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Decision Analysis

"It is interesting and exciting to study decision science because its focus is on developing capacity to make better informed decisions"

16 : 1. INTRODUCTION

Every day we, as humans, make many decisions; and occasionally we make an important *decision* that can have immediate and/or long-term effects on our lives. Such decisions as where to attend school, whether to rent or buy, whether your company should accept a merger proposal, and so on, are important decisions for which we would prefer to make correct choice.

A decision, in general, may be defined as the selection by the decision-maker of an act, considered to be best according to some pre-designated standard, from among the several available options.

16 : 2. DECISION-MAKING PROBLEM

Irrespective of the type of decision-making problem, there are certain essential elements which are common to all such problems. These are :

1. ***Courses of action :*** A decision is made from among a set of defined alternative courses of action. These are also called *actions, acts* or *strategies* and are under control and known to the decision-maker.

2. ***State-of-nature :*** Consequences (or events) of any course of action are dependent upon certain factors beyond the control of the decision-maker. These factors are called states-of-nature (future).

3. ***Uncertainty :*** There is indefiniteness regarding which event or outcome will occur. This uncertainty is indicated in terms of probabilities assigned to events.

4. ***Payoff :*** Each combination of a course of action and an event is associated with a payoff, which measures the *net benefit to the decision-maker that accrues from a given combination of decision alternatives and events.* They are also known as *conditional profit values* or *conditional economic consequences.*

5. ***Payoff table :*** Suppose the problem under consideration has n possible events (states-of-nature) denoted by $E_1, E_2, \ldots, E_n$ and m alternative acts (strategies) denoted by $A_1, A_2, \ldots, A_m$. Then the payoff corresponding to strategy A_j of the decision-maker under the event (state-of-nature) E_i will be denoted by α_{ij} $(i = 1, 2, \ldots, n\,;\, j = 1, 2, \ldots, m)$.

The various payoff elements may be summarized in the form of a payoff table as below :

States-of-nature	*Decision alternatives (Courses of action)*				
	A_1	A_2	A_3	...	A_m
E_1	a_{11}	a_{12}	a_{13}	...	a_{1m}
E_2	a_{21}	a_{22}	a_{23}	...	a_{2m}
E_3	a_{31}	a_{32}	a_{33}	...	a_{3m}
⋮	⋮	⋮	⋮	...	⋮
E_n	a_{n1}	a_{n2}	a_{n3}	...	a_{nm}

16 : 3. DECISION-MAKING PROCESS

The decision-making process involves the following major steps :

Step 1. Determine the various alternative courses of action from which the final decision is to be made.

Step 2. Identify the possible outcomes, called the *states-of-nature* or *events* for the decision problem. The events are beyond the control of the decision-maker.

Step 3. Determine the *payoff* function which describes the consequences resulting from the different combinations of the acts and events. The payoffs may be designated as a_{ij}'s, the payoff resulting from the combination of ith event and jth action.

Step 4. Construct the *regret* or *opportunity loss table.* An opportunity loss occurs due to failure of not adopting the best available course of action. The opportunity loss values are calculated separately for each state-of-nature (or outcome) by first locating the most favourable course of action for that state-of-nature and then determining the departure of the payoff value for that course of action and the payoff value for the best possible course of action that could have been selected.

Consider a fixed state-of-nature E_i $(i = 1, 2, \ldots, n)$ for which the payoff corresponding to the n courses of action are given by $p_{i1}, p_{i2}, \ldots, p_m$. Let M_1 be the payoff for the least possible course of action. Then the opportunity loss table will be as follows :

States-of-nature events	*Conditional opportunity loss (Decision alternatives)*				
	A_1	A_2	A_3	...	A_m
E_1	$M_1 - p_{11}$	$M_1 - p_{12}$	$M_1 - p_{13}$	...	$M_1 - p_{1m}$
E_2	$M_2 - p_{21}$	$M_2 - p_{22}$	$M_2 - p_{23}$	...	$M_2 - p_{2m}$
E_3	$M_3 - p_{31}$	$M_3 - p_{32}$	$M_3 - p_{33}$	...	$M_3 - p_{3m}$
⋮	⋮	⋮	⋮		⋮
E_n	$M_n - p_{n1}$	$M_n - p_{n2}$	$M_n - p_{n3}$	...	$M_n - p_{nm}$

16 : 4. DECISION-MAKING ENVIRONMENT

Decision analysis is used to determine optimum strategies where a decision-maker is faced with several decision alternatives. We may come across several decision-making situations :

(*a*) *Decisions under certainty.* Whenever there exists only one outcome for a decision, we are dealing with this category. Examples of this are linear programming, transportation, assignment and sequencing, etc.

(*b*) *Decisions under conflict.* In many situations, neither states-of-nature are completely known nor are they completely uncertain. Partial knowledge is available and therefore it may be termed as *decision-making under 'partial uncertainty'*. An example of this is the situation of *conflict* involving two or more competitors marketing the same product. Decision-making under conflict will be discussed in competitive games in Chapter 17.

(*c*) *Decisions under uncertainty.* These refer to situations where more than one outcome can result from any single decision. These shall now be discussed.

(*d*) *Decision under risk.* These refer to decision situations wherein the decision-maker chooses from among several possible outcomes where probabilities of occurrence can be stated (or determined) objectively from the past data.

16 : 5. DECISIONS UNDER UNCERTAINTY

Under conditions of uncertainty, only payoffs are known and nothing is known about the likelihood of each state-of-nature. Different persons have suggested several decision rules for making a decision under such situations.

The Laplace Criterion

The *Laplace criterion* uses all the information by assigning equal probabilities to the possible payoffs for each action and then selecting that alternative which corresponds to the maximum *expected payoff.*

The basic steps of this criterion may be summarized as :

Step 1. Assign equal probabilities $(1/n)$ to each payoff of a strategy (having n possible payoffs).

Step 2. Determine the expected payoff value for each alternative.

Step 3. Select that alternative which corresponds to the maximum of the above expected payoffs.

SAMPLE PROBLEM

1601. *A Super Bazar must decide on the level of supplies it must stock to meet the needs of its customers during Diwali days. The exact number of customers is not known, but it is expected to be in one of the four categories; 300, 350, 400 or 450 customers. Four levels of supplies are thus suggested with level j being ideal (from the viewpoint of incurred costs) if the number of customers falls in category j. Deviations from the ideal levels results in additional costs either because extra supplies are stocked needlessly or because demand cannot be satisfied. The table below provides these costs in thousands of rupees.*

Customer category	Supplies level			
	A_1	A_2	A_3	A_4
E_1	7	12	20	27
E_2	10	9	10	25
E_3	23	20	14	23
E_4	32	24	21	17

Solution. The Laplace principle assumes that E_1, E_2, E_3 and E_4 are equally likely. Thus the associated probabilities are given by $P(E_j) = 1/4 \quad (j = 1, 2, 3, 4)$, and the expected costs due to deviations from the best level, for different categories of customers are :

$$E(A_1) = 1/4\,(7 + 10 + 23 + 32) = 18.00$$

$$E(A_2) = 1/4\,(12 + 9 + 20 + 24) = 16.25$$

$$E(A_3) = 1/4\,(20 + 10 + 14 + 21) = 16.25$$

$$E(A_4) = 1/4\,(27 + 25 + 23 + 17) = 23.00$$

Clearly the best level of inventory is specified by the supply level A_2 or A_3.

The Maximin or Minimax Criterion

The *Maximin criterion* is based upon the 'conservative approach' to assume that the worst possible is going to happen. The decision-maker considers each strategy and locates the minimum payoff for each; and then selects that alternative which *maximizes* the *minimum* payoff.

Thus this criterion consists of two steps :

Step 1. Determine the minimum assured payoff for each alternative.

Step 2. Choose that alternative which corresponds to the maximum of above minimum payoffs.

When dealing with the costs, the maximum cost associated with each alternative is considered and the alternative that minimizes this maximum cost is chosen. In this context the criterion used is the *Minimax criterion* and may be carried out in two steps :

Step 1. Determine the maximum possible cost for each alternative.

Step 2. Choose that alternative which corresponds to the minimum of the above costs.

SAMPLE PROBLEM

1602. *A businessman has three alternatives open to him each of which can be followed by any of the four possible events. The conditional payoffs (in Rs.) for each action-event combination are given below :*

Alternative	Payoffs conditional on events			
	A	B	C	D
X	8	0	–10	6
Y	–4	12	18	–2
Z	14	6	0	8

Determine which alternative should the businessman choose, if he adopts the maximin criterion. [C.A. (Nov.) 1996]

Solution. For the given payoff matrix, the minimum assured payoffs for each alternative are :

$$X : -10, \quad Y : -4, \quad \text{and} \quad Z : 0$$

Since the maximum of these minimum payoffs is 0, the alternative Z is selected according to the *maximin principle.*

Example. Consider the payoff matrix of sample problem *1601*. Since the payoff numbers designate the costs, we shall use *minimax* criterion. The maximum possible costs for each alternative are :

$$A_1 : 32, \quad A_2 : 24, \quad A_3 : 21, \quad A_4 : 27.$$

Since the minimum of these maximum costs is 21, the alternative A_3 is chosen.

The Maximax or Minimin Criterion

The *maximax criterion* is based upon 'extreme optimism'. The decision-maker selects that particular strategy which corresponds to the maximum of the *maximum* payoff for each strategy.

The maximax criterion thus consists of the following steps :

Step 1. Determine the maximum possible payoff for each alternative.

Step 2. Select that alternative which corresponds to the maximum of the above maximum payoffs.

Example. In Sample Problem *1602*, the maximum payoffs associated with the different strategies are :

$$X : 8, \quad Y : 18, \quad Z : 14$$

The highest payoff being 18, the strategy *Y* is selected corresponding to the maximax principle.

In decision problems dealing with costs, the minimum for each alternative is considered and then the alternative which minimises the above minimum costs is selected. This is termed as *minimin* principle.

The Savage Criterion

The *Savage criterion* is based on the concept of *regret* (or opportunity loss) and calls for selecting the course of action that minimizes the maximum regret. It is alternatively known as the principle of Minimax regret.

The basic steps of this criterion are :

Step 1. Determine the amounts of regret for payoff of each alternative for a particular event. The regret amount for the *i*th alternative when event *j* occurs, is given by

$$i\text{th regret} = \begin{cases} (\text{maximum payoff—}i\text{th payoff}) \text{ for the } j\text{th event if the payoffs represent profits} \\ (i\text{th payoff—maximum payoff}) \text{ for the } j\text{th event if the payoffs represent costs} \end{cases}$$

Step 2. Determine the maximum regret amount for each alternative.

Step 3. Choose that alternative which corresponds to the minimum regrets.

Remark. The reader may observe that while the other decision rules do not take into account the cost of opportunity lost by making the wrong decision, the minimax regret criterion does so.

Example 1. Considering Sample Problem *1602*, we determine the regrets as shown below, where the regret payoff amounts, when event A occurs, are computed by the relation :

Regret payoff = Maximum payoff from A – Payoff

and similarly for the other three events :

Alternative	Payoff amounts				Regret payoff amount				Maximum regret
	A	*B*	*C*	*D*	*A*	*B*	*C*	*D*	
X	8	0	–10	6	6	12	28	2	28
Y	–4	12	18	–2	18	0	0	10	18*
Z	14	6	0	8	0	6	18	0	18*
Maximum payoff	14	12	18	8					

Since alternatives *Y* and *Z* both correspond to the minimal of the maximum possible regrets, the decision-maker would choose either of these two.

Example 2. Considering Sample Problem *1601*, we determine the regrets as shown below, where the regret amounts, when event E_j occurs, are computed by the relation :

Regret payoff ≙ Payoff – Minimum payoff from E_j, $(j = 1, 2, 3, 4)$

Alternative	Payoff amounts				Regret payoff amount				Maximum regret
	E_1	E_2	E_3	E_4	E_1	E_2	E_3	E_4	
A_1	7	10	23	32	0	1	9	15	15
A_2	12	9	20	24	5	0	6	7	7*
A_3	20	10	14	21	13	1	0	4	13
A_4	27	25	23	17	20	16	9	0	20
Minimum payoff	7	9	14	17					

Since the alternative A_2 corresponds to the minimum of the maximum possible regrets, the decision-maker would choose A_2.

The Hurwicz Criterion

The *Hurwicz criterion* stipulates that a decision-maker's view may fall somewhere between the extreme pessimism of the maximum criterion and the extreme optimism of the maximum criterion. The criterion provides a mechanism by which a balance between extreme pessimism and extreme optimism is made by weighing them with certain degrees of optimism and pessimism.

The basic steps of *Hurwicz criterion* may be summarized as below :

Step 1. Choose an appropriate degree of optimism (or pessimism) of the decision-maker. Let α be his degree of optimism (so that $1 - \alpha$ is his degree of pessimism) $[0 \le \alpha \le 1.]$

Step 2. Determine the maximum as well as minimum payoff for each alternative and obtain the quantities

$$h = \alpha \times \text{maximum} + (1 - \alpha) \times \text{minimum}$$ for each alternative.

Example. The Hurwicz principle is applied to Sample Problem *1601*. Let us choose $\alpha = 0.5$. The necessary calculations are shown below :

Alternative	*Maximum payoff* (*i*)	*Minimum payoff* (*ii*)	$h = \alpha \times (i) + (1 - \alpha) \times (ii)$ $(\alpha = 0.5)$
A_1	32	7	19.5
A_2	24	9	16.5
A_3	21	10	15.5*
A_4	27	17	22.0

Since the payoffs here represent costs according to Hurwicz principle, the optimum solution is to choose alternative A_3.

PROBLEMS

1603. A farmer wants to decide which of the three crops he should plant on his 100-acre farm. The profit from each is dependent on the rainfall during the growing season. The farmer has categorized the amount of rainfall as high, medium and low. His estimated profit for each is shown in the table below :

Rainfall	*Estimated conditional profit (Rs.)*		
	Crop A	*Crop B*	*Crop C*
High	8,000	3,500	5,000
Medium	4,500	4,500	5,000
Low	2,000	5,000	4,000

If the farmer wishes to plant only one crop, decide which should be his 'best crop' using :

(*a*) Maximax criterion. – Crop A

(*b*) Maximin criterion. – Crop C

(*c*) Hurwicz criterion (farmer's degree of optimism being 0.6). – Crop A

(*d*) Laplace criterion. – Crop A

(*e*) Minimax regret criterion. – Crop A or C

1604. The research department of Hindustan Lever has recommended to the marketing department to launch a shampoo of three different types. The marketing manager has to decide one of the types of shampoo to be launched under the following estimated payoffs for various levels of sales :

Types of shampoo	*Estimated levels of sale (units)*		
	15,000	*10,000*	*5,000*
Egg shampoo	30	10	10
Clinic shampoo	40	15	5
Delux shampoo	55	20	3

What will be the marketing manager's decision if (*i*) Maximin, (*ii*) Minimax, (*iii*) Maximax, (*iv*) Laplace, and (*v*) Regret criteria are applied? ***[Sardar Patel B.E. (Mech.) 1984]***

1605. The *ABC* Company is faced with four decision alternatives relating to investments in a capital expansion programme. Since these investments are made in future, the company foresees different market conditions as expressed in the form of states-of-nature. The following table summarizes the decision alternatives, the various states-of-nature and the rate of return associated with each state-of-nature :

Decision	*States-of-nature*		
	θ_1	θ_2	θ_3
D_1	17%	15%	8%
D_2	18%	16%	9%
D_3	21%	14%	9%
D_4	19%	12%	10%

If the company has no information regarding the probability of the occurrence of the three states-of-nature, give the recommended decision for the decision criteria listed below :

(*i*) Maximax criterion.

(*ii*) Maximin criterion.

(*iii*) Minimax regret criterion.

(*iv*) Laplace rational criterion.

(*v*) Hurwicz alpha criterion ($\alpha = 0.75$).

1606. The estimated sales of proposed types of perfumes are as under : ***[C.A. (Nov.) 1995]***

Types of perfumes	*Estimated levels of sales (units)*		
	Rs. 20,000	*Rs. 10,000*	*Rs. 20,000*
A	25	15	10
B	40	20	5
C	60	25	3

(*a*) For each of the following decisions, state the optimal action and specify the value leading to its selection :

(*i*) Maximin (*ii*) Maximax (*iii*) Laplace (*iv*) Minimax regret.

(*b*) What will be the optimal act if the payoff entries represent the costs instead of sales.

1607. Consider the following payoff (profit) matrix :

	θ_1	θ_2	θ_3	θ_4
a_1	5	10	18	25
a_2	8	7	8	23
a_3	21	18	12	21
a_4	30	22	19	15

Solve this using Hurwicz criterion with $\alpha = 0.75$. *[Madurai M.Sc. (Math.) 1989]*

1608. An Informatics Corporation summarizes international financial information reports (on a weekly basis), prints sophisticated data and forecasts which are purchased weekly by mutual funds, banks and insurance companies. This information is very expensive and the demand for the report is limited to a maximum of 30 units per week. The possible demands are 0, 10, 20 or 30 reports per week. The profit per report sold is Rs. 30 and the loss per report unsold at the end of a week is Rs. 20. No production of extra report during a week is possible. Further, there is penalty cost of Rs. 250, for not meeting the demand. Unsold reports cannot be carried over to the next week. Using the payoff table, find out the number of reports to be produced if :

(*i*) Maximin or pessimistic strategy is adopted, (*ii*) Maximax or optimistic strategy is adopted. *[C.A. (May) 1991]*

16 : 6. DECISIONS UNDER RISK

When a decision-maker chooses from among several possible options whose probabilities of occurrence can be stated; he is said to take *decisions under risk.* The probabilities of various outcomes may be determined objectively from the past data. However, past records may not be available to arrive at the objective probabilities. In many cases the decision-maker may, on the basis of his experience and judgment, be able to assign subjective probabilities to the various outcomes. The problem can then be solved as decision problem under risk.

Under conditions of risk, the most popular decision criterion for evaluating the alternatives is the expected monetary value/expected opportunity loss of the expected payoff.

Expected Monetary Value (*EMV*) Criterion

The *Expected Monetary Value (EMV)* for a given course of action is the expected value of the conditional payoff for that action. The conditional payoffs are obtained for each action by considering various act-event combinations. The *EMV* criterion may be summarized as below :

Step 1. List conditional profit for each act-event combinations, along with the corresponding event probabilities.

Step 2. For each act, determine the expected conditional profits.

Step 3. Determine *EMV* for each act.

Step 4. Choose the act which corresponds to the optimal *EMV*.

SAMPLE PROBLEM

1609. *A man has the choice of running either a hot-snack stall or an ice-cream stall at a seaside resort during the summer season. If it is a fairly cool summer, he should make Rs. 5,000 by running the hot-snack stall, but if the summer is quite hot he can only expect to make Rs. 1,000. On the other hand, if he operates the ice-cream stall, his profit is estimated at Rs. 6,500 if the summer is hot, but only Rs. 1,000 if it is cool. There is a 40% chance of the summer being hot. Should he opt for running the hot-snack stall or the ice-cream stall? Give mathematical argument.*

Solution. The following payoff table shows the conditional profits resulting from the given act-event combinations :

Event E_i (*Weather*)	*Probability $P(E_i)$* (*i*)	*Conditional payoff (Rs.) (Action)*	
		Hot-snack stall (*ii*)	*Ice-cream stall* (*iii*)
Cool summer	0.60	5,000	1,000
Hot summer	0.40	1,000	6,500

The expected conditional payoffs are computed as below :

Event E_i (Weather)	Probability (E_i)	Conditional payoff (Rs.) (Action)	
		Hot-snack stall	Ice-cream stall
	(i)	(i) × (ii)	(i) × (iii)
Cool summer	0.6	3,000	600
Hot summer	0.4	400	2,600
	EMV	Rs. 3,400	Rs. 3,200

Since the expected monetary value of running a hot-snack stall is higher, the businessman should opt for running a hot-snack stall.

Expected Opportunity Loss (*EOL*)

An alternative approach to maximizing expected monetary value (*EMV*) is to minimize *expected opportunity loss (EOL)*. Expected opportunity loss or expected value of regrets are calculated in the same manner as the expected payoffs in the *EMV* criterion. Major steps in the *EOL* criterion may be summarized as below :

Step 1. List the conditional profit table for each act-event combination, along with corresponding event probabilities.

Step 2. For each event, determine the *COL* (*conditional opportunity loss*) values by first locating the most favourable act (maximum payoff) for that event; and then taking the difference between that conditional profit value and each conditional profit for that event.

Step 3. For each act, determine the expected *COL* values and sum these values to get the expected opportunity loss (*EOL*) for that act.

Step 4. Choose that act which corresponds to the minimum *COL* value.

The method will now be illustrated.

SAMPLE PROBLEMS

1610. *An investor is given the following investment alternatives and percentage rates of return :*

Investment alternatives	State-of-nature (Market conditions)		
	Low	Medium	High
Regular shares	7%	10%	15%
Risky shares	–10%	12%	25%
Property	–12%	18%	30%

Over the past 300 days, 150 days have been medium market conditions and 60 days have had high market conditions. On the basis of these data, state the optimum investment strategy for the investment. *[Nagpur M.B.A. 1999]*

Solution. According to the given information, the probabilities of low, medium and high market conditions would be $\frac{90}{300}$ or 0.30, $\frac{150}{300}$ or 0.50 and $\frac{60}{300}$ or 0.20 respectively. The expected payoffs for each of the alternatives are calculated and shown in the table below :

Computation of Expected Return

Market conditions	Probability	Strategy		
		Regular shares	Risky shares	Property
Low	0.30	0.07 × 0.30	0.10 × 0.30	0.15 × 0.30
Medium	0.50	–0.10 × 0.50	0.12 × 0.50	0.25 × 0.50
High	0.20	–0.12 × 0.20	0.18 × 0.20	0.30 × 0.20
Expected return		0.136	0.126	0.230

Since the expected return of 23% is the highest for property, the investor should invest in this alternative.

1611. *Your company manufactures goods for a market in which the technology of the products is changing rapidly. The research and development department produced a new product which appears to have potential for commercial exploitation. A further Rs. 60,000 is required for development*

testing. As a result of previous experience of this type of market, it has been possible to derive a probability distribution relating to the proportions of customers who will buy the product, as follows :

Proportion of customers	: 0.04	0.08	0.12	0.16	0.20
Probability	: 0.1	0.1	0.2	0.4	0.2

Determine the expected opportunity losses, given no other information than that stated above, and state whether or not the company should develop the product.

Solution. If the proportion of customers who purchase the new product be p, the conditional profit is given by

$$C.P. = (6,000 - 2,000) \times 100p - 66,000 = (400p - 60) \times 1,000$$

Conditional Profit Table

Proportion of customers	Probability	Alternative strategies	
		Develop the product A_1	Do not develop the product A_2
S_1 (0.04)	0.1	–40,000	0
S_2 (0.08)	0.1	–28,000	0
S_3 (0.12)	0.2	–12,000	0
S_4 (0.16)	0.4	– 4,000	0
S_5 (0.20)	0.2	20,000	0

Opportunity Loss Table

Proportion of customers	Probability	Alternative strategies	
		Develop the product A_1	Do not develop the product A_2
S_1 (0.04)	0.1	0 – (–44,000)	0
S_2 (0.08)	0.1	0 – (–28,000)	0
S_3 (0.12)	0.2	0 – (–12,000)	0
S_4 (0.16)	0.4	4,000 – 4,000	(4,000 – 0)
S_5 (0.20)	0.2	20,000 – 20,000	(20,000 – 0)

$$EOL(A_1) = 0.1 \times 44,000 + 0.1 \times 28,000 + 0.2 \times 12,000 + 0.4 \times 0 + 0.2 \times 0 = \text{Rs. } 9,600$$

$$EOL(A_2) = 0.1 \times 0 + 0.1 \times 0 + 6\,0.2 \times 0 + 0.4 \times 4,000 + 0.2 \times 20,000 = \text{Rs. } 5,600$$

Thus based upon *EOL* criterion, the best decision would be the second alternative, *do not develop* the product.

Expected Value of Perfect Information (*EVPI*)

The *expected profit with perfect information* is the expected return, in the long run, if we have perfect information before a decision is made. The *Expected Value of Perfect Information (EVPI)* may be defined as the maximum amount one would be willing to pay, to acquire perfect information as to which event would occur. *EPPI represents the maximum obtainable EMV* with perfect information as to which event will actually occur (as calculated before perfect information is received). If *EMV** represents the maximum obtainable *EMV* without perfect information, perfect information would increase expected profit from *EMV** up to the value of *EPPI*, so the amount of that increase would be equal to *EVPI*. Thus, we have

$$EVPI = EPPI - EMV^*$$

SAMPLE PROBLEM

1612. *A wholesaler of sports goods has an opportunity to buy 5,000 pairs of skiis that have been declared surplus by the government. The wholesaler will pay Rs. 50 per pair and can obtain Rs. 100 a pair by selling skiis to retailers. The price is well-established, but the wholesaler is in doubt as to just how many pairs he will be able to sell. Any skiis left over, he can sell to discount outlets at Rs. 20 a pair. After a careful consideration of the historical data, the wholesaler assigns probabilities to the demand as follows :*

Retailer's demand	*Probability*
1,000 pairs	*0.6*
3,000 pairs	*0.3*
5,000 pairs	*0.1*

(*a*) *Compute the conditional monetary and expected monetary values.*
(*b*) *Compute the expected profit with a perfect predicting device.*
(*c*) *Compute the EVPI.*

Solution. We are given :

Cost per pair = Rs. 50
Selling price per pair = Rs. 100
Profit per pair = Rs. 50 (if sold)
Disposal selling price = Rs. 20 (if unsold)
Loss on each unsold pair = (50 – 20) = Rs. 30

Conditional profit values are therefore computed by

$$CP = \begin{cases} 50S & \text{when } D > S \\ 50D - 30(S - D) & \text{when } D < S \end{cases}$$

where CP = conditional profit, D = pairs demanded and S = pairs stocked.

(*a*) The resulting conditional payoffs and corresponding expected payoffs are computed in the table below :

('000 Rs.)

Retailer's demand	*Probability*	*Conditional payoffs (Rs.) (Stock per week)*			*Expected payoffs (Rs.) (Stock per week)*		
		1,000 pairs	*3,000 pairs*	*5,000 pairs*	*1,000 pairs*	*3,000 pairs*	*5,000 pairs*
1,000 pairs	0.6	50	–10	–70	30	–6	–42
3,000 pairs	0.3	50	150	90	15	45	27
5,000 pairs	0.1	50	150	250	5	15	25
				EMV	50	54	10

(*b*) The expected profit under perfect information (*EPPI*) is computed below :

('000 Rs.)

Retailer's demand	*Probability*	*Conditional payoffs*			*Under perfect information*	
		1,000 pairs	*3,000 pairs*	*5,000 pairs*	*Maximum payoff*	*Expected payoff*
	(i)	*(ii)*	*(iii)*	*(iv)*	*(v)*	*(i) × (v)*
1,000 pairs	0.6	**50**	–10	–70	50	30
3,000 pairs	0.3	50	**150**	90	150	45
5,000 pairs	0.1	50	150	**250**	250	25
					EPPI	100

(*c*) $EVPI = EPPI - EMV^* = 100 - 54 = 46$

Thus, $EVPI$ = Rs. 46,000.

PROBLEMS

1613. A producer of boats has estimated the following distribution of demand for a particular kind of boat :

No. demanded :	*0*	*1*	*2*	*3*	*4*	*5*	*6*
Probability :	0.14	0.27	0.27	0.18	0.09	0.04	0.01

Each boat cost him Rs. 7,000 and he sells them for Rs. 10,000 each. Any boats that are left unsold at the end of the season must be disposed of for Rs. 6,000 each. How many boats should be in stock so as to maximize his expected profit? *[Sardar Patel M.B.A. 1996]*

1614. A toy camera manufacturer produces two models (Standard and Delux). In preparation for the heavy Christmas selling season, he must decide how many of each model to produce. Variable cost of the Standard camera is Rs. 10 and selling price is Rs. 20, variable cost of the Delux model is Rs. 20 and the selling price is Rs. 35. He estimates demand as follows :

Standard Model		Delux Model	
Demand	*Probability*	*Demand*	*Probability*
6,000	0.30	2,000	0.20
8,000	0.70	4,000	0.80

Any cameras not sold during the season are sold at salvage price of Rs. 5 for the standard and Rs. 10 for the deluxe model. The manufacturer feels that different segments of the market purchase the two different models, thus the probabilities of sales given above are independent. Supposing unlimited production capacity, the two decisions can be made independently.

What are the optimum quantities of each model to produce? What are the two optimum *EMV*'s? *[Mahrishi Dayanand M.B.A. 1995]*

1615. The following payoff table is given :

Action	*Event*			
	E_1	E_2	E_3	E_4
A_1	40	200	–200	100
A_2	200	0	200	0
A_3	0	100	0	150
A_4	–50	400	100	0

(*i*) Calculate the opportunity loss table.

(*ii*) Suppose that the probabilities of the events of this table are $P(E_1) = 0.20$, $P(E_2) = 0.15$, $P(E_3) = 0.40$, $P(E_4) = 0.25$. Calculate the expected payoff and the expected loss of each action.

[C.A. (May) 1997]

1616. *XYZ* company manufactures parts for passenger cars and sells them in lots of 10,000 parts each. The company has a policy of inspecting each lot before it is actually shipped to the retailer. Five inspection categories, established for quality control, represent the percentage of defective items contained in each lot. These are given in the following table. The daily inspection chart for past 100 inspections shows the following rating or breakdown inspection :

The management is considering two possible courses of action :

(*i*) Shut down the entire plant operations and thoroughly inspect each machine.

Rating	*Proportion of defective items*	*Frequency*
Excellent (*A*)	0.02	25
Good (*B*)	0.05	30
Acceptable (*C*)	0.10	20
Fair (*D*)	0.15	20
Poor (*E*)	0.20	5
		Total = 100

(*ii*) Continue production as it now exists but offer the customer a refund for defective items that are discovered and subsequently returned. The first alternative will cost Rs. 600 while the second alternative will cost the company Re. 1 for each defective item that is returned.

What is the optimum decision for the company? Find the *EVPI*. *[M.S. Baroda M.B.A. 1995]*

1617. An engineering firm has installed a machine costing Rs. 4 lakhs and is in the process of deciding on an appropriate number of a certain spare parts required for repairs. The spare parts cost Rs. 4,000 each but are available only if they are ordered now. In case the machine fails and no spares are available, the cost to the company of mending the plant would be Rs. 18,000. The plant has an estimated life experience with similar machines, is as follows :

Failures during 8-yearly period	*0*	*1*	*2*	*3*	*4*	*5*	*6+*
Probability	0.1	0.2	0.3	0.2	0.1	0.1	0

Ignoring any discounting for time value of money, determine the following :

(*a*) The optimal number of units of the spare part on the basis (*i*) minimax principle, (*ii*) minimum principle, (*iii*) Laplace principle, and (*iv*) expected cost principle.

(*b*) The expected number of failures in the 8-year period.

[Poona M.B.A. 1996]

(*c*) *EVPI*.

1618. Under an employment promotion programme it is proposed to allow sale of newspapers on buses during off-peak hour. The vendor can purchase the papers at a special concessional rate of 25

paise and sell it for 40 paise (a piece). Any unsold copy is a dead loss. A vendor has estimated the following probabilities for the number of copies demanded :

No. of copies	:	15	16	17	18	19	20
Probability	:	0.04	0.19	0.33	0.26	0.11	0.07

Prepare a payoff table and find out how many copies should be ordered so that his expected profits will be a maximum. *[Kerala M.Com. 1994; Madras B.E. Comp. Sc. 1991]*

1619. M/s Ramji Lal & Sons are faced with the problem of determining the optimum number of certain magazine to order for sale. The magazine which costs Re. 0.50 per copy sells for Re. 1.00. If the company orders more copies than it can sell, the unsold copies can be returned under the prior wholesale contract for a refund under the following formula : upto first 500 copies, refund is Re. 0.30 for each unsold copy; 501 to 1,000 copies, refund is Re. 0.20 for each copy; and for over 1,000 copies refund is Re. 0.10 each. The sales record of past 100 weeks is given in the following table :

No. of copies sold (per week)	:	4,000	5,000	6,000	7,000	8,000
No. of weeks	:	10	25	35	20	10

(*i*) What is the optimum decision?

(*ii*) Compute the expected maximum profit. *[Nagpur M.B.A. 1997]*

1620. The demand for a seasonal product is as given below :

Demand during the season	:	40	45	50	55	60	65
Probability	:	0.10	0.20	0.30	0.25	0.10	0.05

The product costs Rs. 60 per unit and sells at Rs. 80 per unit. If the units are not sold within the season, they will have no market value.

(*i*) Prepare a payoff and a regret table, (*ii*) Find the expected payoffs and regret, (*iii*) Find the optimum act and *EVPI*. *[Allahabad M.B.A. 1995]*

1621. A TV dealer finds that the cost of holding per unit per week is Rs. 30.00 and the cost of shortage per unit per week is Rs. 70.00. For one particular model of TV, the probability distribution of weekly sale is as follows :

Weekly sales	:	0	1	2	3	4	5	6
Probability	:	0.10	0.10	0.20	0.25	0.15	0.15	0.05

How many units per week should the dealer order? Also, find *EVPI*. *[C.A. (Nov.) 1999]*

1622. A milk producing co-operative union desires to determine how many kilograms of butter it should produce on daily basis to meet the demand. Past records have shown the following pattern of demand :

Quantity demanded (kg.)	:	15	20	25	30	35	40	45
Days when given demand occurred	:	4	16	20	80	40	30	10

Assume that the stock levels are restricted to the range 15-45 kg. (a multiple of 5 kg.) and that butter left unsold at the end of the day must be disposed of due to inadequate storing facilities. Butter costs Rs. 14 per kg. and is sold at Rs. 20 per kg. (*a*) Construct a conditional profit table, (*b*) Determine the action alternative associated with the maximization of expected profit, and (*c*) Determine *EVPI*. *[Gujarat M.B.A. 1996]*

1623. A modern home appliances dealer finds that the cost of holding a mini cooking range in stock for a month is Rs. 200 (insurance, minor deterioration, interest on borrowed capital, etc.). Customer who cannot obtain a cooking range immediately tends to go to other dealers and he estimates that for every customer who cannot get immediate delivery, he loses an average of Rs. 500. The probabilities of a demand of 0, 1, 2, 3, 4, 5 mini cooking ranges in a month are 0.05, 0.10, 0.20, 0.30, 0.20, 0.15 respectively. Determine the optimum stock level of cooking ranges. Also find *EVPI*. *[Delhi M.B.A. 1996]*

1624. A TV dealer finds that the cost of holding a TV in stock for a week is Rs. 50. Customers who cannot obtain new TVs immediately tend to go to other dealers and he estimates that for every customer who cannot get immediate delivery he loses an average of Rs. 200. For one particular model of TV the probabilities of a demand of 0, 1, 2, 3, 4 and 5 TVs in a week are 0.05, 0.10, 0.20, 0.30, 0.20 and 0.15 respectively.

(*i*) How many televisions per week should the dealer order? Assume that there is no time lag between ordering and delivery.

(*ii*) Compute *EVPI*.

(*iii*) The dealer is thinking of spending on a small market survey to obtain additional information regarding the demand levels. How much should he be willing to spend on such a survey? *[Delhi M.B.A. (Nov.) 1997]*

16 : 7. DECISION-TREE ANALYSIS

A *decision-tree* is a graphic display of various decision alternatives and the sequence of events as if they were branches of a tree. In constructing a tree diagram, it is a convention to use the symbol '□' to indicate the *decision point* and '○' to denote the situation of uncertainty or '*event*'. Branches coming out of a decision point are nothing but representation of immediate mutually exclusive alternative *acts* (alternative options) open to the decision-maker. Branches emanating from the 'event' point '○' represent all possible situations (events). These events are not fully under the control of the decision-maker and may represent consumer demand, etc. The basic advantage of a tree diagram is that another act (called *second* act) subsequent to the happening of each event may also be represented. The resulting outcome (payoff) for each act-event combination may be indicated in the tree diagram at the outer end of each branch. A decision-tree diagram is illustrated below :

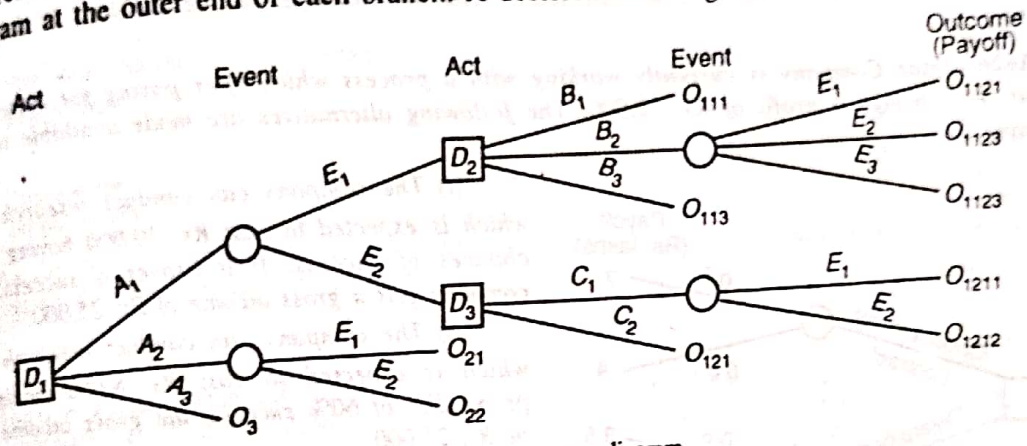

Fig. 16.1. A decision-tree diagram

For example, O_{1211} represents the payoff of the act-event combination $A_1 - E_2 - C_1 - E_1$.

When probabilities of various events are known, they are written along the corresponding branches. Multiplying the probabilities along the branches results in the joint probabilities for the corresponding act-event sequence. Thus in a decision-tree, the decision-maker lists all possible alternatives, possible events and resulting payoff values along with their probabilities for each act-event sequence. This enables him to determine expected payoff values and hence the *EMV* for each act.

A decision-tree is highly useful to a decision-maker in multistage situations which involve a series of decisions each dependent on the preceding one. The modern approach to the analysis of a decision-tree involves the computation of *EMV* for each main branch of the tree. These values then become the conditional expected payoffs for their respective branches; the process is repeated until the *EMV* for a given path has been determined. The optimal path (strategy) is one that corresponds to the maximum *EMV*. This technique, called *Rolling Back Technique*, is explained below :

Since it is impossible to evaluate an *immediate* decision act without first considering all *future* outcomes that result from this decision, one begins the analysis at the *end* of the tree (far right-hand side). The last decision point is of primary importance to us. We analyse this point and take that decision which yields optimal *EMV* and then *roll back* to the last but one decision point, make the same *EMV* analysis for decision and roll-back to the preceding decision point. The *rolling back process* continues till the initial point is reached.

SAMPLE PROBLEMS

1625. *A manager has a choice between (i) A risky contract promising Rs. 7 lakhs with probability 0.6 and Rs. 4 lakhs with probability 0.4, and (ii) A diversified portfolio consisting of two contracts with independent outcomes each promising Rs. 3.5 lakhs with probability 0.6 and Rs. 2 lakhs with probability 0.4.*

Construct a decision-tree for using EMV criteria. Can you arrive at the decision using EMV criteria? *[Poona M.B.A. 1992]*

Solution. The conditional payoff table for the problem may be constructed as below :

(Rs. in lakhs)

Event E_i	Probability (E_1) (i)	Conditional payoffs (Decision) Contract (ii)	Portfolio (iii)	Expected payoff (Decision) Contract (i) × (ii)	Portfolio (i) × (iii)
E_1	0.6	7	3.5	4.2	2.1
E_2	0.4	4	2	1.6	0.8
			EMV	**5.8**	2.0

Using the *EMV* criterion, the manager must go in for the risky contract which will yield him a higher expected monetary value of Rs. 5.8 lakhs. The decision-tree for the problem is given in Fig. 16.2.

1626. *Amar Company is currently working with a process which after paying for materials, labour, etc., brings a profit of Rs. 12,000. The following alternatives are made available to the company :*

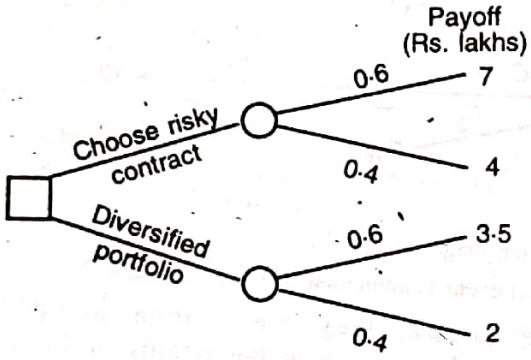

Fig. 16.2. Decision-tree for the Manager

(i) The company can conduct research (R_1) which is expected to cost Rs. 10,000 having 90% chances of success. If it proves a success, the company gets a gross income of Rs. 25,000.

(ii) The company can conduct research (R_2) which is expected to cost Rs. 8,000 having a probability of 60% success, the gross income will be Rs. 25,000.

(iii) The company can pay Rs. 6,000 as royalty for a new process which will bring a gross income of Rs. 20,000.

(iv) The company continues the current process.

Because of limited resources, it is assumed that only one of the two types of research can be carried out at a time.

Use decision-tree analysis to locate the optimal strategy for the company. *[C.A. (May) 1994]*

Solution. The various act-event combinations and resulting payoffs of the problem are introduced in the following decision-tree diagram. The net *EMV* corresponding to various event/decision points are indicated in bold type :

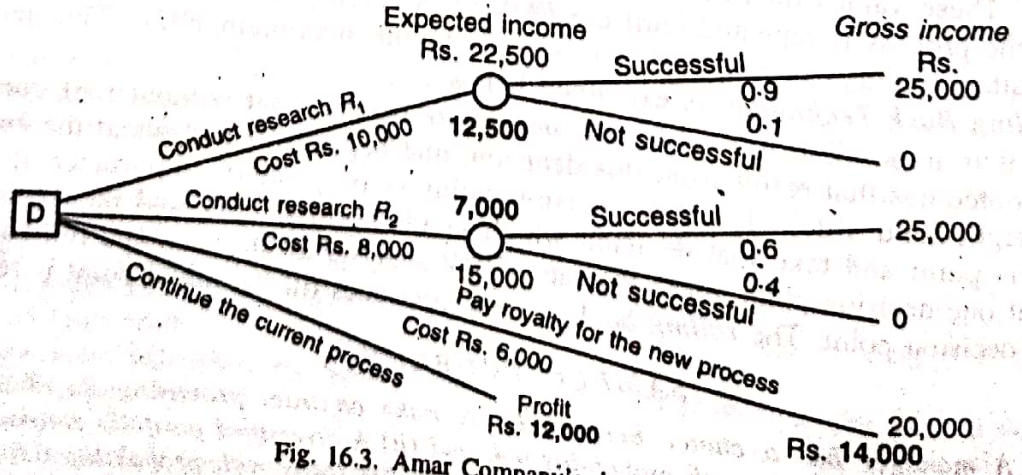

Fig. 16.3. Amar Company's decision-tree

Decision Analysis at Point *D*

Decision	*Event*	*Probability*	*Gross income*	*Expected income*
1. Conduct research R_1	Successful	0.9	(*Rs.*) 25,000	(*Rs.*) 22,500
	Not successful	0.1	0	0
			Total expected income = Rs. 22,500	
			Less cost Rs. 10,000	
			Net *EMV* Rs. 12,500	
2. Conduct research R_2	Successful	0.6	25,000	15,000
	Not successful	0.4	0	0
			Total expected income = Rs. 15,000	
			Less cost Rs. 8,000	
			Net *EMV* Rs. 7,000	
3. Pay royalty for the new process	Certain	1	20,000	Rs. 20,000
			Less cost Rs. 6,000	
			Net *EMV* Rs. 14,000	
4. Continue the current process	Certain	1	12,000	*EMV* Rs. 12,000

As the net *EMV* is highest for the alternative 'pay royalty for the new process', the optimal decision would be to procure new process on royalty basis.

PROBLEMS

1627. A Finance Manager is considering drilling a well. In the past, only 70% of wells drilled were successful at 20 metres depth in that area. Moreover on finding no water at 20 metres, some persons in that area drilled it further upto 25 metres but 20% struck water at that level. The prevailing cost of drilling is Rs. 500 per metre. The Finance Manager estimated that in case he does not get water in his own well, he will have to pay Rs. 15,000 to buy water from outside for the same period of getting water from the well. The following decisions are considered :

(*i*) Do not drill any well,

(*ii*) Drill upto 20 metres, and

(*iii*) If no water is found at 20 metres, drill further upto 25 metres.

Draw an appropriate decision-tree and determine the Finance Manager's optimal strategy. *[C.A. (May) 1992]*

1628. A businessman has two independent investments *A* and *B* available to him, but he lacks the capital to undertake both of them simultaneously. He can choose to take *A* first and then stop, or if *A* is successful then take *B*, or vice-versa. The probability of success on *A* is 0.7, while for *B* it is 0.4. Both investments require an initial capital outlay of Rs. 2,000, and both return nothing if the venture is unsuccessful. Successful completion of *A* will return Rs. 3,000 (over cost), successful completion of *B* will return Rs. 5,000 (over cost).

Draw the decision-tree and determine the best strategy.

1629. A client asks an estate agent to sell three properties (*A*, *B* and *C*) for him and agrees to pay him a 5% commission on each sale. He specifies certain conditions. The estate agent must sell *A* first, and this he must do within 50 days. If and when *A* is sold, the agent receives his 5% commission on that sale. He can then either back out at this stage or nominate and try to sell one of the two remaining properties within 60 days. If he does not succeed in selling the nominated property in that period, he is not given the opportunity to sell the other. If he does sell it in the period, he is given the opportunity to sell the third property on the same conditions. The following table summarizes the prices, selling costs (incurred whenever a sale is attempted) and the estate agent's estimated probability of making a sale :

Property	*Price of property*	*Selling costs*	*Probability of sale*
A	Rs. 12,000	Rs. 400	0.7
B	25,000	225	0.6
C	50,000	450	0.5

(*i*) Draw up an appropriate decision-tree for the estate agent.

(*ii*) What is the estate agent's best strategy under *EMV* approach?