12

Sequencing Problem

"An order is to be observed in all things"

12:1. INTRODUCTION

The selection of an appropriate ***order*** for a series of jobs to be done on a finite number of service facilities, in some pre-assigned order, is called sequencing. A practical situation may correspond to an industry producing a number of products, each of which is to be processed through different machines, of course, finite in number.

12:2. PROBLEM OF SEQUENCING

Consider a problem of machine operator who has to perform three operations, namely (*i*) turning, (*ii*) threading, and (*iii*) knurling on a finite number of different jobs. Let there be six jobs and the time required to perform these operations (in minutes) for each job is known. Also it is given that each job first goes for turning, then for threading, and lastly for knurling. The problem of the machine operator is to decide which job should be processed first, which to process next, and so on, *i.e.*, the order (sequence) of the jobs for the above-mentioned operations in order to minimize the total time required to turn out all the jobs. This is an example of six-job and three-machine sequencing problem. We now consider the general case.

The *general* **sequencing problem** may be defined as : Let there be n jobs to be performed one at a time on each of m machines. The sequence (order) of the machines in which each job should be performed is given. The actual or expected time required by the jobs on each of the machines is also given. The general sequencing problem, therefore, is to find the sequence out of $(n\,!)^m$ possible sequences which minimize the total elapsed time between the start of the job in the first machine and the completion of the last job on the last machine.

Given below are the assumptions underlying a sequencing problem :

1. Each job, once started on a machine, is to be performed up to completion on that machine.
2. The processing time on each machine is known. Such a time is independent of the order of the jobs in which they are to be processed.
3. The time taken by each job in changing over from one machine to another is negligible.
4. A job starts on the machines as soon as the job and the machine both are idle and job is next to the machine and the machine is also next to the job.
5. No machine may process more than one job simultaneously.
6. The order of completion of job has no significance, *i.e.*, no job is to be given priority. The order of completion of jobs is independent of sequence of jobs.

12:3. BASIC TERMS USED IN SEQUENCING

Some of the basic terms in sequencing are :

1. *Number of machines* : It refers to the number of service facilities through which a job must pass before it is assumed to be completed.

2. *Processing order* : It refers to the order (sequence) in which given machines are required for completing the job.

3. *Processing time* : It indicates the time required by a job on each machine.

4. *Total elapsed time* : It is the time interval between starting the first job and completing the last job including the idle time (if any) in a particular order by the given set of machines.

5. *Idle time on a machine* : It is the time for which a machine does not have a job to process, *i.e.*, idle time from the end of job $(i-1)$ to the start of job i.

6. *No passing rule* : It refers to the rule of maintaining the order in which jobs are to be processed on given machines. For example, if n jobs are to be processed through three machines M_1, M_2 and M_3 in the order M_1, M_2, and M_3, then this rule will mean that each job will go to machine M_1 first, then to M_2 and lastly to M_3.

12:4. PROCESSING *n* JOBS THROUGH TWO MACHINES

Let there be n jobs, each of which is to be processed through two machines, say M_1 and M_2 in the order M_1M_2. That is, each job has to pass through the same sequence of operations. In other words, a job is assigned on machine M_1 first and after it has been completely processed on machine M_1, it is assigned to machine M_2. If the machine M_2 is not free at the moment for processing the same job, then the job has to wait in a waiting line for its turn on machine M_2, *i.e.*, passing is not allowed. Let t_{ij} $(i = 1, 2\,;\, j = 1, 2, \ldots, n)$ be the time required for processing ith job on the jth machine.

Since passing is not allowed, therefore machine M_1 will remain busy in processing all the n jobs one-by-one while machine M_2 may remain idle after completion of one job and before starting of another job. Thus the objective is to minimize the idle time of the second machine. Let X_{2j} be the time for which machine M_2 remains idle after finishing $(j-1)$th job and before starting processing the jth job $(j = 1, 2, \ldots, n)$. Clearly, the total elapsed time T is given by

$$T = \sum_{j=1}^{n} t_{2j} + \sum_{j=1}^{n} X_{2j}$$

where some of the X_{2j}'s may be zeros.

The problem now is to minimize T. However, since $\Sigma\, t_{2j}$ is the total time for which machine M_2 has to work and is thus fixed, it does not form a part of the optimization problem. Thus, the problem reduces to that of minimizing ΣX_{2j}. A very convenient procedure for obtaining a sequence of performing jobs to minimize ΣX_{2j} is well illustrated by the following *Gantt Chart* :

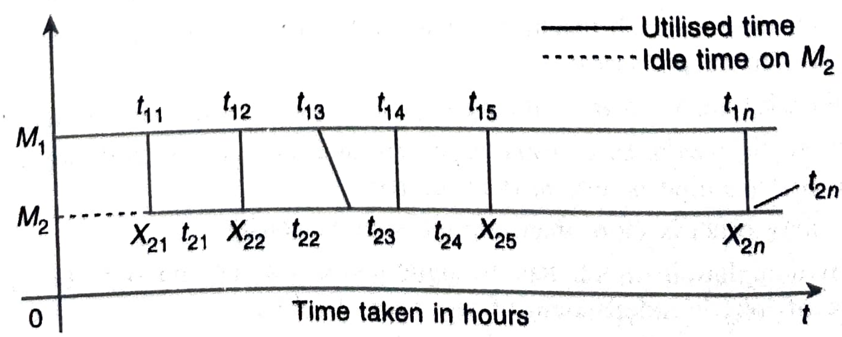

Fig. 12.1 Gantt Chart

From the chart, it is apparent that

$$X_{21} = t_{11}$$

$$X_{22} = \begin{cases} t_{11} + t_{12} - X_{21} - t_{21} & \text{if } t_{11} + t_{12} > X_{21} + t_{21} \\ 0 & \text{otherwise} \end{cases}$$

The expression for X_{22} may be rewritten as

$$X_{22} = \text{Max.} \{t_{11} + t_{12} - X_{21} - t_{21}, 0\}$$

Thus

$$X_{21} + X_{22} = \text{Max.} \{t_{11} + t_{12} - t_{21}, t_{11}\}, \text{ since } X_{21} = t_{11}.$$

Similarly

$$X_{23} = \text{Max.} \{t_{11} + t_{12} + t_{13} - t_{21} - t_{22} - X_{21} - X_{22}, 0\}$$

This gives

$$X_{21} + X_{22} + X_{23} = \text{Max.} \left\{ \left(\sum_{j=1}^{3} t_{1j} - \sum_{j=1}^{2} t_{2j} \right), \sum_{j=1}^{2} X_{2j} \right\}$$

$$= \text{Max.} \left\{ \left(\sum_{j=1}^{3} t_{1j} - \sum_{j=1}^{2} t_{2j} \right), \left(\sum_{j=1}^{2} t_{1j} - t_{2j} \right), t_{11} \right\}$$

In general, we have

$$\sum_{j=1}^{n} X_{2j} = \text{Max.} \left\{ \left(\sum_{j=1}^{n-1} t_{1j} - \sum_{j=1}^{n-1} t_{2j} \right), \left(\sum_{j=1}^{n-1} t_{1j} - \sum_{j=1}^{n-2} t_{2j} \right), \ldots, t_{11} \right\}$$

$$= \underset{1 \le u \le n}{\text{Max.}} \left\{ \sum_{j=1}^{n} t_{1j} - \sum_{j=1}^{n-1} t_{2j} \right\}.$$

Now, if we denote, $\sum_{j=1}^{n} X_{2j}$ by $D_n(S)$, then the problem becomes that of finding the sequence $< S^* >$ for processing the jobs 1, 2, ..., n so as to have the inequality $D_n(S^*) \le D_n(S_0)$ for any sequence $< S_0 >$ other than $< S^* >$. In other words, one has to determine an optimal sequence so as to minimize $D_n(S)$ applying the above rule. This can be achieved iteratively by successively interchanging the consecutive jobs. Each such interchange of jobs gives a value of $D_n(S)$ smaller than or equal to its value before the change.

Optimum Sequence Algorithm

The iterative procedure for determining the optimum sequence for n jobs on 2 machines can be summarized as follows :

Step 1 : List the jobs along with their processing times in a table as shown below :

		J_1	J_2	J_3	...	J_n
Job number	:					
Processing time on machine	M_1 :	t_{11}	t_{12}	t_{13}	...	t_{1n}
	M_2 :	t_{21}	t_{22}	t_{23}	...	t_{2n}

Step 2. Examine the rows for processing times on machines M_1 and M_2, and find the smallest processing time in each row, *i.e.*, find out min. (t_{1j}, t_{2j}) for all j.

Step 3. If the smallest processing time is for the first machine M_1, then place the corresponding job in the first available position in the sequence. If it is for the second machine, then place the corresponding job in the last available position in the sequence.

Step 4. If there is a tie in selecting the minimum of all the processing times, then there may be three situations :

(*a*) Minimum among all processing times is same for the machine, *i.e.*, min $(t_{1j}, t_{2j}) = t_{1k} = t_{2r}$, then process the kth job first and the rth job last.

(*b*) If the tie for the minimum occurs among processing times t_{1j} on machine M_1 only, then select arbitrarily the job to process first.

(*c*) If the tie for the minimum occurs among processing times t_{2j} on machine M_2, then select arbitrarily the job to process last.

Step 5. Cross off the jobs already assigned and repeat *steps 1* through *4*, placing the remaining jobs next to first or next to last, until all the jobs have been assigned.

Step 6. Calculate idle time for machines M_1 and M_2 :

(*a*) Idle time for M_1 = Total elapsed time – (time when the last job in a sequence finishes on M_1)

(*b*) Idle time for M_2 = Time at which the first job in a sequence finishes on M_1

$$+ \sum_{j=2}^{n} \{(\text{time when the } j\text{th job in a sequence starts on } M_2)$$

$$- (\text{time when the } (j-1)\text{th job in a sequence finishes on } M_2)\}.$$

Step 7. The total elapsed time to process all jobs through two machines is as under :

Total elapsed time = Time when the *n*th job in a sequence finishes on machine M_2.

$$= \sum_{j=1}^{n} t_{2j} + \sum_{j=1}^{n} I_{2j}$$

where t_{2j} = Time required for processing *j*th job on machine M_2.

I_{2j} = Time for which machine M_2 remains idle after processing $(j-1)$th job and before starting work on *j*th job.

Note. The procedure outlined above for the processing of *n* jobs on two machines gives us the minimum total *elapsed time*.

SAMPLE PROBLEMS

1201. *In a factory, there are six jobs to perform, each of which should go through two machines A and B, in the order A, B. The processing timings (in hours) for the jobs are given here. You are required to determine the sequence for performing the jobs that would minimize the total elapsed time, T. What is the value of T ?*

Job :	J_1	J_2	J_3	J_4	J_5	J_6
Machine A :	1	3	8	5	6	3
Machine B :	5	6	3	2	2	10

[Madras M.B.A. 2004]

Solution. The smallest processing time in the given problem is 1 on machine A. So, perform J_1 in the beginning as shown below :

J_1					

The reduced set of processing times becomes

Job :	J_2	J_3	J_4	J_5	J_6
Machine *A* :	3	8	5	6	3
Machine *B* :	6	3	2*	2*	10

The minimum processing time in this reduced problem is 2 which coresponds to J_4 and J_5 both on machine *B*. Since the corresponding processing time of J_5 on machine *A* is larger than the corresponding processing time of J_4 on machine *A*, J_5 will be processed in the last and J_4 shall be processed next to last. The updated job sequence is

J_1				J_4	J_5

The remaining processing times are :

Job	:	J_2	J_3	J_6
Machine *A*	:	3*	8	3*
Machine *B*	:	6	3*	10

Now, there is a tie among 3 jobs for the smallest processing time in this reduced problem. These correspond to J_2 and J_6 on machine *A*, and to J_3 on machine *B*. As the corresponding processing time of J_6 on machine *B* is larger than the corresponding processing time of J_2 on machine *B*, J_6 will be processed next to J_1. Now *step 4(a)* applies and J_2 should be placed next. The updated job sequence is

J_1	J_6	J_2	J_3	J_4	J_5

This sequence is the optimum one. The total elapsed time is calculated below :

Job	Machine *A*		Machine *B*		Idle time on *B*
	In	*Out*	*In*	*Out*	
J_1	0	1	1	6	1
J_6	1	4	6	16	—
J_2	4	7	16	22	—
J_3	7	15	22	25	—
J_4	15	20	25	27	—
J_5	20	26	27	29	—

From the above information, we get T = 29 hours.

Idle time of machine *A* is (29 – 26) = 3 hours and that for machine *B* is one hour.

The Gantt chart for the above problem can be illustrated as below :

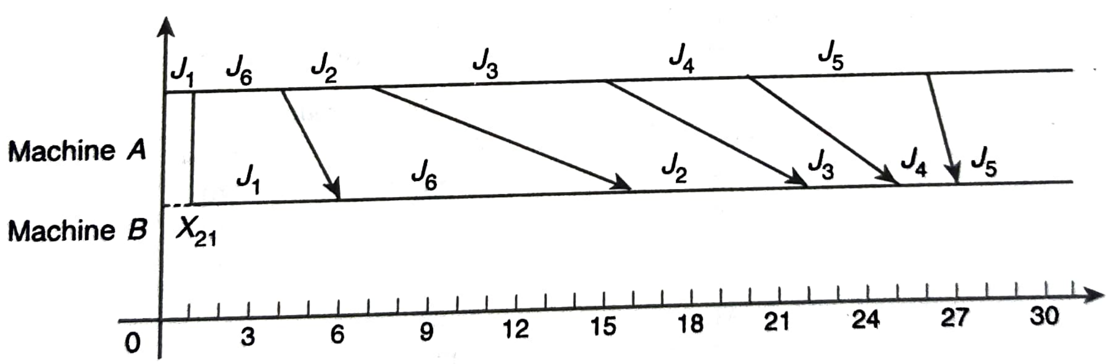

Fig. 12.2

1202. *A book binder has one printing press, one binding machine, and the manuscripts of a number of different books. The time required to perform the printing and binding operations for each book is shown below. Determine the order in which books should be processed, in order to minimize the total time required to turn out all the books :*

Book	:	*1*	*2*	*3*	*4*	*5*	*6*
Printing time (hrs.)	:	30	120	50	20	90	100
Binding time (hrs.)	:	80	100	90	60	30	10

[Panjab B.Com. 2006]

Solution. Here, the books will first go to the printing press and then on the binding machine. If P_i $(i = 1, 2, ..., 6)$ denotes the time in hours on printing press and B_i $(i = 1, 2, ..., 6)$ the binding time for books, then since min. $\{P_i, B_i\}$ = 10 corresponding to B_6, book 6 will be processed in the last. The problem then reduces to the following five and two machines :

Book	:	*1*	*2*	*3*	*4*	*5*
P_i	:	30	120	50	20*	90
B_i	:	80	100	90	50	30

Now, Min. $\{P_i, B_i\}$ = 20 which corresponds to P_4. Therefore, book *4* will be processed just in the beginning. The entries are shown in the sequence cells as below :

4					*6*

After assigning books *4* and *6*, we are now left with 4 books and two machines with their processing times as follows :

Book	:	*1*	*2*	*3*	*5*
P_i	:	30*	120	50	90
B_i	:	80	100	90	30*

Now, the minimum of P_i and B_i is 30 which corresponds to P_1 and B_5, *i.e.*, there is a tie for the minima. So, we place book *1* next to the first and book *5* next to the last, yielding us the sequence.

4	*1*			*5*	*6*

We are now left with the problem of 2 jobs and 2 machines with their respective processing times as follows :

Book	:	*2*	*3*
P_i	:	120	50*
B_i	:	100	90

Here, since smallest printing time is 50 hours for book *3*, we place book *3* in the third cell and the remaining book *2* in the fourth cell and get the following optimal sequence :

4	*1*	*3*	*2*	*5*	*6*

The minimum elapsed time from the start of the first book to the completion of the last book corresponding the optimal sequence is computed as shown in the following table :

Book	*Printing machine*		*Binding machine*		*Idle time of binding machine*
	Time in	*Time out*	*Time in*	*Time out*	
4	0	20	20	80	20
1	20	50	80	160	0
3	50	100	160	250	0
2	100	220	250	350	0
5	220	310	350	380	0
6	310	420	420	430	40

From the above table it is clear that *minimum elapsed time* is 430 hours. *Idle time* for printing machine is 10 hours (from 420 hours to 430 hours) and for binding machine is 20 + 40 = 60 hours.

Remarks 1. It may be noted that the total elapsed time is equal to the sum of the idle time of binding machine and the total processing time on binding machine.

2. The total elapsed time can also be calculated by using Gnatt chart.

PROBLEMS

1203. We have five jobs, each of which must go through the two machines *A* and *B* in the order *AB*. Processing times in hours are given in the table below :

Job (*i*)	:	*1*	2	3	4	5
Machine *A* (A_i)	:	5	1	9	3	10
Machine *B* (B_i)	:	2	6	7	8	4

Determine a sequence for the five jobs that will minimize the elapsed time. *[Panjab M.C.A. 2007]*

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1204. A ready-made garment company has to process 7 items through two stages of production, *viz.*, cutting and sewing. The time taken by each of these items at the different stages are given as :

Item	*1*	*2*	*3*	*4*	*5*	*6*	*7*
Cutting	4	8	3	5	5	12	7
Sewing	3	5	6	4	8	5	8

Find an order in which these seven items are to be processed so as to minimize the total processing time.

[Delhi M.Com. 2010]

1205. A company has six jobs on hand coded 'A' to 'F'. All the jobs have to go through two machines '*M I*' and '*M II*'. The time required for each job on each machine, in hours, is given below :

	A	*B*	*C*	*D*	*E*	*F*
M I	3	12	18	9	15	6
M II	9	18	24	24	3	15

Draw a sequence table scheduling the six jobs on the two machines. *[Punjabi M.B.A. 2005]*

1206. Seven jobs are to be processed on two machines *A* and *B* in the order $A \rightarrow B$. Each machine can process only one job at a time. The processing times (in hours) are as follows :

Job	*1*	*2*	*3*	*4*	*5*	*6*	*7*
Machine *A*	10	12	13	7	14	5	16
Machine *B*	15	11	8	9	6	7	16

Suggest optimum sequence of processing the jobs and the total elapsed time. *[Delhi M.Com. 2006]*

1207. In the machine shop, 8 different products are being manufactured each requiring time on two machines *A* and *B* as given below :

Product	*Time (in min.) on machine A*	*Time (in min.) on machine B*
I	30	20
II	45	30
III	15	50
IV	20	35
V	80	36
VI	120	40
VII	65	50
VIII	10	20

Decide the optimum sequence of processing of different products in order to minimize the total manufacturing time for all the products. Name and discuss the scheduling model used.

[A.I.M.A. P.G. Dip. in Management (June) 1996]

1208. The maintenance crew of a company is divided in two parts C_1 and C_2 which cares for the maintenance of the seven machines $M_1, M_2, \ldots, M_7$ in the factory. Crew C_1 is responsible for replacement of the parts which are worn out while crew C_2 oils and resets the machines back for operation. The time required by crews C_1 and C_2 on different machines is given below :

Machines	*Maintenance time (in hrs.)*	
	Crew C_1	*Crew* C_2
M_1	24	10
M_2	14	20
M_3	22	8
M_4	20	18
M_5	30	10
M_6	28	6
M_7	16	4

In what order the machines should be handled by crew C_1 and C_2 so that the total time taken is minimized. Also determine what shall this time be?

12:5. PROCESSING *n* JOBS THROUGH *k* MACHINES

There is no general method available by which we can obtain optimal sequence(s) in problems involving processing of n jobs on k machines. They can be handled only by enumeration, which is a very lengthy and time-consuming exercise because a total of $(n\,!)^k$ different sequences would require consideration in such a case. However, we do have a method applicable under the condition that no passing of jobs is permissible and if either or both of the conditions stipulated below is/are satisfied.

Let there be n jobs, each of which is to be processed through k machines, say $M_1, M_2, \ldots, M_k$ in the order $M_1, M_2, \ldots, M_k$. The list of jobs with their processing times is :

	Job Number	:	1	2	3	...	n
Processing time on machine	M_1	:	t_{11}	t_{12}	t_{13}	...	t_{1n}
	M_2	:	t_{21}	t_{22}	t_{23}	...	t_{2n}
	M_3	:	t_{31}	t_{32}	t_{33}	...	t_{3n}
	⋮		⋮	⋮	⋮	⋮	⋮
	M_k	:	t_{k1}	t_{k2}	t_{k3}	...	t_{kn}

An optimum solution to this problem can be obtained, if either or both of the following conditions hold :

(*a*) $\min. t_{1j} \geq \max. t_{ij}$, for $i = 2, 3, \ldots, k-1$,

or (*b*) $\min. t_{kj} \geq \max. t_{ij}$, for $i = 2, 3, \ldots, k-1$.

Optimal Sequence Algorithm

The iterative procedure for determining the optimal sequence for 'n' jobs on 'k' machines can be summarized as follows :

Step 1. Find min. t_{1j}, min. t_{kj} and maximum of each of

$$t_{2j}, t_{3j}, \ldots, t_{k-1,j} \qquad \text{for all } j = 1, 2, \ldots, n.$$

Step 2. Check the following :

(*a*) $\min. t_{1j} \geq \max. t_{ij}$, for $i = 2, 3, \ldots, k-1$

or (*b*) $\min. t_{kj} \geq \max. t_{ij}$, for $i = 2, 3, \ldots, k-1$.

Step 3. If the inequalities of *step* 2 are not satisfied, method fails. Otherwise go to next step.

Step 4. Convert the k machine problem into a two-machine problem by introducing two fictitious machines G and H, such that

$$t_{Gj} = t_{1j} + t_{2j} + \ldots + t_{k-1,j}$$

and

$$t_{Hj} = t_{2j} + t_{3j} + \ldots + t_{kj}.$$

Step 5. Determine the optimal sequence for the 'n' jobs and '2' machines equivalent sequencing problem with the prescribed order GH in the same way as discussed earlier. The resulting sequence shall be optimum for the given problem.

Remarks 1. In addition to conditions given in *Step 4*, if

$$t_{2j} + t_{3j} + \ldots + t_{k-1,j} = c,$$

is a fixed positive constant for all $i = 1, 2, \ldots, n$, then determine the optimal sequence for n jobs and two machines M_1 and M_k in the order M_1, M_k by using the optimal sequence algorithm.

2. In addition to the conditions given in *Step 4*, if

$$t_{1j} = t_{kj} \text{ and } t_{Gj} = t_{Hj}, \qquad \text{for } j = 1, 2, \ldots, n,$$

then there will be $n\,!$ optimal sequences, each of which will yield minimum total elapsed time.

3. The above-mentioned procedure of solving the sequencing problems is not a general procedure. The method is applicable only to those sequencing problems in which the minimum cost (time) of processing the jobs through first and/or last machine is greater than or equal to the cost of processing the jobs through mediocre machines. There are many industrial operations, in which the machines are set in some order which does not obey this rule.

SAMPLE PROBLEMS

1209. *Determine the optimal sequence of jobs that minimizes the total elapsed time based on the following information processing time on machines is given in hours and passing is not allowed :*

Job	:	*A*	*B*	*C*	*D*	*E*	*F*	*G*
Machine M_1	:	3	8	7	4	9	8	7
Machine M_2	:	4	3	2	5	1	4	3
Machine M_3	:	6	7	5	11	5	6	12

[Madras B.E. 1999; Delhi M.Com. 2006]

Solution. We are given 7 jobs each of which is to be processed through 3 machines M_1, M_2 and M_3 in the order M_1 M_2 M_3.

Therefore, for $n = 7$ and $k = 3$; we observe that $\min. t_{1j} = 3$, $\min. t_{3j} = 5$ and $\max. t_{2j} = 5$. Since $\min. t_{3j} \geq \max. t_{2j}$ is satisfied, the problem can be converted into that of 7 jobs and 2 machines.

Thus, if G and H are the two machines, such that

$$G_i = t_{1j} + t_{2j}$$

and

$$H_i = t_{2j} + t_{3j}, \quad \text{for } j = 1, 2, \ldots, 7$$

then the problem can be rewritten as the following 7 jobs and 2 machines problem :

Job	*A*	*B*	*C*	*D*	*E*	*F*	*G*
G	3 + 4 = 7	8 + 3 = 11	7 + 2 = 9	4 + 5 = 9	9 + 1 = 10	8 + 4 = 12	7 + 3 = 10
H	4 + 6 = 10	3 + 7 = 10	2 + 5 = 7	5 + 11 = 16	1 + 5 = 6	4 + 6 = 10	3 + 12 = 15

Using the optimal sequence algorithm, the following optimal sequence can easily be obtained :

A	D	G	B	F	C	E

For total elapsed time, we have

		Job						
		A	*D*	*G*	*B*	*F*	*C*	*E*
M_1	In :	0	3	7	14	22	30	37
	Out :	3	7	14	22	30	37	46
M_2	In :	3	7	14	22	30	37	46
	Out :	7	12	17	25	34	39	47
M_3	In :	7	13	24	36	43	49	54
	Out :	13	24	36	43	49	54	59

This table indicates that the minimum total elapsed times is 59 hours. Idle time is 13 hours for M_1, 37 hours for M_2 and 7 hours for M_3.

1210. *We have 4 jobs each of which has to go through the machines M_j ($j = 1, 2, \ldots, 6$) in the order $M_1, M_2, \ldots, M_6$. Processing time (in hours) is given below :*

		Machines					
		M_1	M_2	M_3	M_4	M_5	M_6
Job A	:	18	8	7	2	10	25
Job B	:	17	6	9	6	8	19
Job C	:	11	5	8	5	7	15
Job D	:	20	4	3	4	8	12

Determine a sequence of these four jobs that minimizes the total elapsed time. *[Meerut M.Com. 1998]*

Solution. Here min. t_{1j} = 11 and min. t_{6j} = 12 ; max. of t_{2j}, t_{3j}, t_{4j} and t_{5j} are 8, 9, 6 and 10 respectively.

Since, the conditions min. $t_{2j} \geq$ max. t_{ij} and min. $t_{6j} \geq$ max. t_{ij}, for (j = 1, 2, 3, 4, 5) are satisfied, the given problem can be written as :

Job	:	*A*	*B*	*C*	*D*
Machine *G*	:	45	46	36	39
Machine *H*	:	52	48	40	31

where $G_i = \sum_{j=1}^{5} t_{ij}$ and $H_i = \sum_{j=2}^{6} t_{ij}$

Using the optimal sequence algorithm, the following optimal sequence is easily obtained :

C	*A*	*B*	*D*

The total elapsed time is given in the following table :

Job	*Machines*					
	M_1	M_2	M_3	M_4	M_5	M_6
C	0-11	11-16	16-24	24-29	29-36	36-51
A	11-29	29-37	37-44	44-46	46-56	56-81
B	29-46	46-52	52-61	61-67	67-75	81-100
D	46-66	66-70	70-73	73-77	77-85	100-112

This table shows that the minimum total elapsed time is 112 hours.

1211. *Solve the following sequencing problem when passing out is not allowed :*

Item	*Machine (Processing time in hours)*			
	A	*B*	*C*	*D*
I	15	5	4	15
II	12	2	10	12
III	16	3	5	16
IV	17	3	4	17

Solution. Here, min. A_i = 12, and max. B_i = 5 and max. C_i = 10.

Since min. $A_i \geq$ max. B_i and max. C_i, the given problem can be rewritten as :

Item	:	*I*	*II*	*III*	*IV*
Machine *G*	:	24	24	24	24
Machine *H*	:	24	24	24	24

where $G_i = A_i + B_i + C_i$ and $H_i = B_i + C_i + D_i$.

Since $G_i = H_i$ and $A_i = D_i$ are satisfied, therefore using the optimal sequence algorithm, we get 4! (= 24) sequence, each giving us an optimal sequence :

I II III IV, *I II IV III*, *I III II IV*, *I III IV II*, *I IV II III*, etc.

Each of the above optimal sequence will yield us the same total elapsed time. For the elapsed time, we have

		I	*II*	*III*	*IV*
	Item				
Machine *A*	In	0	15	27	43
	Out	15	27	43	60
Machine *B*	In	15	27	43	60
	Out	20	29	46	63

	Item	I	II	III	IV
Machine C	In	20	29	46	63
	Out	24	39	51	67
Machine D	In	24	39	56	67
	Out	39	51	67	84

Total elapsed time is 84 hours.

PROBLEMS

1212. Given the following data :

(a)

Job	:	1	2	3	4	5	6
Machine A	:	12	10	9	14	7	9
Machine B	:	7	6	6	5	4	4
Machine C	:	6	5	6	4	2	4

(b) Order of processing jobs : *A C B*

(c) Sequence suggested : Jobs 5, 3, 6, 2, 1, 4.

(i) Determine the total elapsed time for the sequence suggested.

(ii) Is the given sequence optimal?

(iii) If your answe is 'No', determine the optimal sequence and the total elapsed time associated with it.

1213. We have five jobs, each of which must go through machines *A*, *B* and *C* in the order *A B C*. Processing times (in hours) are given in the following table :

Job	:	1	2	3	4	5	6	7
Machine A (A_i)	:	7	8	11	14	21	17	8
Machine B (B_i)	:	6	3	1	2	5	4	1
Machine C (C_i)	:	10	9	15	13	18	11	9

[Delhi M.B.A. (PT.) 2009]

1214. Determine the optimal sequence of jobs that minimizes the total elapsed time based on the following information :

Job	:	1	2	3	4	5	6	7
Machine A	:	10	8	12	6	9	11	9
Machine B	:	6	4	6	5	3	4	2
Machine C	:	8	7	5	9	10	6	5

Processing time on machines is given in hours and passing is not allowed. *[Delhi M.Com.2008]*

1215. Find the sequence that minimizes the total time required in performing the following jobs on three machines in the order *ABC* :

Processing time (in hours) on	Job					
	1	2	3	4	5	6
Machine A	8	3	7	2	5	1
Machine B	3	4	5	2	1	6
Machine C	8	7	6	9	10	9

[Meerut M.Sc. (Math.) 1997]

1216. (a) A book binder has one printing press, one binding machine and the manuscripts of a number of different books. The time required to perform the printing and binding operation for each book is known. Determine the order in which the books should be processed in order to minimize the total time required to process all the books. Find also the total time required. (Clearly state any algorithm you might use.)

Processing time (in minutes)

Book	1	2	3	4	5
Printing time	40	90	80	60	50
Binding time	50	60	20	30	40

(b) Suppose that an additional operation is added to the process described in (a), *viz.*, finishing. The time required for this operation is given below :

Finishing time (in minutes)

Book	1	2	3	4	5
Finishing time	80	100	60	70	100

What is the order in which the books should be processed? Find also the minimum total elapsed time.

1217. A readymade garments manufacturer has to process 7 items through two stages of production, *viz.*, cutting and sewing. The time taken for each of these at the different stages are given below in appropriate units :

	Item	*1*	2	*3*	*4*	5	*6*	7
Process time	Cutting	5	7	3	4	6	7	12
	Sewing	2	6	7	5	9	5	8

(*a*) Find an order in which these items are to be processed through these stages so as to minimize the total processing time.

(*b*) Suppose a third stage of production is added, *viz.*, pressing and packing, with processing time for these items as follows :

Item	*1*	2	*3*	*4*	*5*	*6*	7
Processing time (Pressing and packing)	10	12	11	13	12	10	11

Find an order in which these seven items are to be processed so as to minimize the time taken to process all the items through all the three stages. *[IAS 1991]*

1218. Solve the following sequencing problem, giving an optimal solution when passing is not allowed :

	Job				
	A	*B*	*C*	*D*	*E*
Machine M_1	10	12	8	15	16
Machine M_2	3	2	4	1	5
Machine M_3	5	6	4	7	3
Machine M_4	14	7	12	8	10

1219. Solve the following sequencing problem, giving an optimal solution when passing is not allowed :

	Machine (Processing time in hours)			
Job	M_1	M_2	M_3	M_4
1	20	3	3	25
2	12	5	1	11
3	18	4	2	10
4	17	2	4	28

[Madras M.B.A. 1996]

1220. When passing is not allowed, solve the sequencing problem giving an optimal solution :

	Machines (processing time in hours)			
Job	M_1	M_2	M_3	M_4
A	11	4	2	11
B	8	1	8	8
C	12	2	3	12
D	13	2	2	13

[Meerut M.Sc. (Math.) 1999]

12:6. PROCESSING 2 JOBS THROUGH *k* MACHINES

Let there be two jobs *1* and 2 each of which is to be processed on *k* machines say $M_1, M_2, \ldots, M_k$ in two different orders. The technological ordering of each of the two jobs through *k* machines is known in advance. Such ordering may not be same for both the jobs. The exact or expected processing times on all the given machines are known. Each machine can perform only one job at a time. The objective is to determine an optimal sequence of processing the jobs so as to minimize total elapsed time.

The optimal sequence in this case can be obtained by making use of graph. The solution procedure can be summarised in the following steps :

Step 1. Draw two perpendicular lines, horizontal one representing the processing time for job *1* while job 2 remains idle, and the vertical one representing the processing time for job 2 while job *1* remains idle.

Step 2. Mark the processing time for jobs *1* and 2 on the horizontal and vertical lines respectively according to the given order of machines.

Step 3. Construct various blocks starting from the origin (starting point) by pairing the same machines until the end point.

Step 4. Draw the line starting from origin to end point by moving horizontally, vertically and diagonally along a line which makes an angle of 45° with the horizontal line (base). The horizontal segment of this line indicates that first job is under process while second job is idle. Similarly, the vertical segment of the line indicates that the second job is under process while first job is idle. The diagonal segment of the line shows that both the jobs are under process simultaneously.

Step 5. An optimum path is one that minimizes the idle time for both the jobs. Thus, we must choose the path on which diagonal movement is maximum.

Step 6. The total elapsed time is obtained by adding the idle time for either job to the processing time for that job.

SAMPLE PROBLEM

1221. *Use graphical method to minimize the time added to process the following jobs on the machines shown, i.e., for each machine find the job which should be done first. Also calculate the total time elapsed to complete both the jobs :*

Job 1	*Sequence*	A	B	C	D	E
	Time	3	4	2	6	2
Job 2	*Sequence*	B	C	A	D	E
	Time	5	4	3	2	6

[Karnataka B.E. (Prod.) 1994; Meerut M.Sc. (Math.) 1996]

Solution. The solution procedure for solving the above problem can be summarised in the following steps.

Step 1. Draw the set of axes at right angle to each other where *X*-axis represents the processing time of job *1* on different machines while job 2 remains idle and *Y*-axis represents processing time of job 2 while job *1* remains idle.

Step 2. Mark the processing times for jobs *1* and 2 on *X*-axis and *Y*-axis respectively according to the given order of machines as shown in Fig. 12.3 :

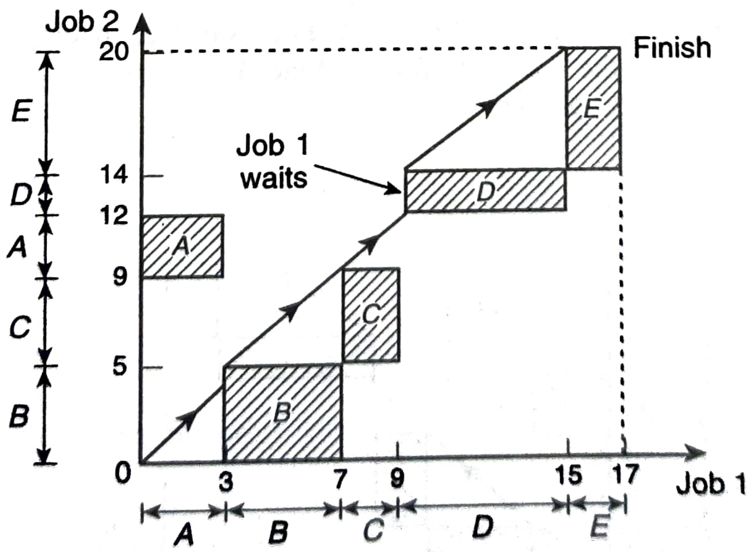

Fig. 12.3

For example, machine *A* takes 3 hours for job *1* and 3 hours for job 2. Construct the rectangle for machine *A* as shown in Fig. 12.3. Similarly construct other rectangles for machine *B, C,D* and *E*.

Step 3. Construct various blocks starting from the origin by pairing the same machines until a point marked 'finish' is obtained.

Step 4. Draw a line starting from origin to the point marked '*finish*' by moving horizontally, vertically and diagonally along a line which makes an angle of 45° with the horizontal axis. Moving horizontally along this line indicates that first job is under process while second job is idle. Similarly, moving vertically along this line indicates that the second job is under process while first job is idle. The diagonal movement along this line shows that both the jobs are under process simultaneously.

Since, simultaneous processing of both the jobs on a machine is not possible, therefore diagonal movement is not allowed. In other words, diagonal movement through rectangles areas is not allowed.

Step 5. An *optimal path* is one that minimizes the idle time for both the jobs. Thus, we must choose the path on which diagonal movement is maximum as shown in Fig. 12.3.

For the elapsed total time, we shall add the idle time for either of the two jobs to the processing time of that job. Now, since the idle time for the chosen path is 5 hours for job *1* and 2 hours for job 2, the total elapsed time is obtained as follows :

Processing time of job *1* + idle time for job *1*

$$= 17 + (2 + 3) = 22 \text{ hours.}$$

Processing time of job 2 + idle time for job 2

$$= 20 + 2 = 22 \text{ hours.}$$

1222. *Using graphical method, calculate the minimum time needed to process job 1 and 2 on five machines A, B, C, D and E, i.e., for each machine find the job which should be done first. Also calculate the total time needed to complete both jobs.*

			Machines				
Job 1	*Sequence*	:	A	B	C	D	E
	Time (hours)	:	6	8	4	12	4
Job 2	*Sequence*	:	B	C	A	D	E
	Time (hours)	:	10	8	6	4	12

Solution. Draw two axes at right angle to each other where *X*-axis represents the processing time of job *1* on different machines while job 2 remains idle, and *Y*- axis represents the processing time of job 2 on different machines while job *1* remains idle.

Mark the processing times for both the jobs *1* and 2 on *X*-axis and *Y*-axis respectively according to the given order of machines as shown in Fig. 12.4. For example, machine *A* takes 6 hours for job *1* as well as for job 2. Construct the rectangle for machine *A* as shown Fig. 12.4. Similarly, construct other rectangles for machines *B*, *C*, *D* and *E*.

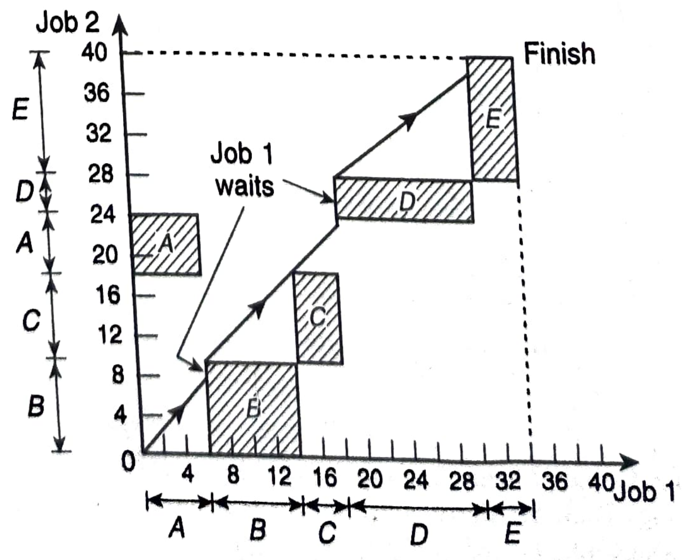

Fig. 12.4

Draw a line starting from origin to the point marked 'finished' by moving horizontally, vertically and diagonally along a line which makes an angle of 45° with the horizontal axis. A horizontal move represents processing of job *1* while job 2 remains idle; a vertical move represents processing of job 2 while job *1* remains idle. The diagonal movement (*i.e.*, a 45° line) shows that both the jobs are under process simultaneously. Since simultaneous processing of both the jobs on a machine is not possible, therefore diagonal movement is not allowed.

An *optimal path* is one that minimizes idle time for both job *1* and 2. This means, this path should coincide with 45° line to the maximum extent. Thus, we must choose the path on which diagonal movement is maximum, as shown in Fig. 12.4. The *total elapsed time* is obtained by adding the idle time for either job to the processing time for that job.

The idle time for the chosen path is : Job *1* = 4 + 6 or 10 hours and Job 2 = 4 hours.

The total elapsed time is obtained as follows :

Processing time of Job *1* + Idle time of Job *1*

$= 34 + (4 + 6) = 44$ hours.

Processing time of Job 2 + Idle time of Job 2

$= 40 + (32 - 28) = 44$ hours.

PROBLEMS

1223. A machine shop has four machines *A*, *B*, *C* and *D*. Two jobs must be processed through each of these machines. The time (in hours) taken on each of the machines and the necessary sequence of jobs through the shop are given below :

Job 1	Sequence	*A*	*B*	*C*	*D*
	Time	2	4	5	1
Job 2	Sequence	*D*	*B*	*A*	*C*
	Time	6	4	2	3

Use graphic method to obtain the total minimum elapsed time.

1224. Use graphic method to find the minimum elapsed total time sequence of 2 jobs and 5 machines, when we are given the following information :

		Machines				
Job 1	Sequence	*A*	*B*	*C*	*D*	*E*
	Time (in hours)	2	3	4	6	2
Job 2	Sequence	*C*	*A*	*D*	*E*	*B*
	Time (in hours)	4	5	3	2	6

[Meerut M.Sc. (Math.) 2000; Karnataka B.E. (Mech.) 1994]

1225. Two jobs are to be processed on four machines *a*, *b*, *c* and *d*. The technological order for these jobs on machines is as follows :

Job 1 :	*a*	*b*	*c*	*d*
Job 2 :	*d*	*b*	*a*	*c*

Processing times are given in the following table :

Job	*Machine* *a*	*b*	*c*	*d*
1	4	6	7	3
2	4	7	5	8

Find the optimal sequence of jobs on each of the machines.

1226. A machine shop has six machines *A, B, C, D, E* and *F*. Two jobs must be processed through ea machines. The times on machines and the necessary sequence of the jobs through the shop are given below :

Order	*1*	*2*	*3*	*4*	*5*	*6*
Job *I*	*A*-20	*C*-10	*D*-10	*B*-30	*E*-25	*F*-16
Job *II*	*A*-10	*C*-30	*B*-15	*D*-10	*F*-15	*E*-20

Determine the optimum sequence for the job in order to minimize the total time necessary to finish the j

12:7. MAINTENANCE CREW SCHEDULING

The problem of scheduling job in a factory can be extended to scheduling the factory maintena crews in such a way so as to minimize their idle time. Let a company have a set of different machi in its plant that need preventive maintenance. The crew team is divided into two groups *A* and First, crew *A* takes the machine and replaces the parts according to the needs. Then crew *B* resets machine and puts it back into operation. If the service times of both the groups on the differ machines are known, we can determine the optimal sequence in which the maintenance jobs be d so that the idle time of the crews is the least.

Like for sequencing problems given earlier, the rules when the service times are given in the ro form may be summarised as :

(*i*) Choose the smallest of all values that appear in the two rows.

(*ii*) If the value is in the first row then the machine corresponding to the value shall be servic by crew *A* in the beginning.

(*iii*) If it is in the second row, then that machine will be serviced by crew *A* in the end.

(*iv*) If there is a tie in the same row, either of the machines involved is selected and assigned t first, or the last, place accordingly as the tie is in the first or the second row.

(*v*) After making the assignment, cross out the machine involved.

(*vi*) Repeat steps (*i*) to (*v*) until all the assignments are made.

SAMPLE PROBLEM

1227. *The maintenance crew of a company is divided in two groups, C_1 and C_2, which cares f the maintenance of the machines. Crew C_1 is responsible for replacement of parts which are worn o while Crew C_2 oils, and resets the machines back for operation. The times (in hours) required crews C_1 and C_2 on different machines which need working on them are as follows :*

Machine :	M_1	M_2	M_3	M_4	M_5	M_6	M_7
Crew C_1 :	8	6	10	11	10	14	4
Crew C_2 :	5	3	7	12	8	6	7

In what order should the machines be handled by crew C_1 and C_2 so that the total time taken minimised ?

Solution. Treating the machines as seven items and crews as the service facilities, the optima sequence can easily be obtained as

M_7	M_4	M_5	M_3	M_6	M_1	M_2

For the total elapsed time, we have

		M_1	M_4	M_5	M_3	M_6	M_1	M_2
Crew C_1	In	0	4	15	25	35	49	57
	Out	4	15	25	35	49	57	63
Crew C_2	In	4	15	27	35	49	57	63
	Out	11	27	35	42	55	62	66