12/08/20

UNIT- I.

PROBABILITY.

Probability :

*. Probability is the measure of the likelihood that an event will occur in a Random Experiment.

*. Probability is quantified as a number between 0 and 1, where, loosely speaking, 0 indicates impossibility and 1 indicater certainty.

*. The higher the probability of an event, the more likely it is that the event will occur.

*. Converting the uncertainty in mathematical method. It's a study about uncertainity.

Ex: the probability of flipping a coin and it being heads is ½, because there is 1 way of getting a head and the total number of possible outcomes is 2.

Experiment :

*. In probability theory, an experiment or trial is any procedure that can be infinitely repeated and has a well-defined set of possible outcomes, known as the sample space.

*. An experiment is said to be random if it has more than one possible outcome, and deterministic if it has only one.

Eg: In a flipping coin, playing cards are the some examples.

Sample space:

*. The set of all possible outcomes is called the sample space of the experiment.

*. This is denoted by S.

*. Sample space can be written using the set notation, { }.

Experiment 1: Tossing a coin ; possible outcomes are head ~~and~~ or tail.

Sample space, S = {head, tail}

Experiment 2: Tossing a die ; Possible outcomes are the numbers 1, 2, 3, 4, 5 and 6.

Sample space, S = {1, 2, 3, 4, 5, 6}

Basic Definitions.

1) Random Experiment:

*. If each trial of experiment conducted under identical conditions.

*. The outcome is not unique, But may be anyone of the possibles outcomes, then such an experiment is called as random experiment.

ex:

*. Tossing a coin, throwing a die, selecting a card from a batch of cards.

2.) Outcome:

*. The result of a random experiment will be called as an outcome.

3.) Trial & Events:

*. Any particular performance of a random experiment is called a trial and outcome or combination of outcomes are turned as events.

ex:

*. If a coin is tossed repeatedly in the result is not unique. We may get any of the true

*. When a die is thrown the possible ~~outco~~ events are 1, 2, 3, 4, 5, 6.

4.) Exhaustive Events:

*. The total number of possible outcomes of a random experiment is known as exhaustive events.

ex:

*. In throwing a die the exhaustive events are {1, 2, 3, 4, 5, 6} and in tossing experiment the exhaustive events are H & T.

(உடன்பாடு)

5.) Mutually Exclusive events:

*. Events are said to be mutually exclusive, if the happening of anyone of them precludes the happening of all the others.

i.e.) *. Mutually exclusive events are things that can't happen at the same time.

ex:

*. In throwing a die, all the six faces numbered 1 to 6 are the mutually exclusive.

6.) Equally Likely Events:

*. Outcome of a trial events are said to be equally likely events. If taking into consideration all the relevant evidences there is no reason except

1 in preference to others.

Ex: In a random tasks of a good coin H & T are equally likely events.

*. When a dice is thrown twice the result of 1st thrown does not affect the result of 2nd throw.

Types of probability:

There are 3 types of probability named,

1.) Mathematical (or) classical probability.

2.) Statistical probability.

3.) Axiomatic probability.

1.) Mathematical Probability:

*. If a random experiment or a trial results in "n" exhaustive, Mutually exclusive and equally likely events. out of if m are favourable to the accurance of E. Then the probability 'P' of occurance of E denoted by.

$$P = \frac{\text{number of favourable cases}}{\text{Total no. of experiments.}}$$

$$\boxed{\therefore P = m/n.}$$

Remarks:

1.) Since $m \geq 0$, $n > 0$ and $m \leq n$

$\therefore P(E) \geq 0$ and $P(E) \leq 1$.

$\Rightarrow 0 \leq P$; $P(E) \leq 1$.

2.) The non happening of the event in is called the complementary event of E and its denoted by $\bar{E}$ (or) nE^c.

The number of cases favourable to E complement is n-m. Then the probability "q" that E will not happen is probability of

$$P(E^c) = \frac{n-m}{n}$$

$$= n/n - m/n = 1 - m/n$$

$$\boxed{E = 1 - P}$$

3.) For sure events the probability value is 1 and for impossible events the probability value is 0. The probability "P" of the happening of a event is also known as the probability of success. The probability "q" of the non-happening of the event as the probability of failure. That is

$$\boxed{P + q = 1}$$

Example :

1.) A die is thrown find the probability of head.

1.) odd no   2.) Even no.

Soln :

$$P(odd) = 3/6 = 1/2.$$

$$P(even) = 3/6 = 1/2.$$

2.) A bag contains six white balls, 4 black balls find the probability of

1.) A ball is selected of random. What is the probability that it is white.

2.) 2 Balls are selected at random. What is the probability that they are black.

3.) 2 balls are selected at random. What is the probability ? They are white and Black.

6 White ; 4 Black.

Answers :

i) $P(1 \text{ white ball}) = {}^6C_1 / {}^{10}C_1$

ii) $P(2 \text{ Black balls}) = \frac{4C_2}{10C_2} = \frac{4\times3/1\times2}{10\times9/1\times2}$

$= 6/45$

iii) $P(1W\ 1B) = \frac{6C_1 \times 4C_1}{10C_2} = 24/45$.

And → Multiplication ; or → Addition.

Limitations of classical Probability:

*. The definition of probability break down in the following cases.

(1.) If the various outcome of random experiment are not equally likely or Equally probable.

(2.) If the exhaustive number of outcome of random experiment is infinite or unknown.

Statistical Probability: (or Empirical probability).

*. If an experiment is performed repeatedly under essentially homogeneous and identical conditions, then the limiting value of the ratio of the number of times the event occurs to the number of trials, as the number of trials becomes indefinitely large, is called the probability of happening of the event, it being assumed that the limit is finite and unique.

*. Symbolically, if in N trials as event E happens M times, then the probability of the happening of E.

denoted by P(E), is given by :

$$\boxed{P(E) = \lim_{N \to \infty} M/N}.$$

Remarks :

*. Since in the relative frequency approach, the probability is obtained objectively by repetitive empirical observations, it is also known as "Empirical probability".

Limitations of Empirical probability :

*. If an experiment is repeated a large number of times, the experimental conditions may not remain identical and homogeneous.

*. The limit in $P(E) = \lim_{N \to \infty} M/N$ may not attain a unique value, however large N may be.

3.) What is the chance that a leap year selected at random will contain 53 Sundays?

Soln:

*. In a leap year we have 366 days. That is 52+2 days. This two days can be (Sun, Mon), (Mon, Tue), (Tue, Wed), (Wed, Thurs), (Thurs, Fri), (Fri, Satur), (Sat, Sun), (Sun, Mon).

Therefore probability of heading 53 Sundays = $2/7$.

4.) 2 unbiased dice are thrown find the following prob.

1.) both the dice show the same number.

2.) The 1st dice shows six.

3.) The total of numbers on the dice is 8.

4.) The total of the numbers on the dice is >8.

5.) The total of the numbers on the dice is 13.

6.) The total of the numbers on the dice is any number from 2-12.

Soln:

while throwing 2 dice following or the possible outcomes

$$S = \left\{\begin{matrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{matrix}\right\}$$

1.) Both are same numbers:

(1, 1) (2, 2) (3, 3) (4, 4) (5, 5) (6, 6)

$P = 6/36$.

2.) 1^{st} dice show six:

(6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6)

$P = 6/36$.

3.) The total no. of on the dice is 8:

(2, 6) (3, 5) (4, 4) (5, 3) (6, 2)

$P = 5/36$.

4.) The total of no. on the dice greater than 8:

(3, 6) (4, 5) (4, 6) (5, 4) (5, 5) (5, 6) (6, 3) (6, 4) (6, 5) (6, 6)

$P = 10/36$.

5.) The total of no. on the dice is 13.

$P = 0/36$.

6.) The total no. of on the dice is any no. from 2-12.

$P = 36/36$

$\boxed{\therefore P = 1}$

5.) 4 cards thrown at random from 52 cards find the probability.

1.) They are a King, a queen, a Jack and an Ace.

2.) 2 are kings and 2 are Queens

3.) 2 are Black and 2 are Red.

4.) There are 2 cards of hearts & 2 cards of diamond.

Soln:

1.) In 52 cards, there are 4 King, 4 Queen, 4 Jack and 4 Ace.

$$\therefore P(\text{King, Queen, Ace, Jack}) = \frac{4C_1 \times 4C_1 \times 4C_1 \times 4C_1}{52C_4}.$$

2.) There are 4 kings and 4 Queens.

$$\therefore P(2K, 2Q) = \frac{4C_2 \times 4C_2}{52C_4}.$$

3.) There are 26 black cards & 26 Red cards.

$$\therefore P(2B, 2R) = \frac{26C_2 \times 26C_2}{52C_4}.$$

4.) There are 13 hearts and 13 diamonds.

$$\therefore P(2H, 2D) = \frac{13C_2 \times 13C_2}{52C_4}.$$

6.) A box contain 6 white, 4 Red and 9 Black balls. 3 Balls are drawn at random. Find the probability that 1.) 2 balls drawn are white. ; 2.) one is of each colour ; 3.) None is red ; 4.) at least one is white.

Soln:

Total no. of Balls are $6+4+9 = 19$.; since 3 balls are drawn at random,

$\therefore$ the exhaustive number of cases are $19C_3$.

1.) If 2 balls of the 3 drawn balls are ought to be white.

If 2 balls should be drawn out of 6 white balls which can be done in $6C_2$ ways and the 3rd ball can be drawn out of the remain in

$19 - 6 = 13$ balls

$$\therefore P(2W \text{ and } 1 \text{ other}) = \frac{6C_2 \times 13C_1}{19C_3}$$

2.) Since the no. of favourable cases of getting one ball of each colour is

$$P(1W, 1R, 1B) = \frac{6C_1 \times 4C_1 \times 9C_1}{19C_3}$$

3.) If none of the drawn balls is red, Balls other than red is $19-4 = 15$.

$$P(3 \text{ other than Red balls}) = \frac{15C_3}{19C_3}$$

4.) Atleast one is white :

$\Rightarrow p+q = 1$.

$\Rightarrow P(\text{atleast one is white}) = 1 - P(\text{none of the three balls is white})$

In order that none of the three balls is white, all the three balls must be drawn out of the red and black balls,

$4+9 = 13$ balls.

$$\therefore P(\text{none of the three balls is white}) = \frac{13C_3}{19C_3}.$$

$$P(\text{atleast one ball is white}) = 1 - \frac{13C_3}{19C_3}.$$

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Assignment-1.

7.) 2 cards are drawn from a 54 cards. 2 of them being that. what is the prob;

i) they are both spades ; ii) they are both kings.

iii) Atleast one of them is Joker ; iv) One of them is Queen and other either is Joker.

Soln:

Total no. of cards = 54 ; and no. of cards drawn are 2 => Total no. of cards = $54C_2$.

i) they are both spades:

No. of spade cards = 13 ; and 2 are drawn from the spade.

$$\therefore P(2S) = \frac{13C_2}{54C_2} = \frac{26}{477}.$$

ii) they are both kings :

No. of kings in 54 cards is 4 from heart, diamond, clover and spade.

$$\therefore P(2K) = \frac{4C_2}{54C_2} = 2/477.$$

iii) Atleast one of them is Joker :

P(atleast one is Joker) = 1 − P(none of the two is Joker).

In order that none of the two cards are Jocker, then the 2 cards taken out of the 13 h, 13 D, 13 C and 13 S ;

$\therefore$ 13+13+13+13 = 52 cards then.

$$P(\text{none of the two is Joker}) = \frac{52C_2}{54C_2}$$

$P(\text{Atleast one is Joker}) = 1 - \frac{52C_2}{54C_2} = \frac{35}{477}$.

4.) One of them is Queen ~~and~~ or other either a Jack or a Joker.

∴ The no. of Queens in 54 cards is 4

$\Rightarrow 4C_1$

The no of Jokers in 54 cards is 2 $\Rightarrow 2C_1$
Jack is $\Rightarrow 4C_1$

$\therefore P(1Q \,\&\, 1J) = \frac{4C_1 \times 2C_1}{54C_2} = \frac{4C_1 \times (4C_1 + 2C_1)}{54C_2} = \frac{4\times 6}{1431} = \frac{24}{1431} = 8/477$.

8.) A bag contains 6 Red, 5 white and 4 Black balls, If two balls are drawn find the probability that none of them is Red.

Soln:

∴ The total no. of balls are

$6+5+4 = 15$ balls, in that 2 balls are drawn that the combination is $15C_2$.

i) None of them is Red:

Balls other than Red $= 15 - 6$
$= 9$

$P(2 \text{ balls other than Red}) = \frac{9C_2}{15C_2} = 12/35$

9.) A bag contains 7 Red and 5 white balls, 4 balls are drawn at random. What is the probability that 1.) All of them are red ; 2.) 2 Red and 2 white balls.

Soln:

The total no. of balls is $7+5 = 12$ balls, and 4 is drawn at random and the number of balls in combination is $12C_4$.

i) All of them are red :

$$P(4R) \text{ (or) } P(\text{All is red}) = \frac{7C_4}{12C_4} = \frac{7}{99}$$

ii) 2 Red and 2 white balls :

$$P(2\text{ red } \& \text{ } 2\text{ white}) = \frac{7C_2 \times 5C_2}{12C_4} = \frac{14}{33}.$$

10.) A box contain 2 Red balls, 3 Blue balls and 4 Black balls, three balls are drawn at random. What is the probability that.

1.) the balls are different colours ; 2.) 2 balls are of the same colour and 3rd is different ; 3.) 3 balls are of the same colour.

Soln:

The total no. of Balls is 9 balls, and 3 drawn from that so the combination is $9C_3$.

i) the balls are different colours :

Red $= 2C_1$; Blue $= 3C_1$; Black $= 4C_1$ because 3 of each is different colour.

$$\therefore P(1R \text{ and } 1\text{Blue and } 1 \text{ black}) = \frac{2C_1 \times 3C_1 \times 4C_1}{9C_3} = \frac{2}{7}.$$

ii) 2 balls are of the same and 3rd is different :

There are 3 combinations in this 2 are same condition that is $2C_2$, $3C_2$ or $4C_2$. ; 3rd is different condition such that $2C_1$, $3C_1$ or $4C_1$.

$$\therefore P(2 \text{ balls are same and } 1 \text{ is different}) = \frac{2C_2 + 3C_2 + 4C_2}{9C_3} \times \frac{2C_1 + 3C_1 + 4C_1}{9C_3}$$

$$= \frac{(2C_2 + 3C_2 + 4C_2) \times (2C_1 + 3C_1 + 4C_1)}{9C_3}$$

$$= \frac{(1+3+6) \times (2+3+4)}{9C_3} = \frac{10 \times 9}{9C_3}$$

$$P(2S+1d) = \frac{90}{9C_3}$$

iii) 3 balls are of the same colour:

∴ The combinations are 2 Red balls is impossible because there are only 2 balls in red and the drawn balls is 3, so it is 0.

Then the blue and black balls are

Blue balls $= 3C_3$ or Black balls $= 4C_3$

$$\therefore P(3 \text{ are in same colour}) = \frac{3C_3 + 4C_3}{9C_3}$$

$$= \frac{1+4}{9C_3} = 5/9C_3 = 5/84.$$

4.) The no 1, 2, 3, 4, 5 and 6 are written on slips of paper and 2 of the slips are drawn. What is the probability that,

1.) The sum of the no drawn is 9; 2.) The sum of the numbers drawn is 5 or less; 3.) That one of the no. drawn is odd or greater than 8; 4.) That the no. are drawn on both odd

Soln:

The total no. of papers is 6 and therefore 2 is drawn, then the total is $6C_2$.

i) The sum of the no drawn is 9:

∴ The conditions are $4+5=9$ or $6+3=9$, which is:

P (sum of numbers is 7) = $\dfrac{\overset{(4+1)}{2C_2} + \overset{(4+3)}{2C_2}}{6C_2}$

$= \dfrac{1+1}{15} = 2/15.$

ii) The sum of the nos drawn is 5 or less :

There are 4 combinations for the nos is 5 or less.

P (5 or less) $= \dfrac{\overset{(2+3)}{2C_2} + \overset{(1+4)}{2C_2} + \overset{(2+1)}{2C_2} + \overset{(1+3)}{2C_2}}{6C_2}$

$= \dfrac{1+1+1+1}{15} = 4/15.$

iii) that one of the no is odd or greater than 3:

That the 6 slips have 3 odd and 3 no greater than 3.

$\therefore$ P(Odd or > 3) $= \dfrac{6C_2}{6C_2}$ [(1, 3, 5) (4, 5, 6)]

$= \dfrac{\cancel{15}^{1}}{\cancel{15}_{1}} = 1.$

iv) The numbers drawn are both odd :

$\therefore$ There is a single combination ; from the 3 odd numbers we would take 2 numbers. So,

P(Both odd) $= \dfrac{3C_2}{6C_2} = \dfrac{\frac{3 \times \cancel{2}}{\cancel{2} \times 1}}{\frac{3\,\cancel{6} \times 5}{\cancel{2} \times 1}}$

$= \dfrac{\cancel{3}^{1}}{\cancel{15}_{5}}$

$= 1/5.$

10.) ii) 2 balls are of the same colour and 3rd is different :

⇒ There are 3 combinations for 2 balls of same colour they are $2C_2$, $3C_2$ and $4C_2$ for red, blue and black balls.

1.) And there is 3 combinations for 1 ball is different and it is taken from $9-2=7 \Rightarrow 7C_1$. for red balls.

$$\therefore P(2R \text{ and } 1 \text{ other}) = \frac{2C_2 \times 7C_1}{9C_3}$$

$$= \frac{1\times 7}{84} = 7/84.$$

2.) If the second combination is with already 1 is taken from other then the combination is $6C_1$

$$\therefore P(2 \text{ Blue and } 1 \text{ other}) = \frac{3C_2 \times 6C_1}{9C_3}$$

$$= \frac{3\times 6}{84} = 18/84.$$

3.) If the third combination is $5C_1$.

$$\therefore P(2 \text{ Black and } 1 \text{ other}) = \frac{4C_2 \times 5C_1}{9C_3}$$

$$= \frac{6\times 5}{84} = 30/84.$$

$$\therefore P(2 \text{ same } \& \ 1 \text{ different colour}) = \frac{7+18+30}{84}.$$

$$= 55/84.$$

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3.) Axiomatic Probability:

P(A) is the probability function defined on a field B of events if the following properties or axioms hold.

1.) For each $A \in B$, P(A) is defined, is real and $P(A) \geq 0$. ......> (Axiom of non-negabivity)

2.) $P(S) = 1$. .....> (Axiom of certainity)

3.) If (A_n) is any finite or infinite sequence of disjoint events in B, then

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i)$$ ...> (Axiom of additivity)

(*) Some theorems on Probability: (5m)

1.) Addition theorem:

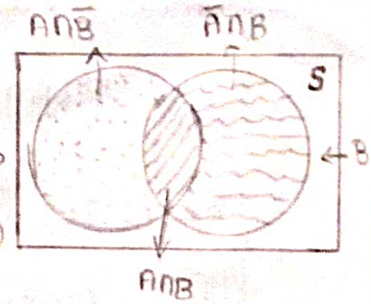
Theorem:

*. If A and B are any two events (subsets of sample space S) and are not disjoint, then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

Proof :

From the Venn diagram, we have



$A \cup B = A \cup (\bar{A} \cap B)$,

where A and $\bar{A} \cap B$ are mutually disjoint, A>

$\therefore P(A \cup B) = P[A \cup (\bar{A} \cap B)]$ → from (Axiom 3)

$= P(A) + P(\bar{A} \cap B)$

$= P(A) + P(\bar{A} \cap B) + P(A \cap B) - P(A \cap B)$ → from diagram

$= P(A) + [P(\bar{A} \cap B) + P(A \cap B)] - P(A \cap B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ ∴ Hence Proved.

2.) Multiplication theorem on Probability : ⊗ ⊗

For two events A and B,

$$P(A\cap B) = P(A).P(B/A) \;;\; P(A)>0$$
$$= P(B).P(A/B) \;;\; P(B)>0$$

*.where $P(B/A)$ represents conditional probability of occurence of B when the event A has already happened and $P(A\backslash B)$ is the conditional probability of happening of A, given that B has already happened.

Proof :

In usual notations, we have

$$P(A) = \frac{n(A)}{n(S)} \;;\; P(B) = \frac{n(B)}{n(S)} \text{ and } P(A\cap B) = \frac{n(A\cap B)}{n(S)} \rightarrow (1)$$

For the conditional event (A/B), the favourable outcomes must be one of the sample points of B, i.e., for the event (A/B), the sample space is B and out of the $n(B)$ sample points, $n(A\cap B)$ pertain to the occurence of the event A. Hence,

$$P(A/B) = \frac{n(A\cap B)}{n(B)}$$

Rewriting (1), we get

$$P(A\cap B) = \frac{n(B)}{n(S)} \times \frac{n(A\cap B)}{n(B)} = P(B).P(A/B) \rightarrow (2)$$

Similarly, we get from (1) :

$$P(A\cap B) = \frac{n(A)}{n(S)} \times \frac{n(A\cap B)}{n(A)} = P(A).P(B/A) \rightarrow (3)$$

From (2) and (3), we get the result ;

$$P(A\cap B) = P(A).P(B/A) \;,\; P(A)>0$$
$$= P(B).P(A/B) \;,\; P(B)>0.$$

Thus, we have proved that " the probability of the simultaneous occurrence of two events A and B is equal to the product of the probability of one of these events and the conditional probability of the other, given that the first one has occurred." Any of the events may be called the first event.

Remarks:

1.) $P(B/A) = \frac{P(A\cap B)}{P(A)}$ and $P(A/B) = \frac{P(A\cap B)}{P(B)}$

*. Thus the conditional Probabilities $P(B/A)$ and $P(A/B)$ are defined if and only if $P(A) \neq 0$ and $P(B) \neq 0$; respectively.

2.) i) For $P(B) > 0$, $P(A/B) \leq P(A)$

Proof:

$n(A\cap B) \leq n(A)$ and $n(B) \leq n(S)$, (Divided, we get)

$$\frac{n(A\cap B)}{n(B)} \leq \frac{n(A)}{n(S)} \Rightarrow P(A/B) \leq P(A)$$

ii) The conditional probability $P(A/B)$ is not defined if $P(B) = 0$.

iii) $P(B/B) = 1$.

Problems.

1a.) 2 dice are thrown what is the probability that. The Sum is 1.) > 8 2.) Neither 7 Nor 11

Soln:

a) If S belongs the sum on 2 dice. we want find the probability that;

i) $P(S>8) = P(S=9) + P(S=10) + P(S=11) + P(S=12)$

$n(S) = \{(1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,5)\ (1,6)$
$(2,1)\ (2,2)\ (2,3)\ (2,4)\ (2,5)\ (2,6)$
$(3,1)\ (3,2)\ (3,3)\ (3,4)\ (3,5)\ (3,6)$
$(4,1)\ (4,2)\ (4,3)\ (4,4)\ (4,5)\ (4,6)$
$(5,1)\ (5,2)\ (5,3)\ (5,4)\ (5,5)\ (5,6)$
$(6,1)\ (6,2)\ (6,3)\ (6,4)\ (6,5)\ (6,6)\}$

$S=9\ ;\ \{(3,6)\ (4,5)\ (5,4)\ (6,3)\}$

$\therefore P(S=9) = 4/36$

$S=10\ ;\ \{(4,6)\ (6,4)\ (5,5)\}$

$\therefore P(S=10) = 3/36$

$S=11\ ;\ \{(5,6)\ (6,5)\}$

$\therefore P(S=11) = 2/36$

$S=12\ ;\ \{(6,6)\}$

$\therefore P(S=12) = 1/36$

$\therefore P(S>8) = 4/36 + 3/36 + 2/36 + 1/36 = \frac{\cancel{10}\,5}{\cancel{36}\,18}$

$P(S>8) = 5/18$

b) Let A denote that event of getting the sum of 7 and B denote the event of getting the sum of 11 with a pair of dice.

$S=7\ ;\ \{(1,6)\ (6,1)\ (2,5)\ (5,2)\ (3,4)\ (4,3)\}$

$P(A) = 6/36$

$S=11\ ;\ \{(5,6)\ (6,5)\}$

$P(B) = 2/36$

$\therefore P(A^c \cap B^c) = 1 - P(A \cup B)$

$= 1 - [P(A) + P(B)]$

$= 1 - 6/36 - 2/36$

$$= \frac{36-6-2}{36} = \frac{36-8}{36}$$

$$= \frac{28}{36} = 7/9.$$

13.) 2 dice are thrown. find the probability of getting an even number on the first die or a total of 8?

Soln:

While tossing 2 dice, we get 36 events. Let A be the event of getting and even number on the 1st dice and Let B be the event of getting the sum of points 8 on the 2 dice. and sample space is $n(S)=36$

$A = \{(2,1)\ (2,2)\ (2,3)\ (2,4)\ (2,5)\ (2,6)$
$(4,1)\ (4,2)\ (4,3)\ (4,4)\ (4,5)\ (4,6)$
$(6,1)\ (6,2)\ (6,3)\ (6,4)\ (6,5)\ (6,6)\}$

$n(A) = 18$

$B = \{(2,6)\ (3,5)\ (4,4)\ (5,3)\ (6,2)\}$

$n(B) = 5$

$A \cap B = \{(2,6)\ (4,4)\ (6,2)\}$

$n(A \cap B) = 3$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{18}{36} + \frac{5}{36} - \frac{3}{36} = \frac{18+5-3}{36}$$

$\boxed{P(A \cup B) = 20/36}$.

14.) The probability that a students passes a physics test is $2/3$ and the Probability that he passes both a Physics test and an English test is $\frac{14}{45}$. The probability that he passes at least one test is $4/5$. What is the probability that he passes the English test?

Soln:

Let us define the following events:

A : The student passes a physics test;

B : The student passes an English test.

$P(A) = 2/3$

$P(A \cap B) = 14/45$

$P(A \cup B) = 4/5$ and we want, $P(B)$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\frac{4}{5} = \frac{2}{3} + P(B) - \frac{14}{45}$$

$$\therefore P(B) = \frac{4 \times 9}{5 \times 9} + \frac{14}{45} - \frac{2 \times 15}{3 \times 15}$$

$$= \frac{36 + 14 - 30}{45} = \frac{20}{45} = \frac{4}{9}$$

$$\boxed{P(B) = 4/9}$$

3) Theorem: 3.3

Probability of the complementary event $\bar{A}$ of A is given by $P(\bar{A}) = 1 - P(A)$

Proof:

A and $\bar{A}$ are mutually disjoint events, so that

$A \cup \bar{A} = S \Rightarrow P(A \cup \bar{A}) = P(S)$

Hence, from axioms (2) and (3) of probability, we have

$P(A) + P(\bar{A}) = P(S) = 1$

$\Rightarrow P(\bar{A}) = 1 - P(A)$.

Cor. 1:

we have $P(A) = 1 - P(\bar{A}) \leq 1$ $[\because P(\bar{A}) \geq 0$, by Axiom 1$]$

Further, since $P(A) \geq 0$ (Axiom 1)

$\Rightarrow 0 \leq P(A) \leq 1$.

Cor. 2 :

$P(\phi) = 0$, since $\phi = \bar{S}$ and

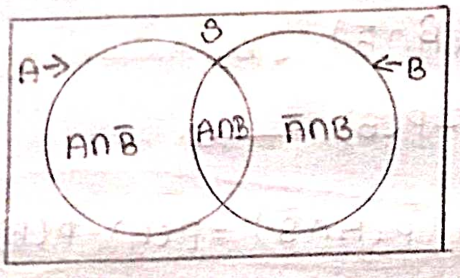
$$P(\phi) = P(\bar{S}) = 1 - P(S)$$
$$= 1 - 1$$
$$= 0.$$

Theorem: 3.4

For any two events A and B, we have

i) $P(\bar{A} \cap B) = P(B) - P(A \cap B)$ ii) $P(A \cap \bar{B}) = P(A) - P(A \cap B)$

Proof :



i) From the venn diagram, we get ; $B = (A \cap B) \cup (\bar{A} \cap B)$,

where $\bar{A} \cap B$ and $A \cap B$ are disjoint events.

Hence by Axiom (3), we get

$$P(B) = P(A \cap B) + P(\bar{A} \cap B)$$
$$P(\bar{A} \cap B) = P(B) - P(A \cap B) \longrightarrow (1)$$

ii) Similarly, we have ; $A = (A \cap B) \cup (A \cap \bar{B})$

where $(A \cap B)$ and $(A \cap \bar{B})$ are disjoint events.

Hence by axiom (3), we get

$$P(A) = P(A \cap B) + P(A \cap \bar{B})$$
$$\Rightarrow P(A \cap \bar{B}) = P(A) - P(A \cap B). \longrightarrow (2)$$

Theorem: 3.5

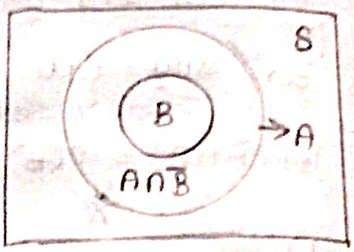
If $B \subset A$, then

i) $P(A \cap \bar{B}) = P(A) - P(B)$,

ii) $P(B) \leq P(A)$

Proof:

i) When $B \subset A$, B and $A \cap \bar{B}$ are mutually exclusive events.



So that $A = B \cup (A \cap \bar{B})$

$\Rightarrow P(A) = P[B \cup (A \cap \bar{B})]$

$= P(B) + P(A \cap \bar{B})$ (By Axiom 3)

$\Rightarrow P(A \cap \bar{B}) = P(A) - P(B). \longrightarrow (1)$

ii) $P(A \cap \bar{B}) \geq 0$ from $P(A \cap \bar{B}) = P(A) - P(B)$

$\Rightarrow P(A) - P(B) \geq 0 \Rightarrow P(B) \leq P(A)$

Hence;

$B \subset A \Rightarrow P(B) \leq P(A) \longrightarrow (2)$

Problems

15) The probabilities of 3 students A, B and C. Solving a Problem in statistics are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$. The problem is given to all the three students. What is the probability that

i) No one will solve the problem.

ii) Only one will solve the problem.

iii) Atleast one will solve the problem.

Soln:

P(A solving of the problem) $= \frac{1}{2}$; $P(A)$

P(B solving of the problem) $= \frac{1}{3}$; $P(B)$

P(C solving of the problem) $= \frac{1}{4}$; $P(C)$

Probability of the persons not solving the problem:

$P(A^c) = 1 - P(A) = 1 - 1/2 = 1/2$

$P(B^c) = 1 - P(B) = 1 - 1/3 = 2/3$

$P(C^c) = 1 - P(C) = 1 - 1/4 = 3/4$

This is based on the event not happening i.e.) the complementary event.

i) No one will solve the problem:

$$= P(A^c) \times P(B^c) \times P(C^c)$$

$$= \cancel{1/2} \times \cancel{2}/\cancel{3} \times \cancel{3}/4 = 1/4.$$

ii) Only one will solve the problem:

In this only one person will solve and the other two persons are not solve.

$$= (P(A) \times P(B^c) \times P(C^c)) + (P(B) \times P(A^c) \times P(C^c)) + (P(C) \times P(A^c) \times P(B^c))$$

$$= (1/2 \times 2/3 \times 3/4) + (1/3 \times 1/2 \times 3/4) + (1/4 \times 1/2 \times 2/3)$$

$$= 1/4 + 1/8 + 1/12 = \frac{6+3+2}{24}$$

$$= 11/24.$$

iii) Atleast one will solve the problem:

$$= 1 - P(\text{none will solve the problem})$$

$$= 1 - 1/4 = \frac{4-1}{4}$$

$$= 3/4.$$

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Baye's Theorem :

Prior Probabilities → New information → Application of Bayes' Theorem → Posterior Probabilities

Theorem :

If $E_1, E_2, E_3, \ldots, E_n$ are mutually disjoint events with $P(E_i) \neq 0$, $(i = 1, 2, 3, \ldots, n)$, then for any arbitrary event A which is a subset of $\bigcup_{i=1}^{n} E_i$ such that $P(A) > 0$, we have

$$P(E_i/A) = \frac{P(E_i)\, P(A/E_i)}{\sum_{i=1}^{n} P(E_i)\, P(A/E_i)}$$

Proof :

* It is given that $E_1, E_2, \ldots E_n$ are mutually disjoint events.
* where $P(E_i) \neq 0$ and also A is a subset of $\bigcup_{i=1}^{n} E_i$

Since $A \subset \bigcup_{i=1}^{n} E_i$, we have

$$A = A \cap \left(\bigcup_{i=1}^{n} E_i\right) = \bigcup_{i=1}^{n} (A \cap E_i)$$

Since $(A \cap E_i) \subset E_i$, $(i = 1, 2, 3, \ldots, n)$ are mutually disjoint events, we have by addition theorem of probability:

$$P(A) = P\left[\bigcup_{i=1}^{n} (A \cap E_i)\right]$$

$$= \sum_{i=1}^{n} P(A \cap E_i) = \sum_{i=1}^{n} P(E_i) \cdot P(A/E_i) \rightarrow (1)$$

by multiplication theorem of probability:

Also we have ; $P(A \cap E_i) = P(A) \cdot P(E_i/A)$

$$\Rightarrow \quad P(E_i/A) = \frac{P(A \cap E_i)}{P(A)} = \frac{P(E_i) \cdot P(A/E_i)}{\sum_{i=1}^{n} P(E_i) \cdot P(A/E_i)} \quad \text{(from (1))}$$

∴ Hence proved.

Q. 16.) (10m) The contents of urns 1, 2 and 3 are as follows:

Box 1 : 1 white, 2 black and 3 red balls

Box 2 : 2 white, 1 black and 1 red balls

Box 3 : 4 white, 5 black and 3 red balls. 1 box is chosen at random and 2 balls are drawn from it. They happened to be white and Red. What is the probability that they come from (1.) Box 1 (2.) Box 2 (3.) Box 3.

Soln:

Let E_1, E_2 and E_3 denote the events that the boxes 1, 2 and 3 is chosen respectively. and

Let A be the event that the 2 balls taken from the selected ~~balls~~ box are white and red.

$$P(E_1) = P(E_2) = P(E_3) = 1/3$$

$$P(A/E_1) = \frac{1C_1 \times 3C_1}{6C_2} = \frac{3}{\frac{\cancel{6}^{3} \times 5}{\cancel{2} \times 1}} = \frac{3}{15} = 1/5$$

$$P(A/E_2) = \frac{2C_1 \times 1C_1}{4C_2} = \frac{2}{\frac{\cancel{4}^{2} \times 3}{\cancel{2} \times 1}} = \frac{2}{6} = 1/3$$

$$P(A/E_3) = \frac{4C_1 \times 3C_1}{12C_2} = \frac{12}{\frac{\cancel{12}^{6} \times 11}{\cancel{2} \times 1}} = \frac{\cancel{12}^{2}}{\cancel{66}_{11}} = \frac{2}{11}$$

Probability for selecting 1 white and 1 Red from the 3 boxes:

Box - 1 ⇒ $$P(E_1/A) = \frac{P(E_1) \cdot P(A/E_1)}{P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2) + P(E_3) \cdot P(A/E_3)}$$

$$= \frac{\frac{1}{3} \times \frac{1}{5}}{\left(\frac{1}{3} \cdot \frac{1}{5}\right) + \left(\frac{1}{3} \cdot \frac{1}{3}\right) + \left(\frac{1}{3} \cdot \frac{2}{11}\right)} = \frac{\frac{1}{15}}{\frac{1}{15} + \frac{1}{9} + \frac{2}{33}}$$

$$= \frac{\frac{1}{15}}{\frac{1062}{4455}} = \frac{1}{15} \times \frac{4455}{1062} = \frac{1}{15} \times \frac{495}{118} = \frac{33}{118}.$$

Box - 2 ⇒ $$P(E_2/A) = \frac{P(E_2) \cdot P(A/E_2)}{P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2) + P(E_3) \cdot P(A/E_3)}$$

$$= \frac{\frac{1}{3}\cdot\frac{1}{3}}{\frac{1062}{4455}} = \frac{1}{9} \times \frac{4455}{1062} = \frac{1}{9_3} \times \frac{495}{118} = \frac{55}{118}$$

Box-3 $\Rightarrow$ $$P(E_3/A) = \frac{P(E_3)\cdot P(A/E_3)}{P(E_1)\cdot P(A/E_1) + P(E_2)\cdot P(A/E_2) + P(E_3)\cdot P(A/E_3)}$$

$$= \frac{\frac{1}{3}\cdot\frac{2}{11}}{\frac{1062}{4455}} = \frac{2}{33}\times\frac{4455}{1062} = \frac{2}{33}\times\frac{495}{118} = \frac{15}{59}.$$

17) From a city population, the probability of selecting (i) a male or a smoker is 7/10 (ii) a male smoker is 2/5, and iii) a male, if a smoker is already selected is 2/3. Find the probability of selecting i) a non-smoker, ii) a male and iii) a smoker, if a male is first selected.

Soln:

A: a male is selected ; B: a smoker is selected.

we are given;

$$P(A\cup B) = 7/10, \quad P(A\cap B) = 2/5, \quad P(A/B) = 2/3$$

i) The probability of selecting a non-smoker is:

$$P(\bar{B}) = 1 - P(B)$$

$\left[\because P(A\cap B) = P(B)\cdot P(A|B) \quad P(B) = \frac{P(A\cap B)}{P(A|B)}\right]$

$$= 1 - \frac{P(A\cap B)}{P(A/B)}$$

$$= 1 - \frac{2/5}{2/3} = 1 - 3/5$$

$$P(\bar{B}) = 2/5$$

$$\therefore P(B) = 1 - P(\bar{B})$$

$$= 1 - 2/5 = 3/5$$

ii) The probability of selecting a male (by Addition theorem) is:

$$P(A) = P(A\cup B) + P(A\cap B) - P(B)$$

$$= 7/10 + 2/5 - 3/5$$

$$P(A) = 1/2.$$

(ii) The probability of selecting a smoker if a male is first selected is :

$$P(B|A) = \frac{P(A\cap B)}{P(A)}$$

$$= \frac{2/5}{1/2} = 2/5 \times 2/1$$

$$P(B/A) = 4/5$$

18.) A consignment of 15 records players contains 4 defectives. The record players are selected at random, one by one, and examined. Those examined are not put back. What is the probability that the 9th one examined is the last defective?

Soln:

Let A be the event of getting exactly 3 defectives in examination of a 8 record players and let B denote the event that the 9th piece examined is a defective one.

Since it is a problem of sampling without replacement and since there are 4 defectives (and 11 non-defectives) out of 15 record players, $P(A) = \frac{4C_3 \times {}^{11}C_5}{15C_8}$.

$P(B/A)$ = probability that the 9th examined record player is defective given that there were 3 defectives in the first 8 pieces examined $= \frac{4-3}{15-8}$

$$= 1/7.$$

Since there is only one defective piece left among the remaining $15-8=7$ record players.

Hence, required probability $= P(A\cap B) = P(A) \cdot P(B/A)$

$$= \frac{4C_3 \times {}^{11}C_5}{15C_8} \times 1/7 = \frac{8}{195}.$$

19.) An MBA applies for an job in two firms X and Y. The probability of his being selected in firm X is 0.7 and being rejected at Y is 0.5. The probability of at least one of his applications being rejected is 0.6. What is probability that he will be selected in one of the firms?

Soln:

Let A be the event that the person is selected in firm X.

Let B be the event that the person is selected in firm Y.

Then in usual notations, we are given:

$P(A) = 0.7 \Rightarrow P(\bar{A}) = 1 - 0.7$
$= 0.3$

$P(B) = 0.5 \Rightarrow P(\bar{B}) = 1 - 0.5$
$= 0.5$

$P(\bar{A} \cup \bar{B}) = 0.6 = P(\bar{A}) + P(\bar{B}) - P(\bar{A} \cap \bar{B})$ —— ①.

The probability that the persons will be selected in one of the two firms X or Y is given by:

$P(A \cup B) = 1 - P(\bar{A} \cap \bar{B})$

$= 1 - [P(\bar{A}) + P(\bar{B}) - P(\bar{A} \cap \bar{B})]$ (From ①)

$= 1 - [0.3 + 0.5 - 0.6]$

$= 1 - 0.2 = 0.8$

20.) Three newspapers A, B and C are published in a certain city. It is estimated from a survey that of the adult population: 20% read A, 16% read B, 14% read C, 8% read both A and B, 5% read both A and C, 4% read both B and C, 2% read all three. Find what percentage read atleast one of the papers?

Soln:

Let E, F and G denote the events that the adult reads newspapers A, B and C respectively. Then we are given;

$$P(E) = \frac{20}{100}, \quad P(F) = \frac{16}{100}, \quad P(G) = \frac{14}{100}, \quad P(E \cap F) = \frac{8}{100}$$

$$P(E \cap G) = \frac{5}{100}, \quad P(F \cap G) = \frac{4}{100} \text{ and } P(E \cap F \cap G) = \frac{2}{100}$$

The required probability that an adult reads at least one of the newspapers (by addition theorem) is given by;

$$P(E \cup F \cup G) = P(E) + P(F) + P(G) - P(E \cap F) - P(F \cap G) - P(E \cap G) + P(E \cap F \cap G)$$

$$= \frac{20}{100} + \frac{16}{100} + \frac{14}{100} - \frac{8}{100} - \frac{5}{100} - \frac{4}{100} + 2/100$$

$$= \frac{35}{100}$$

$$P(E \cup F \cup G) = 0.35$$

~~A card is~~ ∴ Hence 35% of the adult population reads at least one of the newspapers.

Q21.) A card is drawn from a pack of 52 cards. Find the probability of getting a king or a heart or a red card.

Soln:

Let us define the following events:

A : the card drawn is a king

B : the card drawn is a heart

C : the card drawn is a red colour card.

Then A, B and C are not mutually exclusive.

We use;

① ⟹ $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

so we have to find out some of the conditions:

$A \cap B$: the card is king of heart.

$B \cap C$: the card is heart

$C \cap A$: the card is a red king.

$n(A \cap B) = 1$; $n(B \cap C) = 13$; $n(A \cap C) = 2$

$A \cap B \cap C$: the card is king of hearts

$n(A \cap B \cap C) = 1$.

$\therefore P(A) = 4/52$; $P(B) = \frac{13}{52}$; $P(C) = \frac{26}{52}$; $P(A \cap B) = 1/52$

$P(B \cap C) = 13/52$; $P(C \cap A) = 2/52$; $P(A \cap B \cap C) = 1/52$

$\therefore$ Substitute in ①:

$$P(A \cup B \cup C) = \frac{4}{52} + \frac{13}{52} + \frac{26}{52} - 1/52 - \frac{13}{52} - 2/52 + 1/52$$

$$= \frac{44 - 16}{52} = \frac{\cancel{28}^{\cancel{14}\ 7}}{\cancel{52}_{\cancel{26}\ 13}}$$

$P(A \cup B \cup C) = 7/13$.

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22) A box contains 6 red, 4 white and 5 black balls. A person draws 4 balls from the box at random. Find the probability that among the balls drawn there is at least one ball of each colour.

Soln:

The required Event E that "in a draw of 4 balls from the box at random there is atleast one ball of each colour", can materialise in the following mutually disjoint ways:

i) 1 red, 1 white, 2 black balls.

ii) 2 Red, 1 white, 1 black balls

iii) 1 Red, 2 white, 1 black balls.

Hence by addition theorem of probability, the required probability is given by:

$$P(E) = P(i) + P(ii) + P(iii)$$

$$= \frac{{}^6C_1 \times {}^4C_1 \times {}^5C_2}{{}^{15}C_4} + \frac{{}^6C_2 \times {}^4C_1 \times {}^5C_1}{{}^{15}C_4} + \frac{{}^6C_1 \times {}^4C_2 \times {}^5C_1}{{}^{15}C_4}$$

$$= \frac{1}{{}^{15}C_4}\left[\left(6 \times 4 \times \frac{5 \times 4}{2 \times 1}\right) + \left(\frac{6 \times 5}{2 \times 1} \times 4 \times 5\right) + \left(6 \times \frac{4 \times 3}{2 \times 1} \times 5\right)\right]$$

$$= \frac{1}{\frac{15 \times 14 \times 13 \times 12}{4 \times 3 \times 2 \times 1}}\left[(6 \times 4 \times 10) + (15 \times 4 \times 5) + (6 \times 6 \times 5)\right]$$

$$= \frac{1}{1365}\left[240 + 300 + 180\right] = \frac{720}{1365}$$

$$= 0.52747.$$

$$P(E) = 0.5275.$$

23.) It is 8:5 against the wife who is 40 years old living till she is 70 and 4:3 against her husband now 50 living till he is 80. Find the probability that

i) Both will be alive
ii) None will be alive
iii) Only wife will be alive
iv) Only husband will be alive
v) Only one will be alive
vi) Atleast one will be alive.

30 years hence.

Soln:

Let us define the events:

A : wife will be alive and

B : Husband will be alive ; 30 years hence.

Then we are given:

$$P(A) = \frac{5}{8+5} = 5/13 \Rightarrow P(\bar{A}) = 1 - P(A) = 8/13$$

$$P(B) = \frac{3}{4+3} = 3/7 \Rightarrow P(\bar{B}) = 1 - P(B) = 4/7.$$

*. If we assume that A and B are independent so that A and $\bar{B}$, $\bar{A}$ and B, $\bar{A}$ and $\bar{B}$ are also independent, then the required probabilities are given by:

i) $P(A \cap B) = P(A) \cdot P(B)$

$= \frac{5}{13} \times \frac{3}{7} = \frac{15}{91}$ (∵ A and B are independent)

ii) $P(\bar{A} \cap \bar{B}) = P(\bar{A}) \cdot P(\bar{B})$

$= \frac{8}{13} \times \frac{4}{7} = \frac{32}{91}$ (∵ $\bar{A}$ and $\bar{B}$ are independent)

iii) $P(A \cap \bar{B}) = P(A) - P(A \cap B)$

$= \frac{5}{13} - \frac{15}{91} = \frac{20}{91}$ [From ①]

iv) $P(\bar{A} \cap B) = P(B) - P(A \cap B)$

$= \frac{3}{7} - \frac{15}{91} = \frac{24}{91}$ [From ①]

v) $P(A \cap \bar{B}) + P(\bar{A} \cap B) = \frac{20}{91} + \frac{24}{91}$

$= \frac{44}{91}$ [From ③ and ④]

vi) $P(A \cup B) = 1 - P(\bar{A} \cap \bar{B})$

$= 1 - \frac{32}{91} = \frac{91-32}{91}$ [From ②]

$= \frac{59}{91}.$

Assignment - 2.

Q4.) In shuffling a pack of cards, four are accidently dropped, find the chance that the missing cards should be one from each suit.

Soln:

If the four cards from 52 cards are ~~drawn~~ missed. So, the four are from each suit.

$P(H \text{ and } S \text{ and } C \text{ and } D) = \frac{{}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1}{{}^{52}C_4}.$

$$= \frac{13\times13\times13\times13}{\frac{52\times51\times50\times49}{4\times3\times2\times1}} = \frac{13\times13\times13\times13}{270725}$$

$$= \frac{28561}{270725} = \frac{2197}{20825}$$

25.) A problem in statistics is given to three students A, B, C whose chances of solving it are 1/2, 3/4 and 1/4 respectively. What is the probability that the problem will be solved if all of them try independently?

Soln:

Given that;

$P(A) = 1/2$; $P(B) = 3/4$; $P(C) = 1/4$ ∴

we use

$$P(A\cup B\cup C) = P(A) + P(B) + P(C) - P(A\cap B) - P(B\cap C) - P(A\cap C) + P(A\cap B\cap C) \quad \text{(1)}$$

we have to find some combos;

$$P(A\cap B) = P(A)\cdot P(B) = 1/2 \cdot 3/4 = 3/8$$

$$P(B\cap C) = P(B)\cdot P(C) = 3/4 \cdot 1/4 = 3/16$$

$$P(A\cap C) = P(A)\cdot P(C) = 1/2 \cdot 1/4 = 1/8$$

$$P(A\cap B\cap C) = P(A)\cdot P(B)\cdot P(C) = 1/2\cdot 3/4 \cdot 1/4 = \frac{3}{32}.$$

Apply all to (1) ⇒

$$P(A\cup B\cup C) = 1/2 + 3/4 + 1/4 - 3/8 - 3/16 - 1/8 + 3/32$$

$$= \frac{2+3+1}{4} - \frac{6-3-2}{16} + 3/32$$

$$= 6/4 + \frac{(-6-5)}{16} + 3/32$$

$$= \frac{48+3}{32} - \frac{11}{16} = \frac{51}{32} - 11/16 = \frac{51-22}{32}$$

$$P(A\cup B\cup C) = 29/32$$

26.) The probability that a student passes in Statistics examination is 2/3 and the probability that he/she will not pass in Mathematics examination is 5/9. The probability that he/she will pass in atleast one of the examinations is 4/5. Find the probability that he/she will pass in both the examinations.

Soln:

Given that

$$P(S) = 2/3 \; ; \; P(\bar{M}) = 5/9 \; ; \; P(S\cup M) = 4/5.$$

$$P(M) = 1 - P(\bar{M})$$

$$= 1 - 5/9 = \frac{9-5}{9} = 4/9.$$

∴ we conclude that;

$$P(S\cap M) = P(S) + P(M) - P(S\cup M)$$

$$= 2/3 + 4/9 - 4/5 = \cancel{\frac{18+12}{27} - 4/5}$$

$$= \frac{2\times 3}{3\times 3} + 4/9 - 4/5 = \frac{6+4}{9} - \frac{4}{5}$$

$$= \frac{10}{9} - 4/5 = \frac{50-36}{45}$$

$$P(S\cap M) = 14/45$$

27.) A husband and a wife appear in an interview for two vacancies in the same post. The probability of husband's selection is 1/7 and that of wife is 1/5. What is the probability that,

i) both of them will be selected,

ii) only one of them will be selected,

iii) none of them will be selected?

Soln:

$P(H) = 1/7$; $P(W) = 1/5$.

i) Both of them will be selected:

$$P(H \cap W) = P(H) \cdot P(W)$$
$$= 1/7 \cdot 1/5 = \frac{1}{35}$$

ii) Only one of them will be selected:

$$P(H \cap \overline{W}) = P(H) - P(H \cap W)$$
$$= 1/7 - 1/35 = \frac{5-1}{35} = 4/35$$

$$P(\overline{H} \cap W) = P(W) - P(H \cap W)$$
$$= \frac{1}{5} - \frac{1}{35} = \frac{7-1}{35} = \frac{6}{35}$$

$$P(H \cap \overline{W}) + P(\overline{H} \cap W) = \frac{4}{35} + 6/35$$
$$= \frac{\cancel{10}\,2}{\cancel{35}\,7} = 2/7.$$

iii) None of them will be selected:

$$P(\overline{H}) = 1 - P(H) = 1 - 1/7 = \frac{7-1}{7} = 6/7.$$
$$P(\overline{W}) = 1 - P(W) = 1 - 1/5 = \frac{5-1}{5} = 4/5$$

$$P(\overline{H} \cap \overline{W}) = P(\overline{H}) \cdot P(\overline{W})$$
$$= 6/7 \cdot 4/5 = \frac{24}{35}.$$

28.) Bolt factory machines A, B, C manfacture respectively 25%, 35% and 40% of the total. Of their output 5, 4, 2 percent respectively are defective bolts. A bolt is drawn at random from the product and is found to be defective. What are the probabilities that it was manufactured by machines A, B and C.

Soln:

*. That E_1, E_2 and E_3 denote the events that a bolt selected at random is manufactured by the machines A, B and C respectively.

And Let A denote the event of defective.

$P(E_1) = 0.25$

$P(E_2) = 0.35$

$P(E_3) = 0.40$.

*. The probability of drawing a deffective bolt manufacturing by machine A.

$P(A/E_1) = 0.05$; $P(A/E_2) = 0.04$; $P(A/E_3) = 0.02$.

Hence the probability that the defective bolt selected at random manufactured by machine A is given by ;

$$P(E_1/A) = \frac{P(E_1) \cdot P(A/E_1)}{P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2) + P(E_3) \cdot P(A/E_3)}$$

$$= \frac{(0.25)(0.05)}{(0.25)(0.05) + (0.35)(0.04) + (0.40)(0.02)}$$

$$= \frac{0.0125}{0.0125 + 0.014 + 0.008} = \frac{0.0125}{0.0345} = \frac{25}{69}.$$

$P(E_1/A) = 0.3623$.

$$P(E_2/A) = \frac{P(E_2)\cdot P(A/E_2)}{P(E_1)\cdot P(A/E_1) + P(E_2)\cdot P(A/E_2) + P(E_3)\cdot P(A/E_3)}$$

$$= \frac{(0.35)(0.04)}{0.0345} = \frac{0.014}{0.0345}$$

$$P(E_2/A) = \cancel{0.4006}\ 0.4058$$

$$P(E_3/A) = \frac{P(E_3)\cdot P(A/E_3)}{P(E_1)\cdot P(A/E_1) + P(E_2)\cdot P(A/E_2) + P(E_3)\cdot P(A/E_3)}$$

$$= \frac{(0.40)(0.02)}{0.0345} = \frac{0.008}{0.0345}$$

$$P(E_3/A) = 0.2319.$$

Q9.) The contents of Urns A and B are given below A : 4 white, 3 Red and B: 3 White, 7 Red. A ball chose at random and a ball drawn from it. It happen to be white. What is the probability that it come from Urn A.

Soln:

Let E_1, E_2 denotes that the events that a ball chosen at random from A and B urns.

Let A be the event of taking ~~in~~ a ball is white.

$$P(E_1) = P(E_2) = 1/2.$$

The probability of drawn a ball;

$$P(A/E_1) = 4/7 \quad ; \quad P(A/E_2) = 3/10.$$

Hence the probability that a ball is white from urn A is;

$$P(E_1/A) = \frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\cdot P(A/E_2) + P(E_2)\cdot P(A/E_2) + P(E_3)\cdot P(A/E_3)}$$

$$= \frac{\frac{1}{2}\cdot\frac{4}{7}}{\left(\frac{1}{2}\times\frac{4}{7}\right) + \left(\frac{1}{2}\times\frac{3}{10}\right)} = \frac{\frac{2}{7}}{\left(\frac{2}{7}\right) + \left(\frac{3}{20}\right)}$$

$$= \frac{2/7}{\frac{40+21}{140}} = \frac{2}{7} \times \frac{140}{61}$$

$$= \frac{40}{61} = 0.656.$$

30.) The contents of the boxes are given below:

B_1 : 2W, 1B, 3R

B_2 : 3W, 2B, 4R

B_3 : 4W, 3B, 2R ⇒ 1 box chosen at random and 2 balls are drawn, they happen to be red and black. What is the probability both taken from box 2.

Soln:

Let E_1, E_2 and E_3 be the events that a 2 balls are drawn from the boxes 1, 2 and 3.

Let A be the 2 balls taken from the box 2 is red and black.

$$P(E_1) = P(E_2) = P(E_3) = 1/3.$$

Then the probability of taken balls are red and black colour is;

$$P(A/E_1) = \frac{1C_1 \times 3C_1}{6C_2} = \frac{1 \times 3}{\frac{6 \times 5}{2 \times 1}} = \frac{3}{15} = 1/5$$

$$P(A/E_2) = \frac{2C_1 \times 4C_1}{9C_2} = \frac{2 \times 4}{\frac{9 \times 8}{2 \times 1}} = 2/9.$$

$$P(A/E_3) = \frac{3C_1 \times 2C_1}{9C_2} = \frac{3 \times 2}{\frac{9 \times 8}{2 \times 1}} = \frac{3}{18} = 1/6.$$

∴ The probability of both balls taken from box 2.

$$P(E_2/A) = \frac{P(E_2) \cdot P(A/E_2)}{P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2) + P(E_3) \cdot P(A/E_3)}$$

$$= \frac{(1/3)\cdot(2/9)}{(1/3)\cdot(1/5) + (1/3)(2/9) + (1/3)\cdot(1/6)}$$

$$= \frac{\frac{2}{27}}{\left(\frac{1}{15}\right) + \left(\frac{2}{27}\right) + \left(\frac{1}{18}\right)} = \frac{2}{27} \times \frac{15\times 27\times 18}{(27\times 18) + (2\times 15\times 18) + (15\times 27)}$$

$$= \frac{2}{27} \times \frac{7290}{486+540+405} = \frac{2}{27} \times \frac{7290}{1431}$$

$$= \frac{2}{\cancel{27}_{1}} \times \frac{\overset{10}{\cancel{270}}}{53} = \frac{20}{53} = 0.3773.$$

Conditional probability:

*. A and B are two events, the probability of the event A and their assumption that the event B has already happened is denoted by $P(A/B)$ and it is defined below;

$$P(A/B) = \frac{P(AB)}{P(B)}, \quad P(B) > 0.$$

*. The probability of event B and their assumption that the event A has already happened, it denoted by $P(B/A)$ and its defined below;

$$P(B/A) = \frac{P(AB)}{P(A)}, \quad P(A) > 0.$$

Independent events:

*. Two events A and B are said to be independent;

$$P(AB) = P(A).P(B).$$