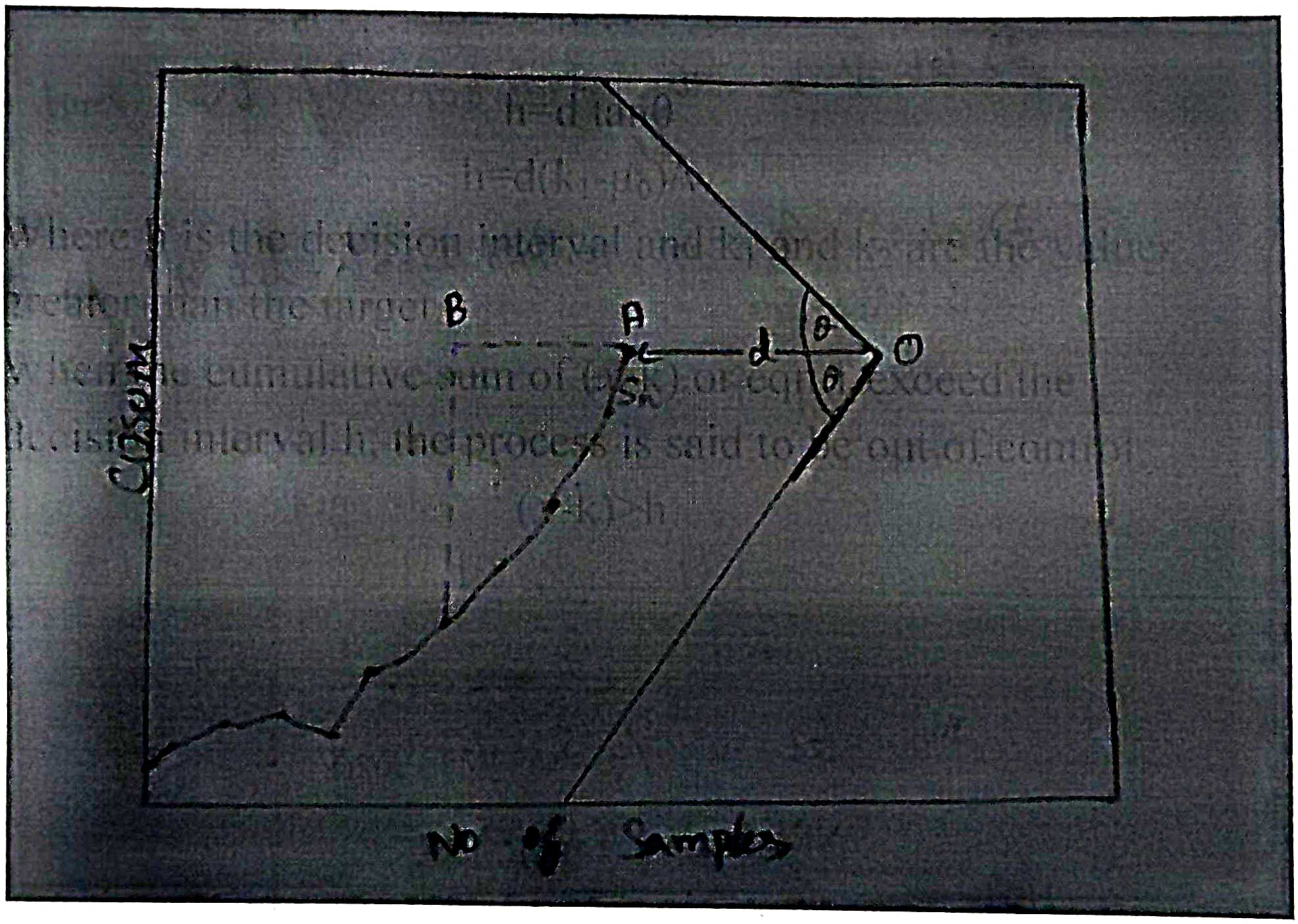
$$h = d \tan\theta$$

$$h = d(k_1 - \mu_0)/w$$

Where h is the decision interval and k_1 and k_2 are the values greater than the target.

❖ When the cumulative sum of ($\bar{x}$-k) or eqn 1 exceed the decision interval h, the process is said to be out of control.

$$(\bar{x} - k) > h$$



θ=inclined angle

A=last CUSUM point

O=vertex of V-mask

$$S_n - S_{n-r} \geq (rw + d)\tan\theta$$

$$\sum (\bar{X} - \mu_0 - w\tan\theta) \geq d\tan\theta$$