9.7 Exponentially Weighted Moving Average Control Charts

- The **exponentially weighted moving average (EWMA)** chart was introduced by Roberts (*Technometrics* 1959) and was originally called a *geometric moving average chart*. The name was changed to reflect the fact that exponential smoothing serves as the basis of EWMA charts.

- Like a cusum chart, an EWMA chart is an alternative to a Shewhart individuals or $\overline{x}$ chart and provides quicker responses to shifts in the process mean then either an individuals or $\overline{x}$ chart because it incorporates information from all previously collected data.

- To construct an EWMA chart, we assume we have k samples of size $n \geq 1$ yielding k individual measurements $x_1, \ldots, x_k$ (if $n = 1$) or k sample means $\overline{x}_1, \ldots, \overline{x}_k$ (if $n > 1$).

- We will work with the simpler case of individual measurements ($n = 1$) when developing the formulas. To work with sample means, replace σ with $\sigma/\sqrt{n}$ in all formulas.

- Let z_i be the value of the exponentially weighted moving average at the i^{th} sample. That is,

$$z_i = \qquad (24)$$

  where $0 < \lambda \leq 1$. λ is called the **weighting constant**.

- We also need to define a **starting value** z_0 before the first sample is taken.
  - If a target value μ is specified, then $z_0 = \mu$.
  - Otherwise, it is typical to use the average of some preliminary data. That is, $z_0 = \overline{x}$.

- Note that the EWMA z_i is a weighted average of all observations that precede it. For example:

$$i = 1 \quad z_1 = \lambda x_1 + (1-\lambda) z_0$$

$$i = 2 \quad z_2 = \lambda x_2 + (1-\lambda) z_1 =$$

$$=$$

$$= \lambda(1-\lambda)^0 x_2 + (1-\lambda)^1 \lambda x_1 + (1-\lambda)^2 z_0$$

$$i = 3 \quad z_3 = \lambda x_3 + (1-\lambda) z_2$$

$$= \lambda x_3 + (1-\lambda)$$

$$= \lambda x_3 + (1-\lambda)$$

$$= (1-\lambda)^0 \lambda x_3 + (1-\lambda)^1 \lambda x_2 + (1-\lambda)^2 \lambda x_1 + (1-\lambda)^3 z_0$$

- In general, by repeated substitution in (24), we recursively can write each z_i (if $0 < \lambda < 1$) as

$$z_i = \lambda \sum_{j=0}^{i-1} (1-\lambda)^j x_{i-j} + (1-\lambda)^i z_0 \qquad (25)$$

- Note that $\frac{\lambda}{2-\lambda}(1-(1-\lambda)^{2i}) \longrightarrow \frac{\lambda}{2-\lambda}$ as i increases. Thus, after the EWMA chart has been running for several samples, the control limits will approach the following steady-state values (called **asymptotic control limits**):

$$\text{UCL} = \mu_0 +$$

$$\text{LCL} = \mu_0 -$$

- It is recommended that exact control limits be used for small values of i because it will greatly improve the performance of the EWMA chart in detecting an off-target process very soon after the EWMA is started.
- *SAS* plots exact control limits by default. Plotting asymptotic control limits is an option.
- **Example**: Suppose $\lambda = .25, L = 3, \sigma = 1$, and $\mu_0 = 0$. Then, using the asymptotic variance, the control limits are

$$UCL = 0 + (3)(1)\sqrt{\frac{.25}{1.75}} \approx \qquad LCL = 0 - (3)(1)\sqrt{\frac{.25}{1.75}} \approx$$

The following table summarizes the EWMA calculation for 16 sample values (with comparison calculations for a tabular cusum with $h = 5$ and $k = .5$). Both the EWMA and the Cusum indicate an out-of-control signal on sample 16.

i	x_i	EWMA z_i $\lambda = .25$	Tabular Cusum with $h=5$, $k=.5$	
0	—	$z_0 = 0$	$C_i^+ = 0$	$C_i^- = 0$
1	1.0	.250	0.5	0.0
2	-0.5	.063	0.0	0.0
3	0.0	.047	0.0	0.0
4	-0.8	-.165	0.0	0.3
5	-0.8	-.324	0.0	0.6
6	-1.2	-.543	0.0	1.3
7	1.5	-.032	1.0	0.0
8	-0.6	-.174	0.0	0.1
9	1.0	.120	0.5	0.0
10	-0.9	-.135	0.0	0.4
11	1.2	.199	0.7	0.0
12	0.5	.274	0.7	0.0
13	2.6	.855	2.8	0.0
14	0.7	.817	3.0	0.0
15	1.1	.887	3.6	0.0
16	2.0	**1.166**	**5.1**	0.0

Sample EWMA calculations of $z_i = .25x_i + .75z_{i-1}$

$$z_1 = (.25)(1) + (.75)(0) = .25$$
$$z_2 = (.25)(-.5) + (.75)(.25) = .0625 \approx .063$$
$$z_3 = (.25)(0) + (.75)(.0625) = .046875 \approx .047$$
$$z_4 = (.25)(-.8) + (.75)(.046875) = -.16484375 \approx -.165$$

Constructing EWMA Charts

The following notation is used in this section:

E_i	exponentially weighted moving average for the i^{th} subgroup
r	EWMA weight parameter ($0 < r \leq 1$)
μ	process mean (expected value of the population of measurements)
σ	process standard deviation (standard deviation of the population of measurements
x_{ij}	j^{th} measurement in i^{th} subgroup, with $j = 1, 2, 3, \ldots, n_i$
n_i	sample size of i^{th} subgroup
$\overline{X}_i$	mean of measurements in i^{th} subgroup. If $n_i = 1$, then the subgroup mean reduces to the single observation in the subgroup
$\overline{\overline{X}}$	weighted average of subgroup means
$\Phi^{-1}(\cdot)$	inverse standard normal function

Plotted Points

Each point on the chart indicates the value of the exponentially weighted moving average (EWMA) for that subgroup. The EWMA for the i^{th} subgroup (E_i) is defined recursively as

$$E_i = r\overline{X}_i + (1 - r)E_{i-1}, \quad i > 0$$

where r is a weight parameter ($0 < r \leq 1$). Some authors (for example, Hunter 1986 and Crowder 1987a,b) use the symbol λ instead of r for the weight. You can specify the weight with the WEIGHT= option in the EWMACHART statement or with the variable _WEIGHT_ in a LIMITS= data set. If you specify a known value (μ_0) for μ, $E_0 = \mu_0$; otherwise, $E_0 = \overline{\overline{X}}$.

The preceding equation can be rewritten as

$$E_i = E_{i-1} + r(\overline{X}_i - E_{i-1})$$

which expresses the current EWMA as the previous EWMA plus the weighted error in the prediction of the current mean based on the previous EWMA.

The EWMA for the i^{th} subgroup can also be written as

$$E_i = r\sum_{j=0}^{i-1}(1 - r)^j\overline{X}_{i-j} + (1 - r)^iE_0$$

which expresses the EWMA as a weighted average of past subgroup means, where the weights decline exponentially, and the heaviest weight is assigned to the most recent subgroup mean.,

Central Line

By default, the central line on an EWMA chart indicates an estimate for μ, which is computed as

$$\hat{\mu} = \overline{\overline{X}} = \frac{n_1 \bar{X}_1 + \cdots + n_N \bar{X}_N}{n_1 + \cdots + n_N}$$

If you specify a known value (μ_0) for μ, the central line indicates the value of μ_0.

Control Limits

You can compute the limits in the following way

- as a specified multiple (k) of the standard error of E_i above and below the central line. The default limits are computed with $k = 3$ (these are referred to as 3σ *limits*).

Table 19.17. Limits for an EWMA Chart

Control Limits
LCL = lower limit = $\overline{\overline{X}} - k\hat{\sigma}r\sqrt{\sum_{j=0}^{i-1}(1-r)^{2j}/n_{i-j}}$
UCL = upper limit = $\overline{\overline{X}} + k\hat{\sigma}r\sqrt{\sum_{j=0}^{i-1}(1-r)^{2j}/n_{i-j}}$

These formulas assume that the data are normally distributed. If standard values μ_0 and σ_0 are available for μ and σ, respectively, replace $\overline{\overline{X}}$ with μ_0 and $\hat{\sigma}$ with σ_0 in Table 19.17. Note that the limits vary with both n_i and i.

If the subgroup sample sizes are constant ($n_i = n$), the formulas for the control limits simplify to

$$\text{LCL} = \overline{\overline{X}} - k\hat{\sigma}\sqrt{r(1-(1-r)^{2i})/n(2-r)}$$
$$\text{UCL} = \overline{\overline{X}} + k\hat{\sigma}\sqrt{r(1-(1-r)^{2i})/n(2-r)}$$

Consequently, when the subgroup sample sizes are constant, the width of the control limits increases monotonically with i. For probability limits, replace k with $\Phi^{-1}(1-\alpha/2)$ in the previous equations. Refer to Roberts (1959) and Montgomery (1991).

As i becomes large, the upper and lower control limits approach constant values:

$$\text{LCL} = \overline{\overline{X}} - k\hat{\sigma}\sqrt{r/n(2-r)}$$
$$\text{UCL} = \overline{\overline{X}} + k\hat{\sigma}\sqrt{r/n(2-r)}$$

Some authors base the control limits for EWMA charts on the asymptotic expressions in the two previous equations.

Various approaches have been proposed for choosing the value of r.

- Hunter (1986) states that the choice "can be left to the judgment of the quality control analyst" and points out that the smaller the value of r, "the greater the influence of the historical data."
- Hunter (1986) also discusses a least squares procedure for estimating r from the data, **assuming an exponentially weighted moving average model for the data.** In this context, the fitted EWMA model provides a forecast of the process that is the basis for dynamic process control. You can use the ARIMA procedure in SAS/ETS software to compute the least squares estimate of r. (Refer to *SAS/ETS User's Guide: Version 6, Second Edition* for information on PROC ARIMA.) Also see "Autocorrelation in Process Data" on page 1522.
- A number of authors have studied the design of EWMA control schemes based on average run length (ARL) computations. The ARL is the expected number of points plotted before a shift is detected. Ideally, the ARL should be short when a shift occurs, and it should be long when there is no shift (the process is in control.) The effect of r on the ARL was described by Roberts (1959), who used simulation methods. The ARL function was approximated and tabulated by Robinson and Ho (1978), and a more general method for studying run-length distributions of EWMA charts was given by Crowder (1987a,b). Unlike Hunter (1986), these authors assume the data are independent and identically distributed; typically the normal distribution is assumed for the data, although the methods extend to nonnormal distributions. A more detailed discussion of the ARL approach follows.

Average run lengths for two-sided EWMA charts are shown in Table 19.18, which is patterned after Table 1 of Crowder (1987a,b). The ARLs were computed using the EWMAARL DATA step function (see page 1602 for details on the EWMAARL function). Note that Crowder (1987a,b) uses the notation L in place of k and the notation λ in place of r.

You can use Table 19.18 to find a combination of k and r that yields a desired ARL for an in-control process ($\delta = 0$) and for a specified shift of δ. Note that δ is assumed to be standardized; in other words, if a shift of Δ is to be detected in the process mean μ, and if σ is the process standard deviation, you should select the table entry with

$$\delta = \Delta/(\sigma/\sqrt{n})$$

where n is the subgroup sample size. Thus, δ can be regarded as the shift in the sampling distribution of the subgroup mean.

For example, suppose you want to construct an EWMA scheme with an in-control ARL of 90 and an ARL of 9 for detecting a shift of $\delta = 1$. Table 19.18 shows that the combination $r = 0.5$ and $k = 2.5$ yields an in-control ARL of 91.17 and an ARL of 8.27 for $\delta = 1$.

Table 19.18. Average Run Lengths for Two-Sided EWMA Charts

		r (weight parameter)					
k	δ	0.05	0.10	0.25	0.50	0.75	1.00
2.0	0.00	127.53	73.28	38.56	26.45	22.88	21.98
2.0	0.25	43.94	34.49	24.83	20.12	18.86	19.13
2.0	0.50	18.97	15.53	12.74	11.89	12.34	13.70
2.0	0.75	11.64	9.36	7.62	7.29	7.86	9.21
2.0	1.00	8.38	6.62	5.24	4.91	5.26	6.25
2.0	1.25	6.56	5.13	3.96	3.59	3.76	4.40
2.0	1.50	5.41	4.20	3.19	2.80	2.84	3.24
2.0	1.75	4.62	3.57	2.68	2.29	2.26	2.49
2.0	2.00	4.04	3.12	2.32	1.95	1.88	2.00
2.0	2.25	3.61	2.78	2.06	1.70	1.61	1.67
2.0	2.50	3.26	2.52	1.85	1.51	1.42	1.45
2.0	2.75	2.99	2.32	1.69	1.37	1.29	1.29
2.0	3.00	2.76	2.16	1.55	1.26	1.19	1.19
2.0	3.25	2.56	2.03	1.43	1.18	1.13	1.12
2.0	3.50	2.39	1.93	1.32	1.12	1.08	1.07
2.0	3.75	2.26	1.83	1.24	1.08	1.05	1.04
2.0	4.00	2.15	1.73	1.17	1.05	1.03	1.02
2.5	0.00	379.09	223.35	124.18	91.17	82.49	80.52
2.5	0.25	73.98	66.59	59.66	58.33	61.07	65.77
2.5	0.50	26.63	23.63	23.28	27.16	33.26	41.49
2.5	0.75	15.41	12.95	11.96	13.96	18.05	24.61
2.5	1.00	10.79	8.75	7.52	8.27	10.57	14.92
2.5	1.25	8.31	6.60	5.39	5.52	6.75	9.46
2.5	1.50	6.78	5.31	4.18	4.03	4.65	6.30
2.5	1.75	5.75	4.46	3.43	3.14	3.43	4.41
2.5	2.00	5.00	3.86	2.92	2.57	2.67	3.24
2.5	2.25	4.43	3.42	2.56	2.18	2.17	2.49
2.5	2.50	4.00	3.07	2.29	1.90	1.83	2.00
2.5	2.75	3.64	2.80	2.08	1.69	1.59	1.67
2.5	3.00	3.36	2.57	1.91	1.52	1.41	1.45
2.5	3.25	3.12	2.39	1.77	1.39	1.29	1.29
2.5	3.50	2.92	2.24	1.64	1.28	1.19	1.19
2.5	3.75	2.74	2.13	1.52	1.20	1.13	1.12
2.5	4.00	2.58	2.04	1.42	1.13	1.08	1.07

(continued)

Table 19.18 *(continued)*

		r (weight parameter)					
k	δ	0.05	0.10	0.25	0.50	0.75	1.00
3.0	0.00	1383.62	842.15	502.90	397.46	374.50	370.40
3.0	0.25	133.61	144.74	171.09	208.54	245.76	281.15
3.0	0.50	37.33	37.41	48.45	75.35	110.95	155.22
3.0	0.75	19.95	17.90	20.16	31.46	50.92	81.22
3.0	1.00	13.52	11.38	11.15	15.74	25.64	43.89
3.0	1.25	10.24	8.32	7.39	9.21	14.26	24.96
3.0	1.50	8.26	6.57	5.47	6.11	8.72	14.97
3.0	1.75	6.94	5.45	4.34	4.45	5.80	9.47
3.0	2.00	6.00	4.67	3.62	3.47	4.15	6.30
3.0	2.25	5.30	4.10	3.11	2.84	3.16	4.41
3.0	2.50	4.76	3.67	2.75	2.41	2.52	3.24
3.0	2.75	4.32	3.32	2.47	2.10	2.09	2.49
3.0	3.00	3.97	3.05	2.26	1.87	1.79	2.00
3.0	3.25	3.67	2.82	2.09	1.69	1.57	1.67
3.0	3.50	3.42	2.62	1.95	1.53	1.41	1.45
3.0	3.75	3.22	2.45	1.84	1.41	1.29	1.29
3.0	4.00	3.04	2.30	1.73	1.31	1.20	1.19
3.5	0.00	12851.0	4106.4	2640.16	2227.34	2157.99	2149.34
3.5	0.25	281.09	381.29	625.78	951.18	1245.90	1502.76
3.5	0.50	53.58	64.72	123.43	267.36	468.68	723.81
3.5	0.75	25.62	25.33	38.68	88.70	182.12	334.40
3.5	1.00	16.65	14.79	17.71	35.97	78.05	160.95
3.5	1.25	12.36	10.37	10.48	17.64	37.15	81.80
3.5	1.50	9.86	8.00	7.25	10.19	19.63	43.96
3.5	1.75	8.22	6.54	5.52	6.70	11.46	24.96
3.5	2.00	7.07	5.55	4.47	4.86	7.33	14.97
3.5	2.25	6.21	4.83	3.77	3.78	5.08	9.47
3.5	2.50	5.55	4.29	3.28	3.10	3.76	6.30
3.5	2.75	5.03	3.87	2.91	2.63	2.94	4.41
3.5	3.00	4.60	3.54	2.63	2.30	2.40	3.24
3.5	3.25	4.25	3.26	2.41	2.05	2.03	2.49
3.5	3.50	3.95	3.03	2.23	1.85	1.76	2.00
3.5	3.75	3.70	2.84	2.10	1.69	1.56	1.67
3.5	4.00	3.47	2.66	1.99	1.55	1.40	1.45

Moving Average Charts

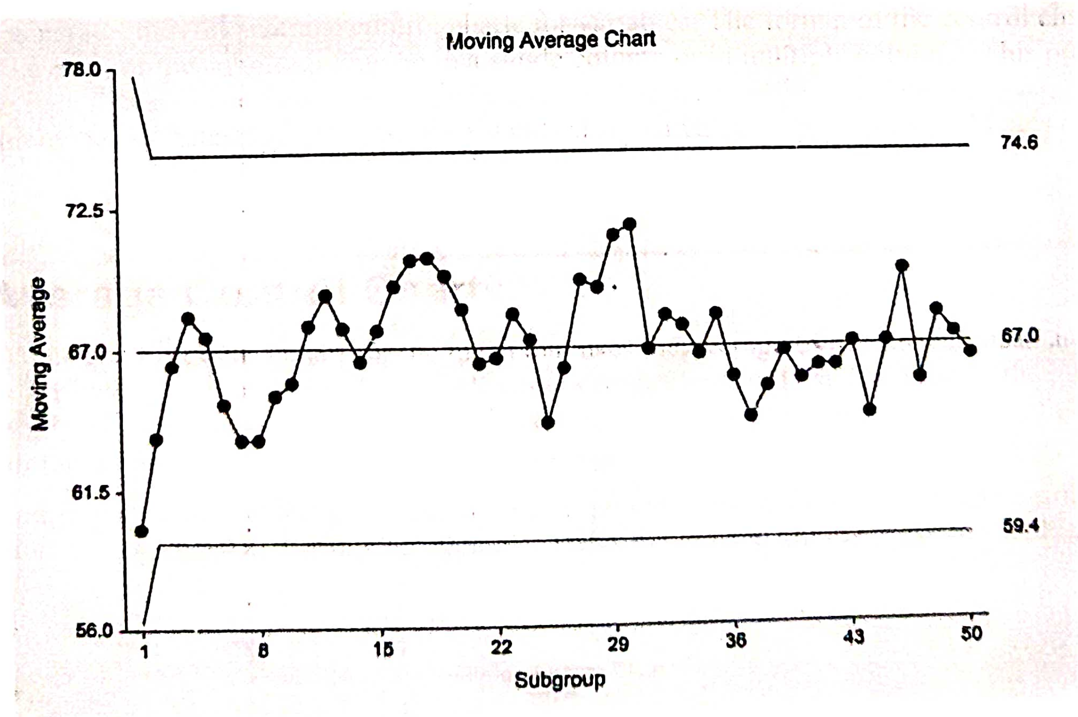
Introduction

This procedure generates moving average control charts for variables. The format of the control charts is fully customizable. The data for the subgroups can be in a single column or in multiple columns. This procedure permits the defining of stages. The target value can be entered directly or estimated from the data, or a sub-set of the data. Sigma may be estimated from the data or a standard sigma value may be entered. Means can be stored to the spreadsheet.

Moving Average Control Charts

The moving average chart is control chart for the mean that uses the average of the current mean and a handful of previous means to produce each moving average. Moving average charts are used to monitor the mean of a process based on samples taken from the process at given times (hours, shifts, days, weeks, months, etc.). The measurements of the samples at a given time constitute a subgroup.

The moving average chart relies on the specification of a target value and a known or reliable estimate of the standard deviation. For this reason, the moving average chart is better used after process control has been established.



Other Control Charts for the Mean and Variation of a Process

When monitoring the mean, the process variation is usually monitored as well using either the range or the standard deviation. Two procedures for monitoring both the mean and the variation are the X-bar and R, and X-bar and s charts.

Two additional control charts available for monitoring the process mean are the cumulative sum (CUSUM) and exponentially weighted moving average (EWMA) charts. The CUSUM and EWMA charts are somewhat similar to the moving average charts in that they take into account the information of previous means at each point. CUSUM and EWMA methods also assume a reliable estimate or known value for the true standard deviation is available.

The moving average chart may also be used when only a single response is available at each time point. Another option for single responses is the individuals and moving range (I-MR) control charts. CUSUM and EWMA charts may also be used for single responses, and are useful when small changes in the mean need to be detected.

Control Chart Formulas

Suppose we have k subgroups, each of size n. Let x_{ij} represent the measurement in the j^{th} sample of the i^{th} subgroup.

Formulas for the Points on the Chart

The i^{th} subgroup mean is calculated using

$$\bar{x}_i = \frac{\sum_{j=1}^{n} x_{ij}}{n}$$

The points of the chart are obtained from the $\bar{x}_i$'s by taking the average of the last w_i subgroups (including the current subgroup), where w_i is the user-specified moving average width.

$$M_i = \frac{\bar{x}_i + \bar{x}_{i-1} + \cdots + \bar{x}_{i-w_i+1}}{w_i}$$

Note that the value of w_i changes during the first few subgroups and then stays constant at the value set by the user.

Estimating the Moving Average Chart Center Line (Grand Mean)

In the Moving Average Charts procedure, the target mean may be input directly, or it may be estimated from a series of subgroups. If it is estimated from the subgroups the formula for the grand average is

$$\bar{\bar{x}} = \frac{\sum_{i=1}^{k}\sum_{j=1}^{n_i} x_{ij}}{\sum_{i=1}^{k} n_i}.$$

If the subgroups are of equal size, the above equation for the grand mean reduces to

$$\bar{\bar{x}} = \frac{\sum_{i=1}^{k} \bar{x}_i}{k} = \frac{\bar{x}_1 + \bar{x}_2 + \cdots + \bar{x}_k}{k}.$$

Estimating Sigma – Sample Ranges

Either the range or the standard deviation of the subgroups may be used to estimate sigma, or a known (standard) sigma value may be entered directly. If the standard deviation (sigma) is to be estimated from the ranges, it is estimated as

$$\hat{\sigma} = \frac{\bar{R}}{d_2}$$

where

$$\bar{R} = \frac{\sum_{i=1}^{k} R_i}{k}$$

$$d_2 = \frac{E(R)}{\sigma} = \frac{\mu_R}{\sigma}$$

The calculation of $E(R)$ requires the knowledge of the underlying distribution of the x_{ij}'s. Making the assumption that the x_{ij}'s follow the normal distribution with constant mean and variance, the values for d_2 are derived through the use of numerical integration. It is important to note that the normality assumption is used and that the accuracy of this estimate requires that this assumption be valid.

When n is one, we cannot calculate R_i since it requires at least two measurements. The procedure in this case is to use the ranges of successive pairs of observations. Hence, the range of the first and second observation is computed, the range of the second and third is computed, and so on. The average of these approximate ranges is used to estimate σ.

Estimating Sigma – Sample Standard Deviations

If the standard deviation (sigma) is to be estimated from the standard deviations, it is estimated as

$$\hat{\sigma} = \frac{\bar{s}}{c_4}$$

where

$$\bar{s} = \frac{\sum_{i=1}^{k} s_i}{k}$$

$$c_4 = \frac{E(s)}{\sigma} = \frac{\mu_s}{\sigma}$$

The calculation of $E(s)$ requires the knowledge of the underlying distribution of the x_{ij}'s. Making the assumption that the x_{ij}'s follow the normal distribution with constant mean and variance, the values for c_4 are obtained from

$$c_4 = \sqrt{\frac{2}{n-1}} \frac{\Gamma\left(\frac{n}{2}\right)}{\Gamma\left(\frac{n-1}{2}\right)}$$

Estimating Sigma – Weighted Approach

When the sample size is variable across subgroups, a weighted approach is recommended for estimating sigma (Montgomery, 2013):

$$\hat{\sigma} = \bar{s} = \left[\frac{\sum_{i=1}^{k} (n_i - 1) s_i^2}{\sum_{i=1}^{k} n_i - k} \right]^{1/2}$$

Moving Average Chart Limits

The lower and upper control limits for the moving-average chart are calculated using the formula

$$LCL_i = \mu_0 - m\left(\frac{\hat{\sigma}}{\sqrt{n_i w_i}}\right)$$

$$LCL_i = \mu_0 + m\left(\frac{\hat{\sigma}}{\sqrt{n_i w_i}}\right)$$

where m is a multiplier (usually set to three) and w_i is the number of rows used in this average. Note that the value of w_i changes during the first few subgroups and then stays constant at the value set by the user.

Data Structure

In this procedure, the data may be in either of two formats. The first data structure option is to have the data in several columns, with one subgroup per row.

Example dataset

S1	S2	S3	S4	S5
2	6	3	8	5
8	8	7	7	9
6	2	2	4	3
5	6	7	6	10
48	2	6	5	0
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.

PROCESS CAPABILITY INDEX

1. Process capability index is a statistical tool to measure the ability of the process to produce output within customers specification limits.

2. It is the ratio of allowable spread to actual spread.

3. $C_p=(USL-LSL)/6\sigma$

   Where

   USL=upper specification limit.

   LSL=lower specification limit.

   σ is an estimate of standard error.

4. The potential of the process can be evaluated using process capability index C_p.

5. It depends on allowable spread and actual spread.

6. Inference:

   If $C_p < 1$, then the process is not capable.

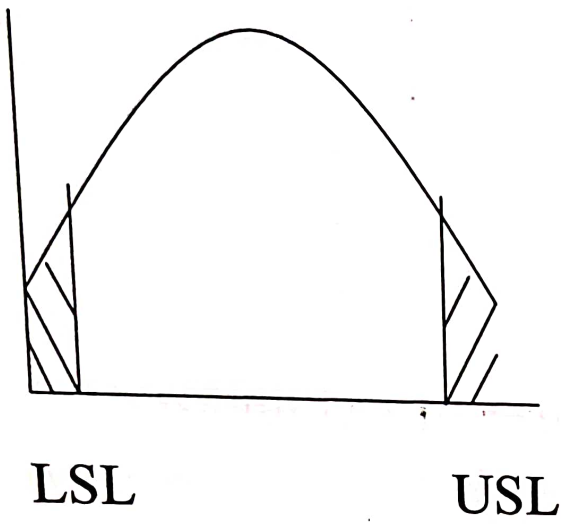
   If $C_p \geq 1$, then the process is capable.

   If $C_p \geq 1.12$, then the process is good.

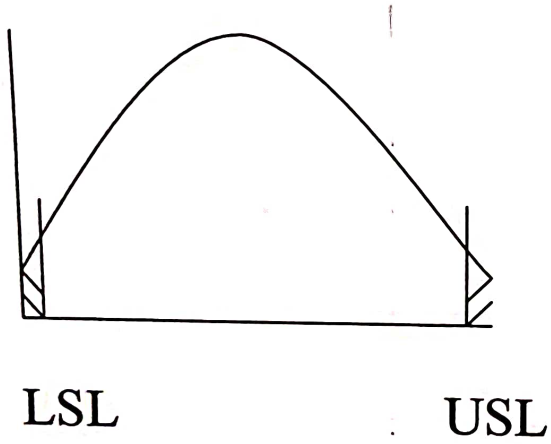
   If $C_p \geq 1.33$, then the process is said to be very good.

7. C_p is unit less measure and can be compared to any two process.

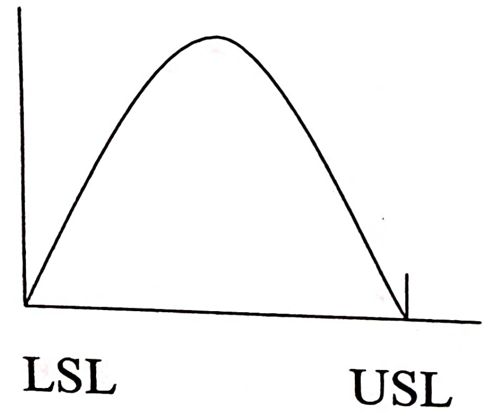
8. Distribution of measurement from the population



$Cp<1.0$



$C_p=1.0$



$C_p>1.0$

PROCESS PERFORMANCE (C_{pk})

1. The process performance is defined as the minimum of the ratio USL-μ to 3σ and the ratio μ-LSL to 3σ.
2. C_{pk} depends on process mean. It also considers the location.
3. $$C_{pk}=\min(USL-\mu/3\sigma, \mu-LSL/3\sigma)$$
4. $C_{pk} \leq C_p$, always.
5. k is called as factor of the process capability. the factor k is

$$k= \frac{(USL+LSL)/2 - \mu}{(USL+LSL)/2}$$

6. If either upper or lower specification limit is given, then the process potential is defined as

   $C_{pu}=USL-\mu/3\sigma$ $C_{pl}=\mu-LSL/3\sigma$

7. The capability ratio is the reciprocal of C_p.

   Process capability=1/ C_p