# TEST OF SIGNIFICANCE (Basic Concepts)

## Introduction:

It is not easy to collect all the information about population and also it is not possible to study the characteristics of the entire population (finite or infinite) due to time factor, cost factor and other constraints. Thus we need sample. Sample is a finite subset of statistical individuals in a population and the number of individuals in a sample is called the sample size.

Sampling is quite often used in our day-to-day practical life. For example in a shop we assess the quality of rice, wheat or any other commodity by taking a handful of it from the bag and then to decide to purchase it or not.

## Parameter and Statistic:

The statistical constants of the population such as mean, ($\mu$), variance ($\sigma^2$), correlation coefficient ($\rho$) and proportion (P) are called ‘ Parameters’ .

Statistical constants computed from the samples corresponding to the parameters namely mean ( $\bar{x}$ ), variance ($s^2$), sample correlation coefficient (r) and proportion (p) etc, are called statistic.

Parameters are functions of the population values while statistic are functions of the sample observations. In general, population parameters are unknown and sample statistics are used as their estimates.

## Sampling Distribution:

The distribution of all possible values which can be assumed by some statistic measured from samples of same size ‘ n’ randomly drawn from the same population of size N, is called as sampling distribution of the statistic (DANIEL and FERREL).

Consider a population with N values .Let us take a random sample of size n from this population, then there are $NC_n = \dfrac{N!}{n!(N-n)!}$ = k (say), possible samples. From each of these k samples if we compute a statistic (e.g mean, variance, correlation coefficient, skewness etc) and then we form a frequency distribution for these k values of a statistic. Such a distribution is called sampling distribution of that statistic.

For example, we can compute some statistic $t = t(x_1, x_2, \ldots x_n)$ for each of these k samples. Then $t_1, t_2 \ldots, t_k$ determine the sampling distribution of the statistic t. In other words statistic t may be regarded as a random variable which can take the values $t_1, t_2 \ldots, t_k$ and we can compute various statistical constants like mean, variance, skewness, kurtosis etc., for this sampling distribution.

The mean of the sampling distribution t is $\bar{t} = \frac{1}{K}[t_1 + t_2 + \ldots\ldots + t_k] = \frac{1}{K}\sum_{i=1}^{k} t_i$

and $\text{var}(t) = \frac{1}{K}[(t_1 - \bar{t})^2 + (t_2 - \bar{t})^2 + \ldots\ldots + (t_k - \bar{t})^2]$

$$= \frac{1}{K}\Sigma(t_i - \bar{t})^2$$

## Standard Error:

The standard deviation of the sampling distribution of a statistic is known as its standard error. It is abbreviated as S.E. For example, the standard deviation of the sampling distribution of the mean $\bar{x}$ known as the standard error of the mean,

Where $v(x) = v\left(\frac{x_1 + x_2 + \ldots\ldots x_n}{n}\right)$

$$= \frac{v(x_1)}{n^2} + \frac{v(x_2)}{n^2} + \ldots\ldots + \frac{v(x_n)}{n^2}$$

$$= \frac{\sigma^2}{n^2} + \frac{\sigma^2}{n^2} + \ldots\ldots\ldots + \frac{\sigma^2}{n^2} = \frac{n\sigma^2}{n^2}$$

The S.E of the mean is $\frac{\sigma}{\sqrt{n}}$

The standard errors of the some of the well known statistic for large samples are given below, where n is the sample size, $\sigma^2$ is the population variance and P is the population proportion and Q = 1-P. $n_1$ and $n_2$ represent the sizes of two independent random samples respectively.

| Sl.No. | Statistic | Standard Error |
|---|---|---|
| 1. | Sample mean $\bar{x}$ | $\frac{\sigma}{\sqrt{n}}$ |
| 2. | Observed sample proportion p | $\sqrt{\frac{PQ}{n}}$ |
| 3. | Difference between of two samples means $(\bar{x}_1 - \bar{x}_2)$ | $\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$ |
| 4. | Difference of two sample proportions $p_1 - p_2$ | $\sqrt{\frac{P_1Q_1}{n_1} + \frac{P_2Q_2}{n_2}}$ |

**Uses of standard error**

i) Standard error plays a very important role in the large sample theory and forms the basis of the testing of hypothesis.

ii) The magnitude of the S.E gives an index of the precision of the estimate of the parameter.

iii) The reciprocal of the S.E is taken as the measure of reliability or precision of the sample.

iv) S.E enables us to determine the probable limits within which the population parameter may be expected to lie.

**Remark:**

S.E of a statistic may be reduced by increasing the sample size but this results in corresponding increase in cost, labour and time etc.,

## Null Hypothesis and Alternative Hypothesis

Hypothesis testing begins with an assumption called a Hypothesis, that we make about a population parameter. A hypothesis is a supposition made as a basis for reasoning. The conventional approach to hypothesis testing is not to construct a single hypothesis about the population parameter but rather to set up two different hypothesis. So that of one hypothesis is accepted, the other is rejected and vice versa.

**Null Hypothesis:**

A hypothesis of no difference is called null hypothesis and is usually denoted by $H_0$ "Null hypothesis is the hypothesis" which is tested for possible rejection under the assumption that it is true " by Prof. R.A. Fisher. It is very useful tool in test of significance. For example: If we want to find out whether the special classes (for Hr. Sec. Students) after school hours has benefited the students or not. We shall set up a null hypothesis that "$H_0$: special classes after school hours has not benefited the students".

**Alternative Hypothesis:**

Any hypothesis, which is complementary to the null hypothesis, is called an alternative hypothesis, usually denoted by $H_1$, For example, if we want to test the null hypothesis that the population has a specified mean $\mu_0$ (say),

i.e., : Step 1: null hypothesis $H_0$: $\mu = \mu_0$

then 2. Alternative hypothesis may be

i) $H_1 : \mu \neq \mu_0$ (ie $\mu > \mu_0$ or $\mu < \mu_0$)

ii) $H_1 : \mu > \mu_0$

iii) $H_1 : \mu < \mu_0$

the alternative hypothesis in (i) is known as a two – tailed alternative and the alternative in (ii) is known as right-tailed (iii) is known as left –tailed alternative respectively. The settings of alternative hypothesis is very important since it enables us to decide whether we have to use a single – tailed (right or left) or two tailed test.

## Level of significance and Critical value:

### Level of significance:

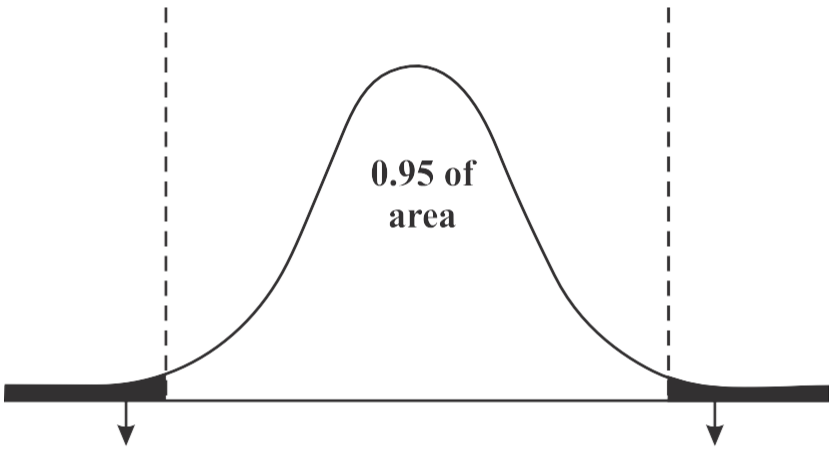
In testing a given hypothesis, the maximum probability with which we would be willing to take risk is called level of significance of the test. This probability often denoted by "α" is generally specified before samples are drawn.

The level of significance usually employed in testing of significance are 0.05( or 5 %) and 0.01 (or 1 %). If for example a 0.05 or 5 % level of significance is chosen in deriving a test of hypothesis, then there are about 5 chances in 100 that we would reject the hypothesis when it should be accepted. (i.e.,) we are about 95 % confident that we made the right decision. In such a case we say that the hypothesis has been rejected at 5 % level of significance which means that we could be wrong with probability 0.05.

The following diagram illustrates the region in which we could accept or reject the null hypothesis when it is being tested at 5 % level of significance and a two-tailed test is employed.

Accept the null hypothesis if the

sample statistics falls in this region




Reject the null hypothesis if the sample

Statistics falls in these two region

Note: Critical Region: A region in the sample space S which amounts to rejection of $H_0$ is termed as critical region or region of rejection.

**Critical Value:**

The value of test statistic which separates the critical (or rejection) region and the acceptance region is called the critical value or significant value. It depends upon i) the level of significance used and ii) the alternative hypothesis, whether it is two-tailed or single-tailed

For large samples the standard normal variate corresponding to the statistic t,

$$Z = \left| \frac{t - E(t)}{S.E.(t)} \right| \sim N(0,1)$$

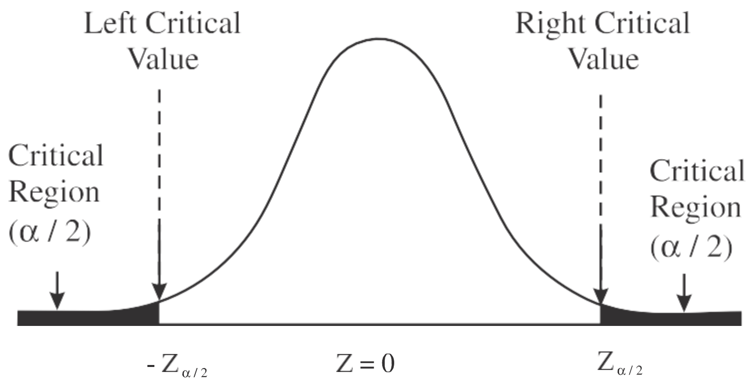
asymptotically as $n \to \infty$

The value of z under the null hypothesis is known as test statistic. The critical value of the test statistic at the level of significance $\alpha$ for a two - tailed test is given by $Z_{\alpha/2}$ and for a one tailed test by $Z_\alpha$. where $Z_\alpha$ is determined by equation $P(|Z| > Z_\alpha) = \alpha$.

$Z\alpha$ is the value so that the total area of the critical region on both tails is $\alpha$ .

$\therefore\ P(Z > Z_\alpha) = \frac{\alpha}{2}$. Area of each tail is $\frac{\alpha}{2}$.

$Z\alpha$ is the value such that area to the right of $Z\alpha$ and to the left of $-Z\alpha$ is $\frac{\alpha}{2}$ as shown in the following diagram.



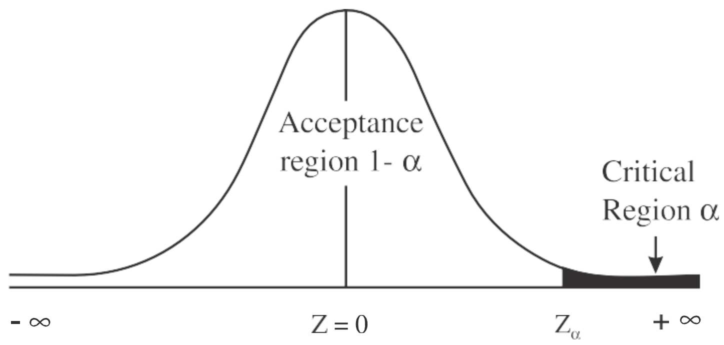

## One tailed and Two Tailed tests:

In any test, the critical region is represented by a portion of the area under the probability curve of the sampling distribution of the test statistic.

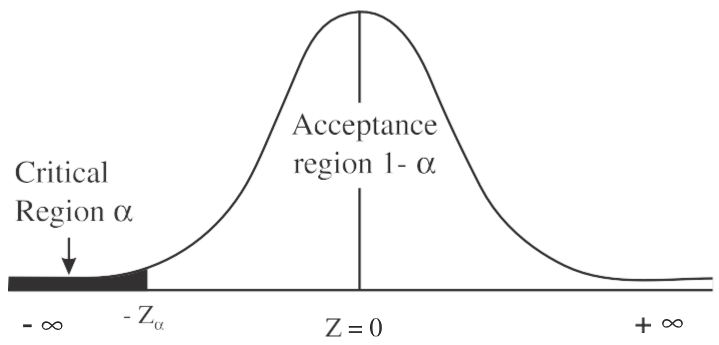
**One tailed test:** A test of any statistical hypothesis where the alternative hypothesis is one tailed (right tailed or left tailed) is called a one tailed test.

For example, for testing the mean of a population $H_0$: $\mu = \mu_0$, against the alternative hypothesis $H_1$: $\mu > \mu_0$ (right – tailed) or $H_1$ : $\mu < \mu_0$ (left –tailed) is a single tailed test. In the right – tailed test $H_1$: $\mu > \mu_0$ the critical region lies entirely in right tail of the sampling distribution of $\bar{x}$, while for the left tailed test $H_1$: $\mu < \mu_0$ the critical region is entirely in the left of the distribution of $\bar{x}$.

**Right tailed test:**




**Left tailed test :**




**Two tailed test:**

A test of statistical hypothesis where the alternative hypothesis is two tailed such as, $H_0 : \mu = \mu_0$ against the alternative hypothesis $H_1 : \mu \neq \mu_0$ ($\mu > \mu_0$ and $\mu < \mu_0$) is known as two tailed test and in such a case the critical region is given by the portion of the area lying in both the tails of the probability curve of test of statistic.

For example, suppose that there are two population brands of washing machines, are manufactured by standard process(with mean warranty period $\mu_1$) and the other manufactured by some new technique (with mean warranty period $\mu_2$): If we want to test if the washing machines differ significantly then our null hypothesis is $H_0 : \mu_1 = \mu_2$ and alternative will be $H_1 : \mu_1 \neq \mu_2$ thus giving us a two tailed test. However if we want to test whether the average warranty period produced by some new technique is more than those produced by standard process, then we have $H_0 : \mu_1 = \mu_2$ and $H_1 : \mu_1 < \mu_2$ thus giving us a left-tailed test.

Similarly, for testing if the product of new process is inferior to that of standard process then we have, $H_0 : \mu_1 = \mu_2$ and $H_1 : \mu_1 > \mu_2$ thus giving us a right-tailed test. Thus the decision about applying a two – tailed test or a single –tailed (right or left) test will depend on the problem under study.

Critical values ($Z\alpha$) of Z

| Level of significance $\alpha$ | 0.05 or 5% | | 0.01 or 1% | |
|---|---|---|---|---|
| | Left | Right | Left | Right |
| Critical values of $Z_\alpha$ for one tailed Tests | – 1.645 | 1.645 | – 2.33 | 2.33 |
| Critical values of $Z_{\alpha/2}$ for two tailed tests | – 1.96 | 1.96 | – 2.58 | 2.58 |

## Type I and Type II Errors:

When a statistical hypothesis is tested there are four possibilities.

1. The hypothesis is true but our test rejects it ( Type I error)
2. The hypothesis is false but our test accepts it (Type II error)
3. The hypothesis is true and our test accepts it (correct decision)
4. The hypothesis is false and our test rejects it (correct decision)

Obviously, the first two possibilities lead to errors. In a statistical hypothesis testing experiment, a Type I error is committed by rejecting the null hypothesis when it is true. On the other hand, a Type II error is committed by not rejecting (accepting) the null hypothesis when it is false.

If we write ,

$\alpha$ = P (Type I error) = P (rejecting $H_0$ | $H_0$ is true)

$\beta$ = P (Type II error) = P (Not rejecting $H_0$ | $H_0$ is false)

In practice, type I error amounts to rejecting a lot when it is good and type II error may be regarded as accepting the lot when it is bad. Thus we find ourselves in the situation which is described in the following table.

| | Accept $H_0$ | Reject $H_0$ |
|---|---|---|
| $H_0$ is true | Correct decision | Type I Error |
| $H_0$ is false | Type II error | Correct decision |

## Test Procedure :

Steps for testing hypothesis is given below. (for both large sample and small sample tests)

1. Null hypothesis : set up null hypothesis $H_0$.
2. Alternative Hypothesis: Set up alternative hypothesis $H_1$, which is complementry to $H_0$ which will indicate whether one tailed (right or left tailed) or two tailed test is to be applied.

3. Level of significance : Choose an appropriate level of significance ($\alpha$), $\alpha$ is fixed in advance.

4. Test statistic (or test of criterian):

   Calculate the value of the test statistic, $Z = \frac{t - E(t)}{S.E.(t)}$ under the null hypothesis, where t is the sample statistic

5. Inference: We compare z the computed value of Z (in absolute value) with the significant value (critical value) $Z\alpha/2$ (or $Z\alpha$). If $|Z| > Z\alpha$, we reject the null hypothesis $H_0$ at $\alpha$ % level of significance and if $|Z| \leq Z\alpha$, we accept $H_0$ at $\alpha$ % level of significance.

**Note:**

1. Large Sample: A sample is large when it consists of more than 30 items.
2. Small Sample: A sample is small when it consists of 30 or less than 30 items.

# TEST Of SIGNIFICANCE
# (Large Sample)

## Introduction:

In practical problems, statisticians are supposed to make tentative calculations based on sample observations. For example

(i) The average weight of school student is 35kg

(ii) The coin is unbiased

Now to reach such decisions it is essential to make certain assumptions (or guesses) about a population parameter. Such an assumption is known as statistical hypothesis, the validity of which is to be tested by analyzing the sample. The procedure, which decides a certain hypothesis is true or false, is called the test of hypothesis (or test of significance).

Let us assume a certain value for a population mean. To test the validity of our assumption, we collect sample data and determine the difference between the hypothesized value and the actual value of the sample mean. Then, we judge whether the difference is significant or not. The smaller the difference, the greater the likelihood that our hypothesized value for the mean is correct. The larger the difference the smaller the likelihood, which our hypothesized value for the mean, is not correct.

## Large samples ($n > 30$):

The tests of significance used for problems of large samples are different from those used in case of small samples as the assumptions used in both cases are different. The following assumptions are made for problems dealing with large samples:

(iii) Almost all the sampling distributions follow normal asymptotically.

(iv) The sample values are approximately close to the population values.

The following tests are discussed in large sample tests.

(i) Test of significance for proportion

(ii) Test of significance for difference between two proportions

(iii) Test of significance for mean

(iv) Test of significance for difference between two means.

## Test of significance for mean:

**Test Procedure:**

**Null and Alternative Hypotheses:**

$H_0 : \mu = \mu_0$.

$H_1: \mu \neq \mu_0$ ($\mu > \mu_0$ or $\mu < \mu_0$)

**Level of significance:**

Let $\alpha = 0.05$ or $0.01$

**Calculation of statistic:**

Under $H_0$, the test statistic is

$$Z_0 = \left| \frac{\bar{x} - E(\bar{x})}{S.E(\bar{x})} \right| = \left| \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} \right|$$

**Expected value:**

$$Z_e = \left| \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} \right| \sim N(0,1)$$

$$= 1.96 \text{ for } \alpha = 0.05 (1.645)$$

or

$$= 2.58 \text{ for } \alpha = 0.01 \ (2.33)$$

**Inference :**

If $Z_0 \leq Z_e$, we accept our null hypothesis and conclude that the sample is drawn from a population with mean $\mu = \mu_0$.

If $Z_0 > Z_e$ we reject our $H_0$ and conclude that the sample is not drawn from a population with mean $\mu = \mu_0$.

**Example 1:**

The mean lifetime of 100 fluorescent light bulbs produced by a company is computed to be 1570 hours with a standard deviation of 120 hours. If $\mu$ is the mean lifetime of all the bulbs produced by the company, test the hypothesis $\mu = 1600$ hours against the alternative hypothesis $\mu \neq 1600$ hours using a 5% level of significance.

**Solution:**

We are given

$\bar{x} = 1570$ hrs $\quad \mu = 1600$hrs $\quad s = 120$ hrs $\quad n=100$

**Null hypothesis:**

$H_0 : \mu = 1600$. ie There is no significant difference between the sample mean and population mean.

**Alternative Hypothesis:**

$H_1: \mu \neq 1600$ (two tailed Alternative)

**Level of significance:**

Let $\alpha = 0.05$

**Calculation of statistics**

Under $H_0$, the test statistic is

$$Z_0 = \left| \frac{\bar{x} - \mu}{s / \sqrt{n}} \right|$$

$$= \left| \frac{1570 - 1600}{\frac{120}{\sqrt{100}}} \right|$$

$$= \frac{30 \times 10}{120}$$

$$= 2.5$$

\

**Expected value:**

$$Z_0 = \left|\frac{\bar{x} - \mu}{s/\sqrt{n}}\right| \sim N(0, 1)$$
$$= 1.96 \text{ for } \alpha = 0.05$$

**Inference :**

Since $Z_0 > Z_e$ we reject our null hypothesis at 5% level of significance and say that there is significant difference between the sample mean and the population mean.

**Example 2:**

A car company decided to introduce a new car whose mean petrol consumption is claimed to be lower than that of the existing car. A sample of 50 new cars were taken and tested for petrol consumption. It was found that mean petrol consumption for the 50 cars was 30 km per litre with a standard deviation of 3.5 km per litre. Test at 5% level of significance whether the company' s claim that the new car petrol consumption is 28 km per litre on the average is acceptable.

**Solution:**

We are given $\bar{x} = 30$ ; $\mu = 28$ ; $n = 50$ ; $s = 3.5$

**Null hypothesis:**

$H_0 : \mu = 28$. i.e The company' s claim that the petrol consumption of new car is 28km per litre on the average is acceptable.

**Alternative Hypothesis:**

$H_1: \mu < 28$ (Left tailed Alternative)

**Level of significance:**

Let $\alpha = 0.05$

**Calculation of statistic:**

Under $H_0$ the test statistics is

$$Z_0 = \left|\frac{\bar{x} - \mu}{s/\sqrt{n}}\right|$$
$$= \left|\frac{30 - 28}{\frac{3.5}{\sqrt{50}}}\right|$$
$$= \frac{2 \times \sqrt{50}}{3.5}$$
$$= 4.04$$

\

**Expected value:**

$$Z_e = \left|\frac{\bar{x}-\mu}{s/\sqrt{n}}\right| \sim N(0,1) \text{ at } \alpha = 0.05$$

$$= 1.645$$

**Inference :**

Since the calculated $Z_0 > Z_e$ we reject the null hypothesis at 5% level of significance and conclude that the company' s claim is not acceptable.

## Test of significance for difference between two means:

**Test procedure**

**Set up the null and alternative hypothesis**

$H_0$: $\mu_1 = \mu_2$ ; $H_1$: $\mu_1 \neq \mu_2$ ($\mu_1 > \mu_2$ or $\mu_1 < \mu_2$)

**Level of significance:**

Let $\alpha$ %

**Calculation of statistic:**

Under $H_0$ the test statistic is

$$Z_0 = \left|\frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}\right|$$

If $\sigma_1^2 = \sigma_2^2 = \sigma^2$ (ie) If the samples have been drawn from the population with common S.D $\sigma$ then under $H_0 : \mu_1 = \mu_2$.

$$Z_0 = \left|\frac{\bar{x}_1 - \bar{x}_2}{\sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}\right|$$

**Expected value:**

$$Z_e = \left|\frac{\bar{x}_1 - \bar{x}_2}{S.E(\bar{x}_1 - \bar{x}_2)}\right| \sim N(0,1)$$

**Inference:**

(i) If $Z_0 \leq Z_e$ we accept the $H_0$ (ii) If $Z_0 > Z_e$ we reject the $H_0$

**Example**

A test of the breaking strengths of two different types of cables was conducted using samples of $n_1 = n_2 = 100$ pieces of each type of cable.

| Cable I | Cable II |
|---|---|
| $\bar{x}_1 = 1925$ | $\bar{x}_2 = 1905$ |
| $\sigma_1 = 40$ | $\sigma_2 = 30$ |

Do the data provide sufficient evidence to indicate a difference between the mean breaking strengths of the two cables? Use 0.01 level of significance.

**Solution:**

We are given

$\bar{x}_1 = 1925$ $\qquad \bar{x}_2 = 1905$ $\qquad \sigma_1 = 40$ $\qquad \sigma_2 = 30$

**Null hypothesis**

$H_0: \mu_1 = \mu_2$ .ie There is no significant difference between the mean breaking strengths of the two cables.

$H_1 : \mu_1 \neq \mu_2$ (Two tailed alternative)

**Level of significance:**

Let $\alpha = 0.01$ or 1%

**Calculation of statistic:**

Under $H_0$ the test statistic is

$$Z_0 = \left| \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \right|$$

$$= \left| \frac{1925 - 1905}{\sqrt{\frac{40^2}{190} + \frac{30^2}{100}}} \right| = \frac{20}{5} = 4$$

**Expected value:**

$$Z_e = \left| \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \right| \sim N(0,1) = 2.58$$

**Inference:**

Since $Z_0 > Z_e$, we reject the $H_0$. Hence the formulated null hypothesis is wrong ie there is a significant difference in the breaking strengths of two cables.

**Example :**

The means of two large samples of 1000 and 2000 items are 67.5 cms and 68.0cms respectively. Can the samples be regarded as drawn from the population with standard deviation 2.5 cms. Test at 5% level of significance.

**Solution:**

We are given

$n_1 = 1000$ ; $n_2 = 2000$ $\bar{x}_1 = 67.5$ cms ; $\bar{x}_2 = 68.0$ cms $\sigma = 2.5$ cms

**Null hypothesis**

$H_0 : \mu_1 = \mu_2$ (i.e.,) the sample have been drawn from the same population.

**Alternative Hypothesis:**

$H_1 : \mu_1 \neq \mu_2$ (Two tailed alternative)

**Level of significance:**

$\alpha = 5\%$

**Calculation of statistic:**

Under $H_e$ the test statistic is

$$Z_0 = \left| \frac{\bar{x}_1 - \bar{x}_2}{\sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \right|$$

$$= \left| \frac{67.5 - 68.0}{2.5\sqrt{\frac{1}{1000} + \frac{1}{2000}}} \right|$$

$$= \frac{0.5 \times 20}{2.5\sqrt{3/5}}$$

$$= 5.1$$

**Expected value:**

$$Z_e = \left| \frac{\bar{x}_1 - \bar{x}_2}{\sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \right| \sim N(0,1) = 1.96$$

**Inference :**

Since $Z_0 > Z_e$ we reject the $H_0$ at 5% level of significance and conclude that the samples have not come from the same population.

# Exercise

**I. Choose the best answer:**

1. Standard error of number of success is given by

(a) $\sqrt{\frac{pq}{n}}$ (b) $\sqrt{npq}$ (c) npq (d) $\sqrt{\frac{np}{q}}$

2. Large sample theory is applicable when

(a) n > 30 (b) n < 30 (c) n < 100 (d) n < 1000

3. Test statistic for difference between two means is

(a) $\frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$ (b) $\frac{p-P}{\sqrt{\frac{PQ}{n}}}$

(c) $\frac{\bar{x}_1-\bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}}$ (d) $\frac{p_1-p_2}{\sqrt{PQ\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$

4. Standard error of the difference of proportions $(p_1 - p_2)$ in two classes under the hypothesis $H_0 : p_1 = p_2$ with usual notation is

(a) $\sqrt{pq\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}$ (b) $\sqrt{p\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}$

(c) $pq\sqrt{\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}$ (d) $\frac{p_1q_1}{n_1}+\frac{p_2q_2}{n_2}$

5. Statistic $z = \frac{\bar{x}-\bar{y}}{\sigma\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$ is used to test the null hypothesis

(a) $H_0: \mu_1 + \mu_2 = 0$ (b) $H_0: \mu_1 - \mu_2 = 0$

(c) $H_0: \mu = \mu_0$ (a constant) (d) none of the above.

**II. fill in the blanks:**

5. If $\hat{P} = \frac{2}{3}$, then $\hat{Q}$ =__________.

6. If $z_0 < z_e$ then the null hypothesis is _____________

7. When the difference is___________, the null hypothesis is rejected.

8. Test statistic for difference between two proportions is_________.

9. The variance of sample mean is_________.

**III. Answer the following**

10. In a test if $z_0 \leq z_e$, what is your conclusion about the null hypothesis?

11. Give the test statistic for

    (a) Proportion

    (b) Mean

    (c) Difference between two means

    (d) Difference between two proportions

12. Write the variance of difference between two proportions

13. Write the standard error of proportion.

14. Write the test procedure for testing the test of significance for

    (a) Proportion (b) mean (c) difference between two proportions

    (d) difference between two mean

15. A coin was tossed 400 times and the head turned up 216 times. Test the hypothesis that the coin is unbiased.

16. A person throws 10 dice 500 times and obtains 2560 times 4, 5 or 6. Can this be attributed to fluctuations of sampling?

17. In a hospital 480 female and 520 male babies were born in a week. Do these figure confirm the hypothesis that males and females are born in equal number?

18. In a big city 325 men out of 600 men were found to be selfemployed. Does this information support the conclusion that the majority of men in this city are self-employed?

19. A machine puts out 16 imperfect articles in a sample of 500. After machine is overhauled, it puts out 3 imperfect articles in a batch of 100. Has the machine improved?

20. In a random sample of 1000 persons from town A , 400 are found to be consumers of wheat. In a sample of 800 from town B, 400 are found to be consumers of wheat. Do these data reveal a significant difference between town A and town B, so far as the proportion of wheat consumers is concerned?

21. 1000 articles from a factory A are examined and found to have 3% defectives. 1500 similar articles from a second factory B are found to have only 2% defectives. Can it be reasonably concluded that the product of the first factory is inferior to the second?

22. In a sample of 600 students of a certain college, 400 are found to use blue ink. In another college from a sample of 900 students 450 are found to use blue ink. Test whether the two colleges are significantly different with respect to the habit of using blue ink.

23. It is claimed that a random sample of 100 tyres with a mean life of 15269kms is drawn from a population of tyres which has a mean life of 15200 kms and a standard deviation of 1248 kms. Test the validity of the claim.

24. A sample of size 400 was drawn and the sample mean B was found to be 99. Test whether this sample could have come from a normal population with mean 100 and variance 64 at 5% level of significance.

25. The arithmetic mean of a sample 100 items drawn from a large population is 52. If the standard deviation of the population is 7, test the hypothesis that the mean of the population is 55 against the alternative that the mean is not 55. Use 5% level of significance.

26. A company producing light bulbs finds that mean life span of the population of bulbs is 1200 hrs with a standard deviation of 125hrs. A sample of 100 bulbs produced in a lot is found to have a mean life span of 1150hrs. Test whether the difference between the population and sample means is statistically significant at 5% level of significance.

27. Test the significance of the difference between the means of the samples from the following data

| | Size of sample | Mean | Standard deviation |
|---|---|---|---|
| Sample A | 100 | 50 | 4 |
| Sample B | 150 | 51 | 5 |

28. An examination was given to two classes consisting of 40 and 50 students respectively. In the first class the mean mark was 74 with a standard deviation of 8, while in the second class the mean mark was 78 with a standard deviation of 7. Is there a significant difference between the performance of the two classes at a level of significance of 0.05?

29. If 60 M.A. Economics students are found to have a mean height of 63.60 inches and 50 M.Com students a mean height of 69.51 inches. Would you conclude that the commerce students are taller than Economics students? Assume the standard deviation of height of post-graduate students to be 2.48 inches.

# Answers:

**I.**

1. (b) 2. (a) 3. (c) 4. (a) 5. (b)

**II.**

6. $\frac{1}{3}$ 7. accepted 8. significant 9. $\frac{p_1 - p_2}{\sqrt{PQ\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$ 10. $\frac{\sigma^2}{n}$

**III.**

16. $z = 1.6$ Accept $H_0$
17. $z = 1.7$ Accept $H_0$
18. $z = 1.265$Accept $H_0$
19. $z = 2.04$ Accept $H_0$
20. $z = 0.106$ Accept $H_0$
21. $z = 4.247$ Reject $H_0$
22. $z = 1.63$ Accept $H_0$
23. $z = 6.38$ Reject $H_0$
24. $z = 0.5529$ Accept $H_0$
25. $z = 2.5$ Reject $H_0$
26. $z = 4.29$ Reject $H_0$
27. $z = 4$ Reject $H_0$
28. $z = 1.75$ Accept $H_0$
29. $z = 2.49$ Reject $H_0$
30. $z = 12.49$ Reject $H_0$

# TESTS OF SIGNIFICANCE
# (Small Samples)

## Introduction:

In the previous chapter we have discussed problems relating to large samples. The large sampling theory is based upon two important assumptions such as

(a) The random sampling distribution of a statistic is approximately normal and

(b) The values given by the sample data are sufficiently close to the population values and can be used in their place for the calculation of the standard error of the estimate.

The above assumptions do not hold good in the theory of small samples. Thus, a new technique is needed to deal with the theory of small samples. A sample is small when it consists of less than 30 items. ( n< 30)

Since in many of the problems it becomes necessary to take a small size sample, considerable attention has been paid in developing suitable tests for dealing with problems of small samples. The greatest contribution to the theory of small samples is that of Sir William Gosset and Prof. R.A. Fisher. Sir William Gosset published his discovery in 1905 under the pen name 'Student' and later on developed and extended by Prof. R.A.Fisher. He gave a test popularly known as ' t-test' .

## t - statistic definition:

If $x_1, x_2, \ldots\ldots x_n$ is a random sample of size n from a normal population with mean $\mu$ and variance $\sigma^2$, then Student' s t-statistic is defined as $t = \dfrac{\bar{x}-\mu}{\dfrac{S}{\sqrt{n}}}$

where $\bar{x} = \dfrac{\sum x}{n}$ is the sample mean

and $S^2 = \dfrac{1}{n-1}\sum(x-\bar{x})^2$

is an unbiased estimate of the population variance $\sigma^2$ It follows student' s t-distribution with $\nu = n - 1$ d.f

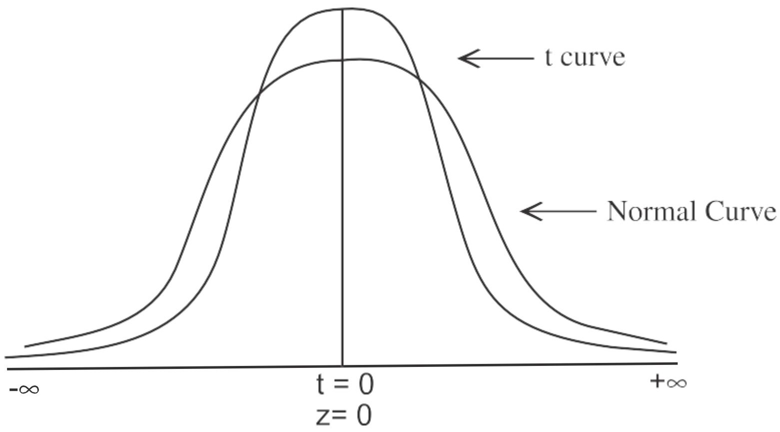
**Assumptions for students t-test:**

1. The parent population from which the sample drawn is normal.
2. The sample observations are random and independent.
3. The population standard deviation $\sigma$ is not known.

**Properties of t- distribution:**

4. t-distribution ranges from $-\infty$ to $\infty$ just as does a normal distribution.
5. Like the normal distribution, t-distribution also symmetrical and has a mean zero.
6. t-distribution has a greater dispersion than the standard normal distribution.
7. As the sample size approaches 30, the t-distribution, approaches the Normal distribution.

**Comparison between Normal curve and corresponding t - curve:**




**Degrees of freedom (d.f):**

Suppose it is asked to write any four number then one will have all the numbers of his choice. If a restriction is applied or imposed to the choice that the sum of these number should be 50. Here, we have a choice to select any three numbers, say 10, 15, 20 and the fourth number is 5: [50 – (10 + 15 + 20)]. Thus our choice of freedom is reduced by one, on the condition that the total be 50. Therefore the restriction placed on the freedom is one and degree of freedom is three. As the restrictions increase, the freedom is reduced.

The number of independent variates which make up the statistic is known as the degrees of freedom and is usually denoted by $\nu$ (Nu)

The number of degrees of freedom for n observations is n - k where k is the number of independent linear constraint imposed upon them.

For the student’ s t-distribution the number of degrees of freedom is the sample size minus one. It is denoted by $\nu = n - 1$ The degrees of freedom plays a very important role in $\chi^2$ test of a hypothesis.

When we fit a distribution the number of degrees of freedom is (n– k-1) where n is number of observations and k is number of parameters estimated from the data.

For e.g., when we fit a Poisson distribution the degrees of freedom is $\nu = n - 1 - 1$.

In a contingency table the degrees of freedom is (r-1) (c -1) where r refers to number of rows and c refers to number of columns.

Thus in a 3 × 4 table the d.f are (3 – 1) (4 – 1) = 6 d.f. In a 2 × 2 contingency table the d.f are (2 – 1) (2 – 1) = 1

In case of data that are given in the form of series of variables in a row or column the d.f will be the number of observations in a series less one ie., $\nu = n - 1$

**Critical value of t:**

The column figures in the main body of the table come under the headings $t_{0.100}$, $t_{0.50}$, $t_{0.025}$, $t_{0.010}$ and $t_{0.005}$. The subscripts give the proportion of the distribution in ' tail' area. Thus for two-tailed test at 5% level of significance there will be two rejection areas each containing 2.5% of the total area and the required column is headed $t_{0.025}$

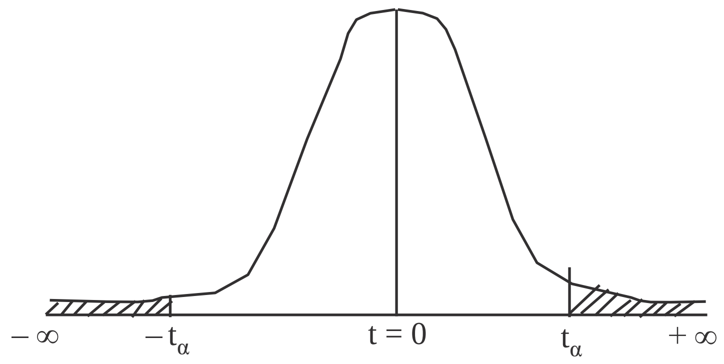
For example,

$t_\nu$ (.05) for single tailed test = $t_\nu$ (0.025) for two tailed test

$t_\nu$ (.01) for single tailed test = $t_\nu$ (0.005) for two tailed test

Thus for one tailed test at 5% level the rejection area lies in one end of the tail of the distribution and the required column is headed $t_{0.05}$.

**Critical value of t – distribution**




**Applications of t-distribution:**

The t-distribution has a number of applications in statistics, of which we shall discuss the following in the coming sections:

(i) t-test for significance of single mean, population variance being unknown.

(ii) t-test for significance of the difference between two sample means, the population variances being equal but unknown.

(a) Independent samples (b) Related samples: paired t-test

## Test of significance for Mean:

We set up the corresponding null and alternative hypotheses as follows:

$H_0 : \mu = \mu_0$; There is no significant difference between the sample mean and population Mean.

$H_1: \mu \neq \mu_0$ ( $\mu < \mu_0$ (or) $\mu > \mu_0$)

**Level of significance:**

5% or 1%

**Calculation of statistic:**

Under $H_0$ the test statistic is

$$t_0 = \left| \frac{\bar{x} - \mu}{\frac{S}{\sqrt{n}}} \right| \text{ or } \left| \frac{\bar{x} - \mu}{s / \sqrt{n-1}} \right|$$

$$\text{where } \bar{x} = \frac{\sum x}{n} \text{ is the sample mean}$$

$$\text{and} \quad S^2 = \frac{1}{n-1} \sum (x - \bar{x})^2 \text{ (or) } s^2 = \frac{1}{n} \sum (x - \bar{x})^2$$

**Expected value :**

$$t_e = \left| \frac{\bar{x} - \mu}{\frac{S}{\sqrt{n}}} \right| \sim \text{student's t-distribution with (n-1) d.f}$$

**Inference :**

If $t_0 \leq t_e$ it falls in the acceptance region and the null hypothesis is accepted and if $t_o > t_e$ the null hypothesis $H_0$ may be rejected at the given level of significance.

**Example 1:**

Certain pesticide is packed into bags by a machine. A random sample of 10 bags is drawn and their contents are found to weigh (in kg) as follows:

50 49 52 44 45 48 46 45 49 45

Test if the average packing can be taken to be 50 kg.

**Solution:**

**Null hypothesis:**

$H_0 : \mu = 50$ kgs in the average packing is 50 kgs.

**Alternative Hypothesis:**

$H_1 : \mu \neq 50$kgs (Two -tailed )

**Level of Significance:**

Let $\alpha = 0.05$

**Calculation of sample mean and S.D**

| X | d = x – 48 | $d^2$ |
|---|---|---|
| 50 | 2 | 4 |
| 49 | 1 | 1 |
| 52 | 4 | 16 |
| 44 | – 4 | 16 |
| 45 | – 3 | 9 |
| 48 | 0 | 0 |
| 46 | – 2 | 4 |
| 45 | – 3 | 9 |
| 49 | + 1 | 1 |
| 45 | – 3 | 9 |
| Total | – 7 | 69 |

$$\bar{x} = A + \frac{\sum d}{n}$$

$$= 48 + \frac{-7}{10}$$

$$= 48 - 0.7 = 47.3$$

$$S^2 = \frac{1}{n-1}\left[\sum d^2 - \frac{(\sum d)^2}{n}\right]$$

$$= \frac{1}{9}\left[69 - \frac{(7^2)}{10}\right]$$

$$= \frac{64.1}{9} = 7.12$$

**Calculation of Statistic:**

Under $H_0$ the test statistic is :

$$t_0 = \left|\frac{\bar{x} - \mu}{\sqrt{S^2 / n}}\right|$$

$$= \left|\frac{47.3 - 50.0}{\sqrt{7.12 / 10}}\right|$$

$$= \frac{2.7}{\sqrt{0.712}} = 3.2$$

**Expected value:**

$$t_e = \left| \frac{\bar{x} - \mu}{\sqrt{S^2 / n}} \right| \text{ follows t distribution with (10–1) d.f}$$

$$= 2.262$$

**Inference:**

Since $t_0 > t_e$, $H_0$ is rejected at 5% level of significance and we conclude that the average packing cannot be taken to be 50 kgs.

**Example 2:**

A soap manufacturing company was distributing a particular brand of soap through a large number of retail shops. Before a heavy advertisement campaign, the mean sales per week per shop was 140 dozens. After the campaign, a sample of 26 shops was taken and the mean sales was found to be 147 dozens with standard deviation 16. Can you consider the advertisement effective?

**Solution:**

We are given

$n = 26$; $\bar{x} = 147$dozens; $s = 16$

**Null hypothesis:**

$H_0$: $\mu = 140$ dozens i.e., Advertisement is not effective.

**Alternative Hypothesis:**

$H_1$: $\mu > 140$ dozens (Right -tailed)

**Calculation of statistic:**

Under the null hypothesis $H_0$, the test statistic is

$$t_0 = \left| \frac{\bar{x} - \mu}{S / \sqrt{n-1}} \right|$$

$$= \left| \frac{147 - 140}{16 / \sqrt{25}} \right| = \frac{7 \times 5}{16} = 2.19$$

**Expected value:**

$$t_e = \left| \frac{\bar{x} - \mu}{s / \sqrt{n-1}} \right| \text{ follows t-distribution with } (26 - 1) = 25 \text{ d.f}$$

$$= 1.708$$

**Inference:**

Since $t_0 > t_e$, $H_0$ is rejected at 5% level of significance. Hence we conclude that advertisement is certainly effective in increasing the sales.

## Test of significance for difference between two means:

**Independent samples:**

Suppose we want to test if two independent samples have been drawn from two normal populations having the same means, the population variances being equal. Let $x_1$, $x_2$, …. $x_{n_1}$ and $y_1$, $y_2$, …… $y_{n_2}$ be two independent random samples from the given normal populations.

**Null hypothesis:**

$H_0 : \mu_1 = \mu_2$ i.e. the samples have been drawn from the normal populations with same means.

**Alternative Hypothesis:**

$H_1 : \mu_1 \neq \mu_2$ ($\mu_1 < \mu_2$ or $\mu_1 > \mu_2$)

**Test statistic:**

Under the $H_0$, the test statistic is

$$t_0 = \left| \frac{\bar{x} - \bar{y}}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \right|$$

$$\text{where } \bar{x} = \frac{\sum x}{n_1};\ \bar{y} = \frac{\sum y}{n_2}$$

$$\text{and } S^2 = \frac{1}{n_1 + n_2 - 2}\left[\sum(x - \bar{x})^2 + \sum(y - \bar{y})^2\right] = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}$$

**Expected value:**

$$t_e = \left| \frac{\bar{x} - \bar{y}}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \right| \text{ follows t-distribution with } n_1 + n_2 - 2 \text{ d.f}$$

**Inference:**

If the $t_0 < t_e$ we accept the null hypothesis. If $t_0 > t_e$ we reject the null hypothesis.

**Example 3:**

A group of 5 patients treated with medicine ‘A’ weigh 42, 39, 48, 60 and 41 kgs: Second group of 7 patients from the same hospital treated with medicine ‘ B’ weigh 38, 42 , 56, 64, 68, 69 and 62 kgs. Do you agree with the claim that medicine ‘ B’ increases the weight significantly?

**Solution:**

Let the weights (in kgs) of the patients treated with medicines A and B be denoted by variables X and Y respectively.

**Null hypothesis:**

$H_0 : \mu_1 = \mu_2$

i.e. There is no significant difference between the medicines A and B as regards their effect on increase in weight.

**Alternative Hypothesis:**

$H_1 : \mu_1 < \mu_2$ (left-tail) i.e. medicine B increases the weight significantly.

**Level of significance :** Let $\alpha = 0.05$

**Computation of sample means and S.Ds**

**Medicine A**

| X | $x - \bar{x}$ $(\bar{x} = 46)$ | $(x - \bar{x})^2$ |
|---|---|---|
| 42 | – 4 | 16 |
| 39 | – 7 | 49 |
| 48 | 2 | 4 |
| 60 | 14 | 196 |
| 41 | – 5 | 25 |
| **230** | **0** | **290** |

$$\bar{x} = \frac{\sum x}{n_1} = \frac{230}{5} = 46$$

**Medicine B**

| Y | $y - \bar{y}$ $(\bar{y} = 57)$ | $(y - \bar{y})^2$ |
|---|---|---|
| 38 | – 19 | 361 |
| 42 | – 15 | 225 |
| 56 | – 1 | 1 |
| 64 | 7 | 49 |
| 68 | 11 | 121 |
| 69 | 12 | 144 |
| 62 | 5 | 25 |
| **399** | **0** | **926** |

$$\bar{y} = \frac{\sum y}{n_2} = \frac{399}{7} = 57$$

$$S^2 = \frac{1}{n_1 + n_2 - 2}\left[\sum (x - \bar{x})^2 + \sum (y - \bar{y})^2\right]$$

$$= \frac{1}{10}[290 + 926] = 121.6$$

**Calculation of statistic:**

Under $H_0$ the test statistic is

$$t_0 = \left| \frac{\bar{x} - \bar{y}}{\sqrt{S^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \right|$$

$$= \left| \frac{46 - 57}{\sqrt{121.6\left(\frac{1}{5} + \frac{1}{7}\right)}} \right|$$

$$= \frac{11}{\sqrt{121.6 \times \frac{12}{35}}}$$

$$= \frac{11}{6.57} = 1.7$$

**Expected value:**

$$t_e = \left| \frac{\bar{x} - \bar{y}}{\sqrt{S^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \right|$$ follows t-distribution with $(5 + 7 - 2) = 10$ d.f

$=1.812$

**Inference:**

Since $t_0 < t_e$ it is not significant. Hence $H_0$ is accepted and we conclude that the medicines A and B do not differ significantly as regards their effect on increase in weight.

**Example 4:**

Two types of batteries are tested for their length of life and the following data are obtained:

| | No. of samples | Mean life (in hrs) | Variance |
|---|---|---|---|
| Type A | 9 | 600 | 121 |
| Type B | 8 | 640 | 144 |

Is there a significant difference in the two means?

**Solution:**

We are given

$n_1 = 9$; $\bar{x}_1 = 600$hrs; $S_1^2 = 121$; $n_2 = 8$; $\bar{x}_2 = 640$hrs; $S_2^2 = 144$

**Null hypothesis:**

$H_0 : \mu_1 = \mu_2$ i.e. Two types of batteries A and B are identical i.e. there is no significant difference between two types of batteries.

**Alternative Hypothesis:**

$H_1 : \mu_1 \neq \mu_2$ (Two- tailed)

**Level of Significance:**

Let $\alpha = 5\%$

**Calculation of statistics:**

Under $H_0$, the test statistic is

$$t_0 = \left| \frac{\bar{x} - \bar{y}}{\sqrt{S^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \right|$$

$$\text{where } S^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}$$

$$= \frac{9 \times 121 + 8 \times 144}{9 + 8 - 2}$$

$$= \frac{2241}{15} = 149.4$$

$$\therefore \quad t_0 = \left| \frac{600 - 640}{\sqrt{149.4\left(\frac{1}{9} + \frac{1}{8}\right)}} \right|$$

$$= \frac{40}{\sqrt{149.4\left(\frac{17}{72}\right)}} = \frac{40}{5.9391} = 6.735$$

**Expected value:**

$$t_e = \left| \frac{\bar{x} - \bar{y}}{\sqrt{S^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \right| \text{ follows t-distribution with } 9 + 8 - 2 = 15 \text{ d.f}$$

$$= 2.131$$

**Inference:**

Since $t_0 \geq t_e$ it is highly significant. Hence $H_0$ is rejected and we conclude that the two types of batteries differ significantly as regards their length of life.

**Related samples –Paired t-test:**

In the t-test for difference of means, the two samples were independent of each other. Let us now take a particular situations where

(i) The sample sizes are equal; i.e., $n_1 = n_2 = n$ (say), and

(ii) The sample observations $(x_1, x_2, \ldots\ldots x_n)$ and $(y_1, y_2, \ldots\ldots y_n)$ are not completely independent but they are dependent in pairs.

That is we are making two observations one before treatment and another after the treatment on the same individual. For example a business concern wants to find if a particular media of promoting sales of a product, say door to door canvassing or advertisement in papers or through T.V. is really effective. Similarly a pharmaceutical company wants to test the efficiency of a particular drug, say for inducing sleep after the drug is given. For testing of such claims gives rise to situations in (i) and (ii) above, we apply paired t-test.

**Paired – t –test:**

Let $d_i = X_i - Y_i$ $(i = 1, 2, \ldots\ldots n)$ denote the difference in the observations for the $i^{th}$ unit.

**Null hypothesis:**

$H_0 : \mu_1 = \mu_2$ ie the increments are just by chance

**Alternative Hypothesis:**

$H_1 : \mu_1 \neq \mu_2$ $(\mu_1 > \mu_2$ (or) $\mu_1 < \mu_2)$

**Calculation of test statistic:**

$$t_0 = \left| \frac{\bar{d}}{S/\sqrt{n}} \right|$$

$$\text{where } \bar{d} = \frac{\Sigma d}{n} \text{ and } S^2 = \frac{1}{n-1}\Sigma(d-\bar{d})^2 = \frac{1}{n-1}\left[\Sigma d^2 - \frac{(\Sigma d)^2}{n}\right]$$

**Expected value:**

$$t_e = \left| \frac{\bar{d}}{S/\sqrt{n}} \right| \text{ follows t-distribution with } n-1 \text{ d.f}$$

**Inference:**

By comparing $t_0$ and $t_e$ at the desired level of significance, usually 5% or 1%, we reject or accept the null hypothesis.

**Example 5:**

To test the desirability of a certain modification in typists desks, 9 typists were given two tests of as nearly as possible the same nature, one on the desk in use and the other on the new type.

The following difference in the number of words typed per minute were recorded:

| Typists | A | B | C | D | E | F | G | H | I |
|---|---|---|---|---|---|---|---|---|---|
| Increase in number of words | 2 | 4 | 0 | 3 | – 1 | 4 | – 3 | 2 | 5 |

Do the data indicate the modification in desk promotes speed in typing?

**Solution:**

**Null hypothesis:**

$H_0 : \mu_1 = \mu_2$ i.e. the modification in desk does not promote speed in typing.

**Alternative Hypothesis:**

$H_1 : \mu_1 < \mu_2$ (Left tailed test)

**Level of significance:** Let $\alpha = 0.05$

| Typist | d | $d^2$ |
|---|---|---|
| A | 2 | 4 |
| B | 4 | 16 |
| C | 0 | 0 |
| D | 3 | 9 |
| E | –1 | 1 |
| F | 4 | 16 |
| G | –3 | 9 |
| H | 2 | 4 |
| I | 5 | 25 |
| | $\sum d = 16$ | $\sum d^2 = 84$ |

$$\bar{d} = \frac{\sum d}{n} = \frac{16}{9} = 1.778$$

$$S = \sqrt{\frac{1}{n-1}\left[\sum d^2 - \frac{(\sum d)^2}{n}\right]}$$

$$= \sqrt{\frac{1}{8}\left[84 - \frac{(16)^2}{9}\right]} = \sqrt{6.9} = 2.635$$

**Calculation of statistic:**

Under $H_0$ the test statistic is

$$t_0 = \left|\frac{\bar{d}.\sqrt{n}}{S}\right| = \frac{1.778\times 3}{2.635} = 2.024$$

**Expected value:**

$$t_e = \left|\frac{\bar{d}.\sqrt{n}}{S}\right| \text{ follows t-distribution with } 9-1=8\,\text{d.f}$$

$$= 1.860$$

**Inference:**

When $t_o < t_e$ the null hypothesis is accepted. The data does not indicate that the modification in desk promotes speed in typing.

**Example 6:**

An IQ test was administered to 5 persons before and after they were trained. The results are given below:

| Candidates | I | II | III | IV | V |
|---|---|---|---|---|---|
| IQ before training | 110 | 120 | 123 | 132 | 125 |
| IQ after training | 120 | 118 | 125 | 136 | 121 |

Test whether there is any change in IQ after the training programme (test at 1% level of significance)

**Solution:**

**Null hypothesis:**

$H_0 : \mu_1 = \mu_2$ i.e. there is no significant change in IQ after the training programme.

**Alternative Hypothesis:**

$H_1 : \mu_1 \neq \mu_2$ (two tailed test)

**Level of significance :**

$\alpha = 0.01$

| x | 110 | 120 | 123 | 132 | 125 | Total |
|---|---|---|---|---|---|---|
| y | 120 | 118 | 125 | 136 | 121 | - |
| d = x – y | – 10 | 2 | – 2 | – 4 | 4 | – 10 |
| $d^2$ | 100 | 4 | 4 | 16 | 16 | 140 |

$$\bar{d} = \frac{\sum d}{n} = \frac{-10}{5} = -2$$

$$S^2 = \frac{1}{n-1}\left[\sum d^2 - \frac{(\sum d)^2}{n}\right]$$

$$= \frac{1}{4}\left[140 - \frac{100}{5}\right] = 30$$

**Calculation of Statistic:**

Under $H_0$ the test statistic is

$$t_0 = \left|\frac{\bar{d}}{S/\sqrt{n}}\right|$$

$$= \left|\frac{-2}{\sqrt{30/5}}\right|$$

$$= \frac{2}{2.45}$$

$$= 0.816$$

**Expected value:**

$$t_e = \left|\frac{\bar{d}}{S^2/\sqrt{n}}\right| \text{ follows t-distribution with } 5-1 = 4 \text{ d.f}$$

$$= 4.604$$

**Inference:**

Since $t_0 < t_e$ at 1% level of significance we accept the null hypothesis. We therefore, conclude that there is no change in IQ after the training programme.

## Chi square statistic:

Various tests of significance described previously have mostly applicable to only quantitative data and usually to the data which are approximately normally distributed. It may also happens that we may have data which are not normally distributed. Therefore there arises a need for other methods which are more appropriate for studying the differences between the expected and observed frequencies. The other method is called Non-parametric or distribution free test. A non- parametric test may be defined as a statistical test in which no hypothesis is made about specific values of parameters. Such non-parametric test has assumed great importance in statistical analysis because it is easy to compute.

**Definition:**

The Chi- square ($\chi^2$) test (Chi-pronounced as ki) is one of the simplest and most widely used non-parametric tests in statistical work. The $\chi^2$ test was first used by Karl Pearson in the year 1900. The quantity $\chi^2$ describes the magnitude of the discrepancy between theory and observation. It is defined as

$$\chi^2 = \sum_{i=1}^{n} \left[\frac{(Oi - Ei)^2}{Ei}\right]$$

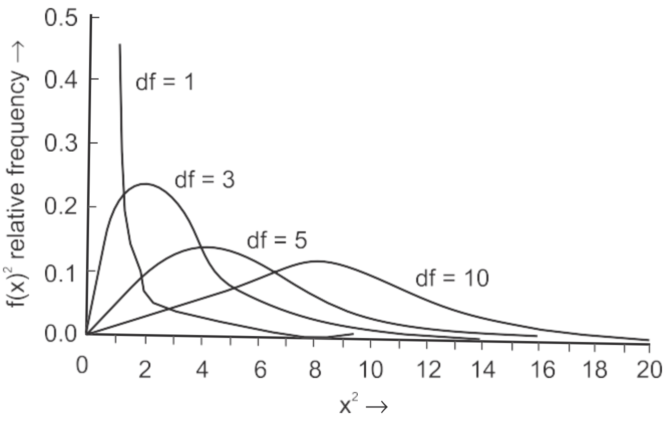
Where O refers to the observed frequencies and E refers to the expected frequencies.

**Note:**

If $\chi^2$ is zero, it means that the observed and expected frequencies coincide with each other. The greater the discrepancy between the observed and expected frequencies the greater is the value of $\chi^2$.

**Chi square - Distribution:**

The square of a standard normal variate is a Chi-square variate with 1 degree of freedom i.e., If X is normally distributed with mean μ and standard deviation σ, then $\left(\frac{x-\mu}{\sigma}\right)^2$ is a Chi-square variate ($\chi^2$) with 1 d.f. The distribution of Chi-square depends on the degrees of freedom. There is a different distribution for each number of degrees of freedom.




**properties of Chi-square distribution:**

1. The Mean of $\chi^2$ distribution is equal to the number of degrees of freedom (n)
2. The variance of $\chi^2$ distribution is equal to 2n
3. The median of $\chi^2$ distribution divides, the area of the curve into two equal parts, each part being 0.5.
4. The mode of $\chi^2$ distribution is equal to (n-2)
5. Since Chi-square values always positive, the Chi square curve is always positively skewed.
6. Since Chi-square values increase with the increase in the degrees of freedom, there is a new Chi-square distribution with every increase in the number of degrees of freedom.
7. The lowest value of Chi-square is zero and the highest value is infinity ie $\chi^2 \geq 0$.
8. When Two Chi- squares $\chi_1^2$ and $\chi_2^2$ are independent $\chi^2$ distribution with $n_1$ and $n_2$ degrees of freedom and their sum $\chi_1^2 + \chi_2^2$ will follow $\chi^2$ distribution with $(n_1 + n_2)$ degrees of freedom.

9. When n (d.f) > 30, the distribution of $\sqrt{2\chi^2}$ approximately follows normal distribution. The mean of the distribution $\sqrt{2\chi^2}$ is $\sqrt{2n-1}$ and the standard deviation is equal to 1.

**Conditions for applying $\chi^2$ test:**

The following conditions should be satisfied before applying $\chi^2$ test.

10. N, the total frequency should be reasonably large, say greater than 50.
11. No theoretical cell-frequency should be less than 5. If it is less than 5, the frequencies should be pooled together in order to make it 5 or more than 5.
12. Each of the observations which makes up the sample for this test must be independent of each other.
13. $\chi^2$ test is wholly dependent on degrees of freedom.

## Test of independence

Let us suppose that the given population consisting of N items is divided into r mutually disjoint (exclusive) and exhaustive classes $A_1, A_2, \ldots, A_r$ with respect to the attribute A so that randomly selected item belongs to one and only one of the attributes $A_1, A_2, \ldots, A_r$ .Similarly let us suppose that the same population is divided into c mutually disjoint and exhaustive classes $B_1, B_2, \ldots, B_c$ w.r.t another attribute B so that an item selected at random possess one and only one of the attributes $B_1, B_2, \ldots, B_c$. The frequency distribution of the items belonging to the classes $A_1, A_2, \ldots, A_r$ and $B_1, B_2, \ldots, B_c$ can be represented in the following r × c manifold contingency table.

**r × c manifold contingency table**

| B<br>A | $B_1$ | $B_2$ | □ | $B_j$ | □ | $B_c$ | Total |
|---|---|---|---|---|---|---|---|
| $A_1$ | $(A_1B_1)$ | $(A_1B_2)$ | □ | $(A_1B_j)$ | □ | $(A_1B_c)$ | $(A_1)$ |
| $A_2$ | $(A_2B_1)$ | $(A_2B_2)$ | □ | $(A_2B_j)$ | □ | $(A_2B_c)$ | $(A_2)$ |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| $A_i$ | $(A_iB_1)$ | $(A_iB_2)$ | □ | $(A_iB_j)$ | □ | $(A_iB_c)$ | $(A_i)$ |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| $A_r$ | $(A_rB_1)$ | $(A_rB_2)$ | □ | $(A_rB_j)$ | □ | $(A_rB_c)$ | $(A_r)$ |
| Total | $(B_1)$ | $(B_2)$ | □ | $(B_j)$ | □ | $(B_c)$ | $\sum Ai =$<br>$\sum Bj = N$ |

$(A_i)$ is the number of persons possessing the attribute $A_i$, (i = 1, 2, … r), $(B_j)$ is the number of persons possing the attribute $B_j$,(j = 1,2,3,….c) and $(A_i B_j)$ is the number of persons possessing both the attributes $A_i$ and $B_j$ (i = 1, 2, …r, j = 1,2,…c).

Also $\sum A_i = \sum B_j = N$

Under the null hypothesis that the two attributes A and B are independent, the expected frequency for $(A_iB_j)$ is given by

$$= \frac{(Ai)\ (Bj)}{N}$$

**Calculation of statistic:**

Thus the under null hypothesis of the independence of attributes,the expected frequencies for each of the cell frequencies of the above table can be obtained on using the formula

$$\chi_0^2 = \Sigma \left( \frac{(O_i - E_i)^2}{E_i} \right)$$

**Expected value:**

$$\chi_e^2 = \Sigma \left( \frac{(O_i - E_i)^2}{E_i} \right)$$ follows $\chi^2$-distribution with (r – 1) (c – 1) d.f

**Inference:**

Now comparing $\chi_0^2$ with $\chi_e^2$ at certain level of significance ,we reject or accept the null hypothesis accordingly at that level of significance.

**2 × 2 contingency table :**

Under the null hypothesis of independence of attributes, the value of $\chi^2$ for the 2 × 2 contingency table

| | | | Total |
|---|---|---|---|
| | a | b | a + b |
| | c | d | c + d |
| Total | a + c | b + d | N |

is given by

$$\chi_0^2 = \frac{N(ad - bc)^2}{(a + c)(b + d)(a + b)(c + d)}$$

**Yate' s correction**

In a 2 × 2 contingency table, the number of d.f. is (2 – 1) (2 – 1) = 1. If any one of the theoretical cell frequency is less than 5,the use of pooling method will result in d.f = 0 which is meaningless. In this case we apply a correction given by F.Yate (1934) which is usually known as “Yates correction for continuity”. This consisting adding 0.5 to cell frequency which is less than 5 and then adjusting for the remaining cell frequencies accordingly. Thus corrected values of $\chi^2$ is given as

$$\chi^2 = \frac{N\left[\left(a \mp \frac{1}{2}\right)\left(d \mp \frac{1}{2}\right) - \left(b \pm \frac{1}{2}\right)\left(c \pm \frac{1}{2}\right)\right]^2}{(a+c)(b+d)(a+b)(c+d)}$$

**Example :**

1000 students at college level were graded according to their I.Q. and the economic conditions of their homes. Use $\chi^2$ test to find out whether there is any association between economic condition at home and I.Q.

| Economic Conditions | IQ | | Total |
|---|---|---|---|
| | High | Low | |
| Rich | 460 | 140 | 600 |
| Poor | 240 | 160 | 400 |
| Total | 700 | 300 | 1000 |

**Solution:**

**Null Hypothesis:**

There is no association between economic condition at home and I.Q. i.e. they are independent.

$$E_{11} = \frac{(A)(B)}{N} = \frac{600 \times 700}{1000} = 420$$

The table of expected frequencies shall be as follows.

| | | | Total |
|---|---|---|---|
| | 420 | 180 | 600 |
| | 280 | 120 | 400 |
| Total | 700 | 300 | 1000 |

| Observed Frequency O | Expected Frequency E | $(O-E)^2$ | $\left(\frac{(O-E)^2}{E}\right)$ |
|---|---|---|---|
| 460 | 420 | 1600 | 3.81 |
| 240 | 280 | 1600 | 5.714 |
| 140 | 180 | 1600 | 8.889 |
| 160 | 120 | 1600 | 13.333 |
| | | | 31.746 |

$$\chi_0^2 = \Sigma \left( \frac{(O-E)^2}{E} \right) = 31.746$$

**Expected Value:**

$$\chi_e^2 = \Sigma \left( \frac{(O-E)^2}{E} \right)$$ follows $\chi^2$-distribution with (2 – 1) (2 –1) = 1 d.f.

= 3.84

**Inference:**

$\chi_0^2 > \chi_e^2$, hence the hypothesis is rejected at 5% level of significance. ∴ There is association between economic condition at home and I.Q.

**Example:**

Out of a sample of 120 persons in a village, 76 persons were administered a new drug for preventing influenza and out of them, 24 persons were attacked by influenza. Out of those who were not administered the new drug ,12 persons were not affected by influenza.. Prepare

(i) 2 × 2 table showing actual frequencies.

(ii) Use chi-square test for finding out whether the new drug is effective or not.

**Solution:**

The above data can be arranged in the following 2 × 2 contingency table

**Table of observed frequencies**

| New drug | Effect of Influenza | | Total |
|---|---|---|---|
| | Attacked | Not attacked | |
| Administered | 24 | 76 – 24 = 52 | 76 |
| Not administered | 44 – 12 = 32 | 12 | 120 – 76 = 44 |
| Total | 120 – 64 = 56<br>24 + 32 = 56 | 52 + 12 = 64 | 120 |

**Null hypothesis:**

'Attack of influenza' and the administration of the new drug are independent.

**Computation of statistic:**

$$\chi_0^2 = \frac{N(ad - bc)^2}{(a+c)(b+d)(a+b)(c+d)}$$

$$= \frac{120\,(24 \times 12 - 52 \times 32)^2}{56 \times 64 \times 7\mathbf{6} \times 44}$$

$$= \frac{120\,(-1376)^2}{56 \times 64 \times 76 \times 44} = \frac{120\,(1376)^2}{56 \times 64 \times 76 \times 44}$$

$$= \text{Anti } \log[\log 120 + 2\log 1376 - (\log 56 + \log 64 + \log 76 + \log 44)]$$

$$= \text{Antilog } (1.2777) = 18.95$$

**Expected Value:**

$\chi_e^2 = \Sigma\left(\frac{(O-E)^2}{E}\right)$ follows $\chi^2$-distribution with (2 – 1) (2 –1) = d.f.

= 3.84

**Inference:**

Since $\chi_0^2 > \chi_e^2$ $H_0$ is rejected at 5 % level of significance. Hence we conclude that the new drug is definitely effective in controlling (preventing) the disease (influenza).

**Example :**

Two researchers adopted different sampling techniques while investigating the same group of students to find the number of students falling in different intelligence levels. The results are as follows

| Researchers | No. of students | | | | Total |
|---|---|---|---|---|---|
| | Below average | Average | Above average | Genius | |
| X | 86 | 60 | 44 | 10 | 200 |
| Y | 40 | 33 | 25 | 2 | 100 |
| Total | 126 | 93 | 69 | 12 | 300 |

Would you say that the sampling techniques adopted by the two researchers are independent?

**Solution:**

**Null Hypothesis:**

The sampling techniques adopted by the two researchers are independent.

$$E(86) = \frac{126 \times 200}{300} = 84$$

$$E(60) = \frac{93 \times 200}{300} = 62$$

$$E(44) = \frac{69 \times 200}{300} = 46$$

The table of expected frequencies is given below.

| | Below average | Average | Above average | Genius | Total |
|---|---|---|---|---|---|
| X | 84 | 62 | 46 | 200 – 192 = 8 | 200 |
| Y | 126 –84 = 42 | 93 – 62 = 31 | 69 – 46 = 23 | 12 – 8 = 4 | 100 |
| Total | 126 | 93 | 69 | 12 | 300 |

**Computation of chi-square statistic:**

| Observed Frequency O | Expected Frequency E | (O – E) | $(O-E)^2$ | $\left(\frac{(O-E)^2}{E}\right)$ |
|---|---|---|---|---|
| 86 | 84 | 2 | 4 | 0.048 |
| 60 | 62 | – 2 | 4 | 0.064 |
| 44 | 46 | – 2 | 4 | 0.087 |
| 10 | 8 | 2 | 4 | 0.500 |
| 40 | 42 | – 2 | 4 | 0.095 |
| 33 | 31 | 2 | 4 | 0.129 |
| $\begin{bmatrix}25\\2\end{bmatrix}$ 27 | $\begin{bmatrix}23\\4\end{bmatrix}$ 27 | 0 | 0 | 0 |
| 300 | 300 | 0 | | 0.923 |

$$\chi_o^2 = \Sigma\left(\frac{(O-E)^2}{E}\right) = 0.923$$

**Expected Value:**

$$\chi_e^2 = \Sigma\left(\frac{(O-E)^2}{E}\right)$$ follows $\chi^2$-distribution with (4 – 1) (2 –1)

= 3 – 1 = 2 d.f.

= 5.991

**Inference:**

Since $\chi_0^2 < \chi_e^2$, we accept the null hypothesis at 5 % level of significance. Hence we conclude that the sampling techniques by the two investigators, do not differ significantly.

## Exercise

**I. Choose the best answer:**

1. Student' s ' t' distribution was pioneered by

(a) Karl Pearson (b) Laplace (c) R.A. Fisher (d) William S.Gosset

2. t - distribution ranges from

(a) $-\infty$ to 0 (b) 0 to $\infty$ (c) $-\infty$ to $\infty$ (d) 0 to 1

3. The difference of two means in case of a small samples is tested by the formula

(a) $t = \frac{\overline{x_1} - \overline{x_2}}{s}$ (b) $\frac{\overline{x_1} - \overline{x_2}}{s} \sqrt{\frac{n_1 + n_2}{n_1 + n_2}}$

(c) $t = \frac{\overline{x_1} - \overline{x_2}}{s} \sqrt{\frac{n_1 n_2}{n_1 + n_2}}$ (d) $t = \sqrt{\frac{n_1 n_2}{n_1 + n_2}}$

4. While testing the significance of the difference between two sample means in case of small samples, the degree of freedom is

(a) $n_1 + n_2$ (b) $n_1 + n_2 - 1$ (c) $n_1 + n_2 - 2$ (d) $n_1 + n_2 + 2$

5. Paired t-test is applicable when the observations in the two samples are

(a) Paired (b) Correlated (c) equal in number (d) all the above

6. The mean difference between a paired observations is 15.0 and the standard deviation of differences is 5.0 if n = 9, the value of statistic t is

(a) 27 (b) 9 (c) 3 (d) zero

7. When observed and expected frequencies completely coincide $\chi^2$ will be

(a) –1 (b) + 1 (c) greater than 1 (d) 0

8. For $\nu = 2$, $\chi^2_{0.05}$ equals

(a) 5.9 (b) 5.99 (c) 5.55 (d) 5.95

9. The calculated value of $\chi^2$ is

(a) always positive (b) always negative

(c ) can be either positive or negative (d) none of these

10. The Yate’ s corrections are generally made when the cell frequency is

(a) 5 (b) < 5 (c) 1 (d) 4

11. The $\chi^2$ test was derived by

(a) Fisher (b) Gauss (c) Karl Pearson (d) Laplace

12. Degrees of freedom for Chi-square in case of contingency table of order (4 × 3) are

(a) 12 (b) 9 (c) 8 (d) 6

13. Customarily the larger variance in the variance ratio for F-statistic is taken

(a) in the denominator (b) in the numerator

(c) either way (d) none of the above

14. The test statistic $F = \frac{S_1^2}{S_2^2}$ is used for testing

(a) $H_0 : \mu_1 = \mu_2$ (b) $H_0 : \sigma_1^2 = \sigma_2^2$ (c) $H_0 : \sigma_1 = \sigma_2$ (d) $H_0 : \sigma^2 = \sigma_0^2$

15. Standard error of the sample mean in testing the difference between population mean and sample mean under t- statistic

(a) $\frac{\sigma^2}{\sqrt{n}}$ (b) $\frac{s}{\sqrt{n}}$ (c) $\frac{\sigma}{\sqrt{n}}$ (d) $\frac{s}{n}$

## II. fill in the blanks:

16. The assumption in t- test is that the population standard deviation is _____________

17. t- values lies in between _____________

18. Paired t- test is applicable only when the observations are ______________

19. Student t- test is applicable in case of____________samples

20. The value of $\chi^2$ statistic depends on the difference between___________and___________ frequencies

21. The value of $\chi^2$ varies from____________to ____________

22. Equality of two population variances can be tested by ____________

23. The $\chi^2$ test is one of the simplest and most widely used__________test.

24. The greater the discrepancy between the observed and expected frequency__________the value of $\chi^2$.

25. In a contingency table $\nu$__________.

26. The distribution of the $\chi^2$ depends on the___________.

27. The variance of the $\chi^2$ distribution is equal to________the d.f.

28. One condition for application of $\chi^2$ test is that no cell frequency should be __________

29. In a $3 \times 2$ contingency table, there are_________cells

30. F- test is also known as___________ratio test.

**III. Answer the following**

31. Define students ‘ t’ – statistic

32. State the assumption of students ‘ t’ test

33. State the properties of t- distribution

34. What are the applications of t- distribution

35. Explain the test procedure to test the significance of mean in case of small samples.

36. What do you understand by paired ‘ t’ test > What are its assumption.

37. Explain the test procedure of paired – t- test

38. Define Chi square test

39. Define Chi square distribution

40. What is $\chi^2$ test of goodness of fit.

41. What are the precautions are necessary while applying $\chi^2$ test?

42. Write short note on Yate’ s correction.

43. Explain the term ‘ Degrees of freedom’

44. Define non-parametric test

45. Define $\chi^2$ test for population variance

46. Ten flower stems are chosen at random from a population and their heights are found to be (in cms) 63 , 63, 66, 67, 68, 69, 70, 70, 71 and 71. Discuss whether the mean height of the population is 66 cms.

47. A machine is designed to produce insulating washers for electrical devices of average thickness of 0.025cm. A random sample of 10 washers was found to have an average thickness of 0.024cm with a standard deviation of 0.002cm. Test the significance of the deviation.

48. Two types of drugs were used on 5 and 7 patients for reducing their weight. Drug A was imported and drug B indigenous. The decrease in the weight after using the drugs for six months was as follows:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Drug A : | 10 | 12 | 13 | 11 | 14 | | |
| Drug B : | 8 | 9 | 12 | 14 | 15 | 10 | 9 |

Is there a significant difference in the efficiency of the two drugs? If not, which drug should you buy?

49. The average number of articles produced by two machines per day are 200 and 250 with standard deviations 20 and 25 respectively on the basis of records 25 days production. Can you conclude that both the machines are equally efficient at 1% level of significance.

50. A drug is given to 10 patients, and the increments in their blood pressure were recorded to be 3, 6, -2 , +4, -3, 4, 6, 0, 0, 2. Is it reasonable to believe that the drug has no effect on change of blood pressure?

51. The sales data of an item in six shops before and after a special promotional campaign are as under:

| | | | | | | |
|---|---|---|---|---|---|---|
| Shops: | A | B | C | D | E | F |
| Before Campaign: | 53 | 28 | 31 | 48 | 50 | 42 |
| After Campaign: | 58 | 29 | 30 | 55 | 56 | 45 |

Can the campaign be judges to be a success? Test at 5% level of significance.

52. A survey of 320 families with 5 children each revealed the following distribution.

| | | | | | | |
|---|---|---|---|---|---|---|
| No. of boys | 5 | 4 | 3 | 2 | 1 | 0 |
| No. of Girls | 0 | 1 | 2 | 3 | 4 | 5 |
| No. of families | 14 | 56 | 110 | 88 | 40 | 12 |

Is the result consistent with the hypothesis that the male and female births are equally probable?

53. The following mistakes per page were observed in a book.

| | | | | | | |
|---|---|---|---|---|---|---|
| No. of mistakes per page | 0 | 1 | 2 | 3 | 4 | Total |
| No. of pages | 211 | 90 | 19 | 5 | 0 | 325 |

Fit a Poisson distribution and test the goodness of fit.

54. Out of 800 persons, 25% were literates and 300 had travelled beyond the limits of their district 40% of the literates were among those who had not travelled. Test of 5% level whether there is any relation between travelling and literacy.

55. You are given the following

| Fathers | Intelligent Boys | Not intelligent boys | Total |
|---|---|---|---|
| Skilled father | 24 | 12 | 36 |
| Unskilled Father | 32 | 32 | 64 |
| Total | 56 | 44 | 100 |

Do these figures support the hypothesis that skilled father have intelligent boys?

56. A random sample of size 10 from a normal population gave the following values

65 , 72, 68, 74, 77, 61,63, 69 , 73, 71

Test the hypothesis that population variance is 32.

57. A sample of size 15 values shows the s.d to be 6.4. Does this agree with hypothesis that the population s.d is 5, the population being normal.

58. In a sample of 8 observations, the sum of squared deviations of items from the mean was 94.5. In another sample of 10 observations, the value was found to be 101.7 test whether the difference in the variances is significant at 5% level.

59. The standard deviations calculated from two samples of sizes 9 and 13 are 2.1 and 1.8 respectively. May the samples should be regarded as drawn from normal populations with the same standard deviation?

60. Two random samples were drawn from two normal populations and their values are

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| A | 66 | 67 | 75 | 76 | 82 | 84 | 88 | 90 | 92 | - | - |
| B | 64 | 66 | 74 | 78 | 82 | 85 | 87 | 92 | 93 | 95 | 97 |

Test whether the two populations have the same variance at 5% level of significance.

61. An automobile manufacturing firm is bringing out a new model. In order to map out its advertising campaign, it wants to determine whether the model will appeal most to a particular age – group or equal to all age groups. The firm takes a random sample from persons attending a pre-view of the new model and obtained the results summarized below:

| Person who | Age groups | | | | |
|---|---|---|---|---|---|
| | Under 20 | 20-39 | 40-50 | 60 and over | Total |
| Liked the car | 146 | 78 | 48 | 28 | 300 |
| Disliked the car | 54 | 52 | 32 | 62 | 200 |
| Total | 200 | 130 | 80 | 90 | 500 |

What conclusions would you draw from the above data?

# Answers:

**I.**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1. (d) | 2. (c) | 3. (c) | 4. (c) | 5. (d) | 6. (b) | 7. (d) | 8. (b) |
| 9. (a) | 10. (c) | 11 (c) | 12. (d) | 13. (b) | 14. (b) | 15. (b) | |

**II.**

| | | |
|---|---|---|
| 16. not known | 17. - ∞ to ∞ | 18. paired |
| 19. small | 20. observed, expected | 21. 0, ∞ |
| 22. F- test | 23. non parametric | 24. greater |
| 25. (r – 1 ) ((–1)) | 26. degrees of freedom | 27. d.f. twice |
| 28. less than 5 | 29. 6 | 30. variance |

**III.**

| | |
|---|---|
| 46. t = 1.891 $H_0$ is accepted | 47. t = 1.5 $H_0$ is accepted |
| 48. t = 0.735 $H_0$ is accepted | 49. t = 7.65 $H_0$ is rejected |
| 50. t = 2, $H_0$ is accepted | 51. t = 2.58 $H_0$ is rejected |
| 52. $\chi^2$ = 7.16 $H_0$ is accepted | 53. $\chi^2$ = 0.068 $H_0$ is accepted |
| 54. $\chi^2$ = 0.0016 $H_0$ is accepted | 55. $\chi^2$ = 2.6 $H_0$ is accepted |
| 56. $\chi^2$ = 7.3156 $H_0$ is accepted | 57. $\chi^2$ = 24.58 $H_0$ is rejected |
| 58. $\chi^2$ = 24.576 $H_0$ is rejected | 59. F = 1.41 $H_0$ is accepted |
| 60. F = 1.415 $H_0$ is accepted | 61. $\chi^2$ = 7.82 , $H_0$ is rejected |