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Inteval Estimation - 1

Interval Estimation

In interval estimation the estimate ~~of~~ for the parameter lies between ~~of~~ the two limits within ~~with~~ which the estimates for the parameter lies are known as confidence limits (or) fiducial limits and the interval bounded by these two limits as confidence interval or fiducial interval, the confidence interval depends upon the confidence i.e required to set up.

The probability that the associate with interval is called the confidence level. It shows how confidently we can say that the interval estimate with include the population parameter. The higher the probability, the more is the confidence although any confidence level can bee consider; ~~of~~ but a most commonly used confidence levels are 90%, 95%, 98% and 99% in the light of above discussion the interval estimation is defined as,

$P(C_1 < \theta < C_2$, for given value of statistic $t) = 1-\alpha$

but where α is the level of significance. (C_1, C_2) is the interval within which the unknown parameter θ is excepted to lies is known as confidence interval (or) fiducial interval.

C_1, C_2 are respectively known as lower limit and the upper limit of the confidence interval C_1, C_2 and $1-\alpha$ is the confidence co-efficient depending upon the desire decission of the estimate For example $\theta = 0.01$ gives 99% confidence limits

Confidence Intervals for large Samples :

It has been proved that under regularity conditions the first derivative of the logrithims of the likelihood function with respect to parameter θ i.e

$\frac{\partial \log L}{\partial \theta}$ is assymptotatically normal with mean 0 and variance given by

$$V\left(\frac{\partial \log L}{\partial \theta}\right) = E\left[\frac{-\partial^2 \log L}{\partial \theta^2}\right]$$

Hence for large n $Z = \frac{\partial \log L / \partial \theta}{\sqrt{Var\left(\frac{\partial \log L}{\partial \theta}\right)}} \sim N(0,1)$

The result enable us to obtain confidence interval for the parameter θ in large samples.

Thus for large samples the confidence interval for θ with confidence co-eff $(1-\alpha)$ is obtained by converting ~~the~~ by inequalities in

$P(|Z| \leq \lambda_\alpha) = 1-\alpha$ where λ_α is given by

$$\frac{1}{\sqrt{2\pi}} \int_{-\lambda_\alpha}^{\lambda_\alpha} \exp\left(-u^2/2\right) du = 1-\alpha$$

Example : 1

Important Question (2)

Obtain $100(1-\alpha)\%$ confidence limits (for large samples) for the parameter λ of the ~~po~~ Poisson distribution

$$f(x, \lambda) = \frac{e^{-\lambda} \lambda^x}{x!} ; x = 0, 1, 2 \ldots$$

Soln:

we have the likelihood function $L = \prod_{i=1}^{n} f(x_i, \theta)$

$$L = \frac{e^{-n\lambda} \lambda^{\sum_{i=1}^{n} x_i}}{x_1! \, x_2! \ldots x_n!}$$

Taking log on both sides

$$\log L = -n\lambda + \sum_{i=1}^{n} x_i \log \lambda - \sum_{i=1}^{n} \log (x_i)!$$

$$\frac{\partial \log L}{\partial \lambda} = \left(-n + \sum_{i=1}^{n} x_i / \lambda\right)$$

$\sum x_i = n\bar{x}$

$$= (-n + n\bar{x}/\lambda) = -n\,(1 - \bar{x}/\lambda)$$

$$= n\,(\bar{x}/\lambda - 1)$$

$$\frac{\partial^2 \log L}{\partial \lambda^2} = -\frac{n\bar{x}}{\lambda^2}$$ (4)

$$V\left(\frac{\partial \log L}{\partial \lambda}\right) = E\left(\frac{\partial^2 \log L}{\partial \theta^2}\right) = E\left[-\left(-\frac{n\bar{x}}{\lambda^2}\right)\right]$$

$$E\left(\frac{n\bar{x}}{\lambda^2}\right) = \frac{n}{\lambda^2} E(\bar{x}) = {}^{n}/_{\lambda^2}\,\lambda = {}^{n}/_{\lambda}$$

$$V\left(\frac{\partial \log L}{\partial \theta}\right) = {}^{n}/_{\lambda}$$

The confidence interval for large sample is given by

$$Z = \frac{\left(\partial \log L/\partial \lambda\right)}{\sqrt{V\left(\frac{\partial \log L}{\partial \lambda}\right)}} \sim N(0,1)$$

$$= \frac{n\left(\frac{\bar{x}}{\lambda} - 1\right)}{\sqrt{n/\lambda}} \sim N(0,1)$$

$$= \frac{\sqrt{n}\sqrt{n}\left(\frac{\bar{x}-\lambda}{\lambda}\right)\sqrt{\lambda}}{\sqrt{n}} \sim N(0,1)$$

$$= \sqrt{n}\left(\frac{\bar{x}-\lambda}{\sqrt{\lambda}}\right) \sim N(0,1)$$

$$z = \sqrt{n/\lambda}\,(\bar{x}-\lambda) \sim N(0,1).$$ $P[|z| \leq \lambda_\alpha] = 1-\alpha$

Hence $100(1-\alpha)\%$ confidence interval for λ is given by (large samples)

$P[|z| \leq \lambda_\alpha] = 1-\alpha$

$$P\{|\sqrt{(n/\lambda)}\,(\bar{x}-\lambda)| \leq \lambda_\alpha\} = 1-\alpha$$

$P[|z| \leq \lambda_\alpha] = 1-\alpha$

Hence the required limits for λ are the root of the equation

$$|\sqrt{(n/\lambda)}\,(\bar{x}-\lambda)| \leq \lambda_\alpha$$

$$\Rightarrow (n/\lambda)(\bar{x}-\lambda)^2 = \lambda_\alpha^2$$

$$n(\bar{x}-\lambda)^2 = \lambda_\alpha^2 \cdot \lambda$$

$$n(\bar{x}^2 + \lambda^2 - 2\bar{x}\lambda) - \lambda_\alpha^2 \cdot \lambda = 0$$

$$\Rightarrow \lambda^2 + \bar{x}^2 - 2\bar{x}\lambda - \frac{\lambda_\alpha^2 \lambda}{n} = 0$$

$$\Rightarrow \lambda^2 - \lambda\left(2\bar{x} + \frac{\lambda_\alpha^2}{n}\right) + \bar{x}^2 = 0$$

Here $a = 1$, $b = -\left(2\bar{x} + \frac{\lambda_\alpha^2}{n}\right)$ and $c = \bar{x}^2$

$$\lambda = \frac{\left(2\bar{x} + \frac{\lambda_\alpha^2}{n}\right) \pm \sqrt{\left(-\left(2\bar{x} + \frac{\lambda_\alpha^2}{n}\right)\right)^2 - 4(1)(\bar{x}^2)}}{2(1)}$$

$$= \frac{1}{2}\left(2\bar{x} + \frac{\lambda_\alpha^2}{n}\right) \pm \frac{1}{2}\sqrt{4\bar{x}^2 + \frac{\lambda_\alpha^4}{n^2} + 2.2\bar{x} \cdot \frac{\lambda_\alpha^2}{n} - 4\bar{x}^2}$$

$$= \frac{1}{2}\left(2\bar{x} + \frac{\lambda_\alpha^2}{n}\right) \pm \sqrt{\frac{\lambda_\alpha^4}{4n^2} + \frac{4\bar{x}\lambda_\alpha^2}{4n}}$$

$$= \frac{1}{2}\left(2\bar{x} + \frac{\lambda_\alpha^2}{n}\right) \pm \sqrt{\frac{\lambda_\alpha^4}{4n^2} + \frac{\bar{x}\lambda_\alpha^2}{n}}$$

For example 95% confidence interval for λ is given by $\lambda_\alpha = 1.96$

$$= \frac{1}{2}\left(2\bar{x} + \frac{1.96^2}{n}\right) \pm \sqrt{\frac{(1.96)^4}{4n^2} + \frac{(1.96)^2\bar{x}}{n}}$$

$$= \frac{1}{2}\left(2\bar{x} + \frac{3.84}{n}\right) \pm \sqrt{\frac{3.69}{n^2} + \frac{3.84\bar{x}}{n}}$$

Example: 2

Show that for a distribution $dF(x) = \theta e^{-x\theta}$; $0 < x < \infty$ central confidence limits for large samples with 95% confidence co-efficients are given by

$$\theta = \left(1 \pm \frac{1.96}{\sqrt{n}}\right) / \bar{x}.$$

Solution:

Here the likelihood function is $L = \theta^n \exp\{-\theta \sum_{i=1}^n x_i\}$

Taking log on both sides

$$\log L = n \log \theta - \theta \sum_{i=1}^n x_i$$

$$\frac{\partial \log L}{\partial \theta} = \frac{n}{\theta} - \sum_{i=1}^n x_i$$

$$= \frac{n}{\theta} - n\bar{x}$$

$$= n(1/\theta - \bar{x})$$

$$\frac{\partial^2 \log L}{\partial \theta^2} = -\frac{n}{\theta^2}$$

$$\mathrm{Var}\left(\frac{\partial \log L}{\partial \theta^2}\right) = E\left(-\frac{\partial^2 \log L}{\partial \theta^2}\right) = E\left(-\left(\frac{-n}{\theta^2}\right)\right)$$

$$= E(n/\theta^2) = n/\theta^2$$

Hence for large samples

$$Z = \frac{\partial \log L / \partial \theta}{\sqrt{V(\partial^2 \log L / \partial \theta)}} \sim N(0,1)$$

$$= \frac{n(1/\theta - \bar{x})}{\sqrt{n/\theta^2}} \sim N(0,1)$$

$$= \frac{\sqrt{n}\sqrt{n}\left(\frac{1 - \bar{x}\theta}{\theta}\right)}{\sqrt{n/\theta^2}} \sim N(0,1)$$

$$= \sqrt{n}(1 - \bar{x}\theta) \sim N(0,1)$$

Hence 95% confidence limits for θ are given

$$P\{|Z| \leq \lambda\alpha\} = 1 - \alpha$$

$$P\{-1.96 \leq \sqrt{n}(1 - \bar{x}\theta) \leq 1.96\} = 0.95 \quad - (1)$$

$$\sqrt{n}(1 - \bar{x}\theta) \leq 1.96$$

$$(1 - \bar{x}\theta) \leq \frac{1.96}{\sqrt{n}}$$

$$-\bar{x}\theta \leq \frac{1.96}{\sqrt{n}} - 1$$

$$-\theta \leq \left(\frac{1.96}{\sqrt{n}} - 1\right)\frac{1}{\bar{x}}$$

0.45
0.05
0.42
0.08

$$\left(1-\frac{1.96}{\sqrt{n}}\right)\frac{1}{\bar{x}} \le \theta \rightarrow (2)$$

$$\left(1-\frac{1.96}{\sqrt{n}}\right)\frac{1}{\bar{x}} \le \theta$$

$$1-\frac{1.96}{\sqrt{n}} \le \theta(\bar{x})$$

$$-\frac{1.96}{\sqrt{n}} \le 1-\bar{x}\theta$$

$$-1.96 \le \sqrt{n}(1-\bar{x}\theta)$$

$$\sqrt{n}-1.96 \frac{1}{\sqrt{n}}\frac{1}{\bar{x}} \le \theta$$

$$\sqrt{n}-1.96 \le \sqrt{n}(\theta\bar{x})$$

$$\sqrt{n}-1.96 \le \sqrt{n}\,\theta\bar{x}$$

$$\sqrt{n}(-1+\bar{x}\theta)$$

$$\sqrt{n}(1-\bar{x}\theta)$$

$$-1.96 \le \sqrt{n}(1-\bar{x}\theta)$$

$$\frac{-1.96}{\sqrt{n}} \le 1-\bar{x}\theta$$

$$\frac{-1.96}{\sqrt{n}}-1 \le -\bar{x}\theta$$

$$-\left(\frac{1.96}{\sqrt{n}}+1\right)\frac{1}{\bar{x}} \le -\theta$$

$$\theta \ge \left(\frac{1.96}{\sqrt{n}}+1\right)\frac{1}{\bar{x}}$$

$$\left(\frac{1.96}{\sqrt{n}}+1\right)\frac{1}{\bar{x}} \ge \theta$$

$$\theta \le \left(\frac{1.96}{\sqrt{n}}+1\right)\frac{1}{\bar{x}} - (3)$$

Hence from (1), (2) and (3) the central 95% confidence limit for θ is given by $\theta = \left(1 \pm \frac{1.96}{\sqrt{n}}\right)\Big/\bar{x}$

Interval Estimation - 1

Confidence Interval for mean of normal population : [Single mean]

Let $x_1, x_2 \ldots x_n$ be a random sample from the normal population $N(\mu, \sigma^2)$ where μ is unknown and σ^2 is known, then the statistic $Z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} \sim N(\mu, \sigma^2)$, where $\bar{x}$ is the sample mean has population mean μ has the normal population $N(0,1)$. Hence the confidence interval with probability $(1-\alpha)$ is given by

$$P\left[-Z_{\alpha/2} \le Z \le Z_{\alpha/2}\right] = 1-\alpha$$

$$P\left[-Z_{\alpha/2} \le \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} \le Z_{\alpha/2}\right] = 1-\alpha$$

$$P\left[-Z_{\alpha/2}\, \sigma/\sqrt{n} \le \bar{x}-\mu \le \sigma/\sqrt{n}\, Z_{\alpha/2}\right] = 1-\alpha$$

$$P\left[-\bar{x}-Z_{\alpha/2}\, \sigma/\sqrt{n} \le -\mu \le -\bar{x}+Z_{\alpha/2}\, \sigma/\sqrt{n}\right] = 1-\alpha$$

$$P\left[\bar{x}+Z_{\alpha/2}\, \sigma/\sqrt{n} \ge \mu \ge \bar{x}-Z_{\alpha/2}\, \sigma/\sqrt{n}\right] = 1-\alpha$$

$$P\left[\bar{x}-Z_{\alpha/2}\, \sigma/\sqrt{n} \le \mu \le \bar{x}+Z_{\alpha/2}\, \sigma/\sqrt{n}\right] = 1-\alpha$$

$(1-\alpha)100\%$ confidence interval for the population mean is $\left(\bar{x}-Z_{\alpha/2}\, \sigma/\sqrt{n},\ \bar{x}+Z_{\alpha/2}\, \sigma/\sqrt{n}\right)$

$$\left(\bar{x} \pm Z_{\alpha/2}\, \sigma/\sqrt{n}\right)$$

Properties:

→ For large samples if σ^2 is known then we can replace σ by $\hat{\sigma}$ and hence the confidence interval is given by $\left(\bar{x} \pm Z_{\alpha/2}\, \hat{\sigma}/\sqrt{n}\right)$

→ 95% confidence limits for the population mean is given by $\left(\bar{x} \pm 1.96\, \sigma/\sqrt{n}\right)$

→ 99% confidence limits for the population mean is given by $\left(\bar{x} \pm 2.58\, \sigma/\sqrt{n}\right)$

→ 98% confidence limits for the population mean is given by $\left(\bar{x} \pm 2.33\, \sigma/\sqrt{n}\right)$.

Confidence interval for the difference between the means:

Suppose variance of both populations are known. Let x_1 and x_2 be a random variable having normal population $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$. Let $\bar{x}_1$ and $\bar{x}_2$ be the means of the sample of sizes n_1 and n_2 respectively taken from these two normal populations. We know that sampling distribution of $\bar{x}_1$ and $\bar{x}_2$ are also normal and are given by $N(\mu_1, \sigma_1^2/n_1)$ and $N(\mu_2, \sigma_2^2/n_2)$. Since $\bar{x}_1$ and $\bar{x}_2$ are indepen-dent the distribution of $\bar{x}_1 - \bar{x}_2$ is also

$$N\left(\mu_1 - \mu_2, \sigma_1^2/n_1 + \sigma_2^2/n_2\right)$$

$\therefore$ The distribution of $Z = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}} \sim N(0,1)$

Let $Z = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}}$ then $P\left[-Z_{\alpha/2} \le Z \le Z_{\alpha/2}\right] = 1-\alpha$

$$P\left[-Z_{\alpha/2} \le \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}} \le Z_{\alpha/2}\right] = 1-\alpha$$

$$P\left[-Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \le (\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2) \le Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}\right] = 1-\alpha$$

$$P\left[-(\bar{x}_1 - \bar{x}_2) - Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \le -(\mu_1 - \mu_2) \le -(\bar{x}_1 - \bar{x}_2) + Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}\right] = 1-\alpha$$

$$P\left[(\bar{x}_1 - \bar{x}_2) + Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \ge (\mu_1 - \mu_2) \ge (\bar{x}_1 - \bar{x}_2) - Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}\right] = 1-\alpha$$

$$P\left[(\bar{x}_1 - \bar{x}_2) - Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \le (\mu_1 - \mu_2) \le (\bar{x}_1 - \bar{x}_2) + Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}\right] = 1-\alpha$$

$$\left[(\bar{x}_1 - \bar{x}_2) - Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}},\ (\bar{x}_1 - \bar{x}_2) + Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}\right]$$

$(1-\alpha)100\%$ confidence interval for a difference between the mean is $(\bar{x}_1 - \bar{x}_2) \pm Z_{\alpha/2}\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}$

Properties :

$\Rightarrow$ 95% confidence limits for the difference between the mean is

$$Z = (\bar{x}_1 - \bar{x}_2) \pm 1.96\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

(11)

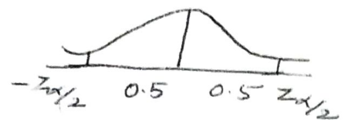
⇒ 99% confidence limits for the difference between the mean is

$$Z = (\bar{x}_1 - \bar{x}_2) \pm 2.58 \sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}$$

⇒ 98% confidence limits for the difference between the mean is

$$Z = (\bar{x}_1 - \bar{x}_2) \pm 2.33 \sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}$$

Work out



95% confidence interval	98% C.I	99% C.I
$\alpha = 5\%$	$\alpha = 2\%$	$\alpha = 1\%$
$\alpha = 0.05$	$\alpha = 0.02$	$\alpha = 0.01$
$\alpha/2 = 0.025$	$\alpha/2 = 0.01$	$\frac{\alpha}{2} = 0.005$
0.500	0.50	0.500
0.025	0.01	0.005
0.475	0.49	0.495
1.96	2.33	2.58

Confidence Interval for proportion:

There are many suitations in which we may interested with estimating proportion, probability or percentage of population

(In such cases, for a large sample we assume the population has binomial with paramketer p. for large n we know that)

$$Z = \frac{p - P}{\sqrt{PQ/n}} \sim N(0,1)$$

For the area under normal probability curve,

$$P\left[-Z_{\alpha/2} \leq Z \leq Z_{\alpha/2}\right] = 1 - \alpha$$

$$P\left[-Z_{\alpha/2} \le \frac{p-P}{\sqrt{pq/n}} \le Z_{\alpha/2}\right] = 1-\alpha$$

$$P\left[-Z_{\alpha/2}\sqrt{pq/n} \le p-P \le Z_{\alpha/2}\sqrt{pq/n}\right] = 1-\alpha$$

$$P\left[-Z_{\alpha/2}\sqrt{pq/n} - p \le -P \le Z_{\alpha/2}\sqrt{pq/n} - p\right] = 1-\alpha$$

$$P\left[Z_{\alpha/2}\sqrt{pq/n} + p \ge P \ge -Z_{\alpha/2}\sqrt{pq/n} + p\right] = 1-\alpha$$

$$P\left[p - Z_{\alpha/2}\sqrt{pq/n} \le P \le p + Z_{\alpha/2}\sqrt{pq/n}\right] = 1-\alpha$$

$$\left[p - Z_{\alpha/2}\sqrt{pq/n},\ p + Z_{\alpha/2}\sqrt{pq/n}\right]$$

$$p \pm Z_{\alpha/2}\sqrt{pq/n}$$

where $p = x/n$ $\quad q = 1-p = 1 - x/n$

$(1-\alpha)100\%$ of confidence interval for a population proportion is $p \pm Z_{\alpha/2}\sqrt{pq/n}$

Properties:

$\Rightarrow$ 95% of confidence ~~interval~~ limit for a proportion is given by, $p \pm 1.96\sqrt{pq/n}$

$\Rightarrow$ 98% of confidence limit for a proportion is given by $p \pm 2.\cancel{58}^{\,33}\sqrt{pq/n}$

$\Rightarrow$ 99% of confidence limit for proportion is given by $p \pm 2.\cancel{33}^{\,58}\sqrt{pq/n}$

Confidence interval for difference b/w 2 proportion:

Let P_1 and P_2 be two binomial parameter we want to estimate the difference between 2 binomial parameters P_1 and P_2 based on independent random sample of size n_1 and n_2. For large n_1 and n_2

$$Z = \frac{(P_1 - P_2) - (P_1 - P_2)}{\sqrt{\frac{P_1 q_1}{n_1} + \frac{P_2 q_2}{n_2}}} \sim N(0,1)$$

Area under normal probability curve is given by $P\left[-Z_{\alpha/2} \le Z \le Z_{\alpha/2}\right] = 1 - \alpha$

$$P\left[-Z_{\alpha/2} \le \frac{(P_1 - P_2) - (P_1 - P_2)}{\sqrt{\frac{P_1 q_1}{n_1} + \frac{P_2 q_2}{n_2}}} \le Z_{\alpha/2}\right] = 1 - \alpha$$

$$P\left[-Z_{\alpha/2} \cdot \sqrt{\frac{P_1 q_1}{n_1} + \frac{P_2 q_2}{n_2}} \le (P_1 - P_2) - (P_1 - P_2) \le Z_{\alpha/2} \sqrt{\frac{P_1 q_1}{n_1} + \frac{P_2 q_2}{n_2}}\right] = 1 - \alpha$$

$$P\left[-(P_1 - P_2) - Z_{\alpha/2} \sqrt{\frac{P_1 q_1}{n_1} + \frac{P_2 q_2}{n_2}} \le -(P_1 - P_2) \le -(P_1 - P_2) + Z_{\alpha/2} \sqrt{\frac{P_1 q_1}{n_1} + \frac{P_2 q_2}{n_2}}\right] = 1 - \alpha$$

$$P\left[(P_1 - P_2) + Z_{\alpha/2} \sqrt{\frac{P_1 q_1}{n_1} + \frac{P_2 q_2}{n_2}} \ge P_1 - P_2 \ge (P_1 - P_2) - Z_{\alpha/2} \sqrt{\frac{P_1 q_1}{n_1} + \frac{P_2 q_2}{n_2}}\right] = 1 - \alpha$$

$$\left[(P_1 - P_2) - Z_{\alpha/2} \sqrt{\frac{P_1 q_1}{n_1} + \frac{P_2 q_2}{n_2}}, (P_1 - P_2) + Z_{\alpha/2} \sqrt{\frac{P_1 q_1}{n_1} + \frac{P_2 q_2}{n_2}}\right]$$

$$(P_1 - P_2) \pm Z_{\alpha/2} \sqrt{\frac{P_1 q_1}{n_1} + \frac{P_2 q_2}{n_2}}$$

where $(1-\alpha)100\%$ confidence interval for the difference b/w 2 proportion is

$$(P_1 - P_2) \pm Z_{\alpha/2} \sqrt{\frac{P_1 q_1}{n_1} + \frac{P_2 q_2}{n_2}}$$

Properties:

$\Rightarrow$ 95% of confidence limit for a difference b/w 2 proportion is 14

$$(P_1 - P_2) \pm 1.96 \sqrt{\frac{P_1 q_1}{n_1} + \frac{P_2 q_2}{n_2}}$$

$\Rightarrow$ 98% of confidence limit for a diff b/w 2 proportion is

$$(P_1 - P_2) \pm 2.33 \sqrt{\frac{P_1 q_1}{n_1} + \frac{P_2 q_2}{n_2}}$$

$\Rightarrow$ 99% of confidence limit for a diff b/w 2 proportion is

$$(P_1 - P_2) \pm 2.58 \sqrt{\frac{P_1 q_1}{n_1} + \frac{P_2 q_2}{n_2}}$$

Problems:

Test of Significance for Single mean:

A sample of 900 members as a mean 3.4 cm and S.D 2.61 cm. The sample from a large population of mean 3.25 cm and S.D 2.61 cm find the 95% and 98% fiducial limit (confidence limit) of means.

Soln.

$n = 900$

$\bar{x} = 3.4$ cm ; $\mu = 3.25$ cm

$\sigma = 2.61$ cm ; $S = 2.61$ cm

98% confidence limit for the population mean are given by

$$\bar{x} \pm 2.33 \, \sigma/\sqrt{n}$$

$$3.4 + 2.33 \times \frac{2.61}{30}$$

$3.4 - 2.33 \times \frac{2.61}{30} = 3.60271 ; 3.19729$

95% confidence limit for the population mean (μ)

$\bar{x} \pm 1.96\ \sigma/\sqrt{n}$

$= 3.4 \pm 1.96 \times \frac{2.61}{30}$

$= 3.4 + 1.96 \times \frac{2.61}{30} ; 3.4 - 1.96 \times \frac{2.61}{30}$

$= 3.57052 ; 3.22948.$

Test of Significance of diff two means

In certain factory there are 2 independent process manufacturing the same item the average weight in a sample of 250 item reduce from one process is found to be 120 Ozs (Occurences) with a S.D of 12 Ozs while the corresponding figures in a sample of 400 items from the other process of 124 and 14 find the 99% confidence limit for the difference in the average weight of item reduced by 2 process respectively.

Soln:

Usual notation

we are given $n_1 = 250$, $n_2 = 400$

$\bar{x}_1 = 120$ (Ozs) $\bar{x}_2 = 124$ (Ozs) $S_1 = 12$ (Ozs) $= \hat{\sigma}_1$

$S_2 = 14$ (Ozs) $= \hat{\sigma}_2$

The fiducial confidence limits for the difference of 2 means is given by

$$(\bar{x}_1 - \bar{x}_2) \pm 2.58 \sqrt{\frac{\hat{\sigma}_1^2}{n_1} + \frac{\hat{\sigma}_2^2}{n_2}}$$

$$= 120 - 124 \pm 2.58 \sqrt{\frac{(12)^2}{250} + \frac{(14)^2}{400}}$$

$$= 120 - 124 + 2.58 \sqrt{\frac{144}{250} + \frac{196}{400}} \; ; \; 120 - 124 - 2.58 \sqrt{\frac{144}{250} + \frac{196}{400}}$$

$$= -1.33622 \; ; \; -6.66377$$

$$= 11.336221 \quad = 16.663771$$

$\therefore$ 6.67 and 1.33

Test of Significant for single proportion

A random Sample of 500 apples are taken from a range consignment and 60 were found to be bad. Obtain the 95%, 98% confidence limit for the % of bad apple in consignment.

Soln

we have

p = proportion of bad apple in a Sample

$$= \frac{60}{500} = 0.12.$$

$$q = 1 - p = 1 - 0.12 = 0.88$$

confidence limit for 95% for single proportion is given by

$$= p \pm 1.96 \sqrt{\frac{(0.12)(0.88)}{500}}$$

$$= 0.12 + 1.96 \sqrt{\frac{(0.12)(0.88)}{500}} \; ; \; 0.12 - 1.96 \sqrt{\frac{(0.12)(0.88)}{500}}$$

$$= 0.148 \; ; \; 0.091$$

Confidence limit for 98%

$= 0.12 + 2.58\sqrt{\frac{(0.12)(0.88)}{500}} \; ; \; 0.12 - 2.58\sqrt{\frac{(0.12)(0.88)}{500}}$

$= 0.12 + 0.037 \; ; \; 0.12 - 0.037$

$= 0.157 \; ; \; 0.083$

Test of significant b/w 2 different proportion

In a large city A, 20% of random Sample of 900 School children had defective eye-sight. In another large city B 15% of random Sample of 1600 children had same defective. Obtain 95% confidence limit for the difference in the population proportion.

Soln:

$n_1 = 900 \qquad n_2 = 1600$

$P_1 = \frac{20}{100} = 0.2 \qquad P_2 = \frac{15}{100} = 0.15$

$q_1 = 1 - P_1 = 1 - 0.2 = 0.8 \qquad q_2 = 1 - P_2 = 1 - 0.15 = 0.85$

95% confidence limit for the difference

$P_1 - P_2 \pm 1.96\sqrt{\frac{(0.2)(0.8)}{900} + \frac{(0.15)(0.85)}{1600}}$

$= 0.2 - 0.15 \pm 1.96\,(0.0160)$

$= 0.2 - 0.15 \pm 0.03136$

$= 0.05 \pm 0.03136$

$= 0.08136 \; ; \; 0.05.$