UNIT III

- Probability : Definition – types- types of events- theories – Binomial, Poisson - Normal distribution

Meaning

- **Probability** is the branch of mathematics concerning numerical descriptions of how likely an event is to occur, or how likely it is that a proposition is true.
- The **probability** of an event is a number between 0 and 1, where, roughly speaking, 0 indicates impossibility of the event and 1 indicates certainty.
- The value is expressed from zero to one. Probability has been introduced in Maths to predict how likely events are to happen.

Meaning

- The value is expressed from zero to one. Probability has been introduced in Maths to predict how likely events are to happen.
- In **probability**, the set of outcomes from an experiment is known as an **Event**. for **example** you conduct an experiment by tossing a coin.
- The outcome of this experiment is the coin landing 'heads' or 'tails'. These can be said to be the **events** connected with the experiment.

Types of Events in Probability:

- Some of the important probability events are:
- Impossible and Sure Events
- Simple Events
- Compound Events
- Independent and Dependent Events
- Mutually Exclusive Events
- Exhaustive Events
- Complementary Events
- Events Associated with “OR”
- Events Associated with “AND”
- Event E1 but not E2

Types

- Impossible and Sure Events

If the probability of occurrence of an event is 0, such an event is called an **impossible event** and if the probability of occurrence of an event is 1, it is called a **sure event**.

In other words, the empty set φ is an impossible event and the sample space S is a sure event.

- Simple Events

Any event consisting of a single point of the sample space is known as a **simple event** in probability. For example, if S = {56 , 78 , 96 , 54 , 89} and E = {78} then E is a simple event.

Types

- Compound Events

Contrary to the simple event, if any event consists of more than one single point of the sample space then such an event is called a **compound event**. Considering the same example again, if S = {56 ,78 ,96 ,54 ,89}, E_1 = {56 ,54 }, E_2 = {78 ,56 ,89 } then, E_1 and E_2 represent two compound events.

- Independent Events and Dependent Events

If the occurrence of any event is completely unaffected by the occurrence of any other event, such events are known as an **independent event** in probability and the events which are affected by other events are known as **dependent events**.

Types

- Mutually Exclusive Events

If the occurrence of one event excludes the occurrence of another event, such events are mutually **exclusive events** i.e. two events don't have any common point. For example, if S = {1 , 2 , 3 , 4 , 5 , 6} and E_1, E_2 are two events such that E_1 consists of numbers less than 3 and E_2 consists of numbers greater than 4.

- So, E1 = {1,2} and E2 = {5,6} .
- Then, E1 and E2 are mutually exclusive.
- Exhaustive Events

A set of events is called **exhaustive** if all the events together consume the entire sample space.

Types

Complementary Events

- For any event E_1 there exists another event E_1' which represents the remaining elements of the sample space S.
- $\mathbf{E_1 = S - E_1'}$
- If a dice is rolled then the sample space S is given as S = {1 , 2 , 3 , 4 , 5 , 6 }. If event E_1 represents all the outcomes which is greater than 4, then E_1 = {5, 6} and E_1' = {1, 2, 3, 4}.
- Thus E_1' is the complement of the event E_1.
- Similarly, the complement of E_1, E_2, E_3……….E_n will be represented as E_1', E_2', E_3'……….E_n'

Events Associated with "OR"

- If two events E_1 and E_2 are associated with **OR** then it means that either E_1 or E_2 or both. The union symbol **(U)** is used to represent OR in probability.
- Thus, the event E_1U E_2 denotes E_1 OR E_2.
- If we have mutually exhaustive events E_1, E_2, E_3 ………E_n associated with sample space S then,
- E_1 U E_2 U E_3U ………E_n = S

Types

- Events Associated with "AND"
- If two events E_1 and E_2 are associated with **AND** then it means the intersection of elements which is common to both the events. The intersection symbol **(**∩**)** is used to represent AND in probability.
- Thus, the event $E_1 \cap E_2$ denotes E_1 and E_2.
- Event E_1 but not E_2
- It represents the difference between both the events. Event E_1 but not E_2 represents all the outcomes which are present in E_1 but not in E_2. Thus, the event E_1 but not E_2 is represented as
- $E_1, E_2 = E_1 – E_2$

Terms

Some common terms related to probability

Experiment: Is a situation involving chance or probability that leads to results called outcomes.

Outcome: A possible result of a random experiment.

Equally likely outcomes: All outcomes with equal probability.

Terms

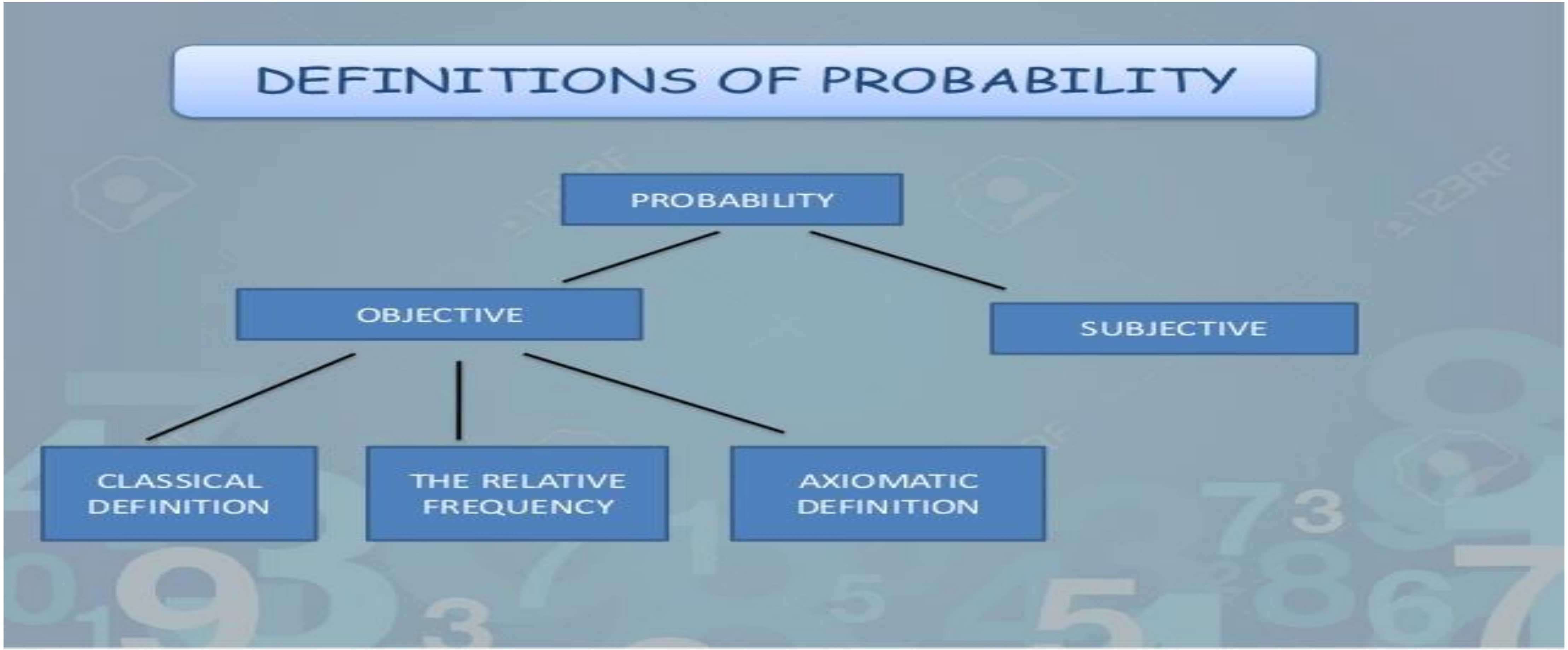
Some common terms related to probability (contd.)

Sample space: The set of outcomes of an experiment is known as sample space.

Event: One or more outcomes in an experiment.

Sample point: Each element of the sample space is called a sample point.

Defintion…



DEFINITIONS OF PROBABILITY
PROBABILITY
OBJECTIVE
SUBJECTIVE
CLASSICAL DEFINITION
THE RELATIVE FREQUENCY
AXIOMATIC DEFINITION

Classical..

CLASSICAL DEFINITION OF PROBABILITY

- If a random experiment can produce 'n' mutually exclusive and equally likely outcomes and if 'm' out of these outcomes are favorable to occurrence of certain event 'A' , then probability of event A , is defined as the ratio m/n.

  P(A)=(no of favourableoutcome)/total outcome

- EXAMPLE:

a) The roll of a die: There are 6 equally likely outcomes. The probability of each is 1/6.

b) The toss of two coins: The four possible outcomes are {H,H}, {H,T}, {T,H} and {T,T}. The probability of each is 1/4.

Limitation

LIMITATION OF CLASSICAL DEFINITION

- This definition is only applicable if events are equally likely.
- No. of possible outcomes should always be finite.

Relative…

THE RELATIVE FREQUENCY DEFINITION

- When a random experiment is repeated a large no of times , say n , under identical conditions and if an event A is observed to occur ‘m’ times ,then probability of event A is defined as the limit of R.F m/n as n tends to infinity.

$$P(A)=\lim_{n\to\infty} (m/n)$$

when n increases infinitely, the ratio m/n tends to become stable.

Axiomatic

AXIOMATIC DEFINITION

- In mathematics, an axiom is a result that is accepted without the need for proof.
- In this case, we say that this is the axiomatic definition of probability because we define probability as a function that satisfies the three axioms given below.

If we do a certain experiment, which has a sample space Ω, we define the probability as a function that associates a certain probability, $P(A)$ with every event A, satisfying the following properties.

- The probability of any event A is positive or zero. Namely $P(A)\geq 0$. The probability measures, in a certain way, the difficulty of event A happening: the smaller the probability, the more difficult it is to happen.

Contn…

- The probability of the sure event is 1. Namely $P(\Omega)=1$. And so, the probability is always greater than 0 and smaller than 1: probability zero means that there is no possibility for it to happen (it is an impossible event), and probability 1 means that it will always happen (it is a sure event).

- The probability of the union of any set of two by two incompatible events is the sum of the probabilities of the events. That is, if we have, for example, events A,B,C, and these are two by two incompatible, then

  $P(A \cup B \cup C)=P(A)+P(B)+P(C)$.

Subjective…

SUBJECTIVE PROBABILITY

- Also known as personalistic probability.
- It is the measure of strength of a person's belief regarding the occurrence of an event A.
- This is based on whatever evidences are available to the individual.
- The disadvantage of subjective probability is that two or persons faced with the same evidence ,may arrive at different probabilities.

Laws of probability

LAWS OF PROBABILITY

1. $0 \leq P(E) \leq 1$ The probability of an event E is between 0 and 1 inclusive.
2. P( ) = 0

   The probability of an empty set is zero.

   Consequence: IF $P(A \cap B) = 0$ then

   it implies A and B are mutually exclusive.
3. $P(S)=1$ The probability of the sample space is 1.

Addition Law

ADDITION LAW

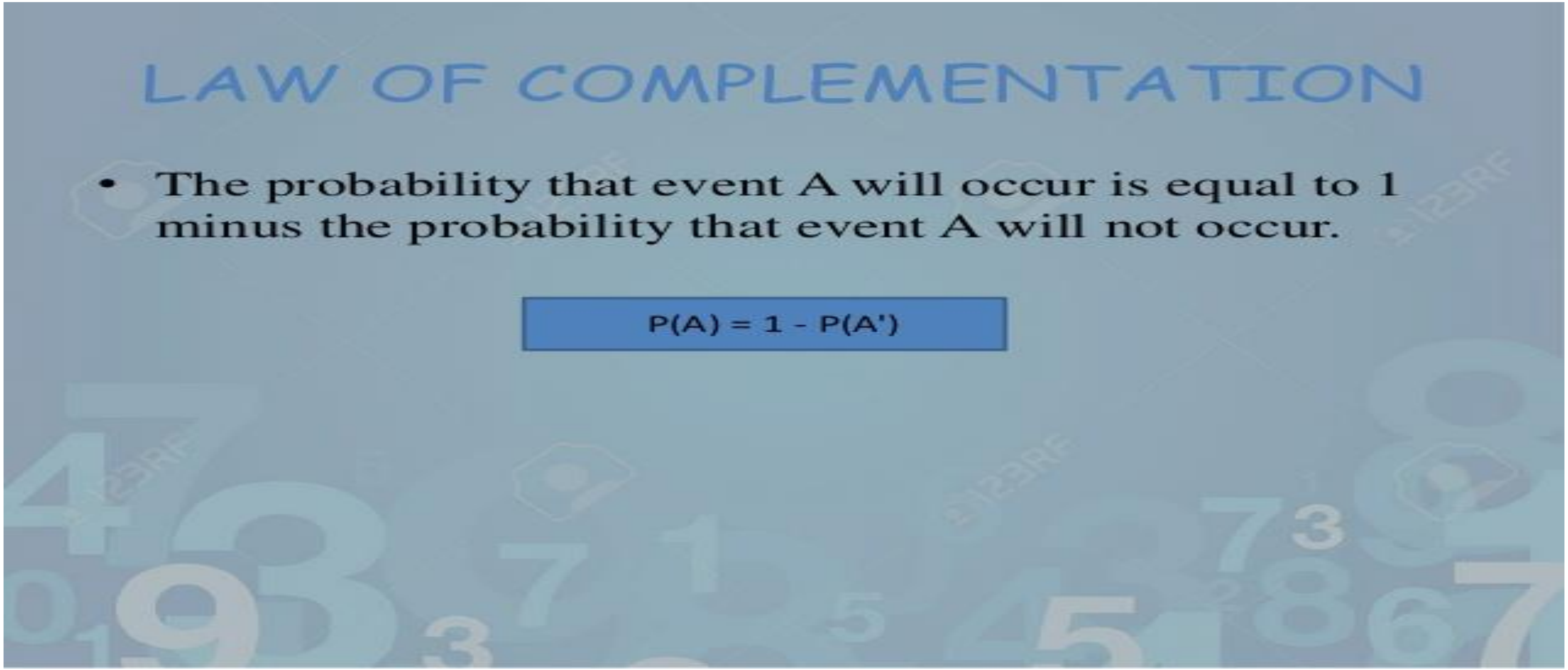
- The probability that Event A or Event B occurs is equal to the probability that Event A occurs plus the probability that Event B occurs minus the probability that both Events A and B occur.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

for mutually exclusive events:

$$P(A \cup B) = P(A) + P(B)$$

LAW OF COMPLEMENTATION



Types of Theoretical Distribution

- Binomial Distribution
- Poisson distribution
- Normal distribution or Expected Frequency distribution

Binomial Distribution:

- The prefix ‘Bi’ means two or twice. A binomial distribution can be understood as the probability of a trail with two and only two outcomes.
- It is a type of distribution that has two different outcomes namely, ‘success’ and ‘failure’. Also, it is applicable to discrete random variables only.
- Thus, the binomial distribution summarized the number of trials, survey or experiment conducted. It is very useful when each outcome has an equal chance of attaining a particular value.
- The binomial distribution has some assumptions which show that there is only one outcome and this outcome has an equal chance of occurrence.

Binomial Distribution

The **three different criteria** of binomial distributions are:

- *The number of the trial or the experiment must be fixed*.
- *Every trial is independent*. None of your trials should affect the possibility of the next trial.
- *The probability always stays the same and equal*. The probability of success may be equal for more than one trial.

Binomial distribution

- The formula for the binomial is given as

$$p(x) = \frac{N!}{x!(N-x)!} p^x q^{(N-x)}$$

where *N* = the size of the sample, *p* = the probability of a successful outcome, *q* = 1 - *p*, and *x* = the number of "successes" in question.

Poisson Distribution

- The Poisson Distribution is a theoretical discrete probability distribution that is very useful in situations where the events occur in a continuous manner.
- Poisson Distribution is utilized to determine the probability of exactly x_0 number of successes taking place in unit time.
- At first, we divide the time into n number of small intervals, such that $n \to \infty$ and p denote the probability of success, as we have already divided the time into infinitely small intervals so $p \to 0$. So the result must be that in that condition is n x $p = \lambda$ *(a finite constant).*

Properties of Poisson Model :

- The event or success is something that can be counted in whole numbers.
- The probability of having success in a time interval is independent of any of its previous occurrence.
- The average frequency of successes in a unit time interval is known.
- The probability of more than one success in unit time is very low.

Poisson Distribution

- Poisson distribution

The distribution that we seek would tell us the probability of 0, 1, 2, 3, ... calls per day. This probability is given by the Poisson distribution as

$$p(x) = \frac{e^{-\mu}\mu^{x}}{x!}$$

Normal Distribution

- The Normal Distribution defines a probability density function *f(x)* for the continuous random variable *X* considered in the system.
- The random variables which follow the normal distribution are ones whose values can assume any known value in a given range.
- We can hence extend the range to $-\infty$ to $+\infty$. Continuous Variables are such random variables and thus, the Normal Distribution gives you the probability of your value being in a particular range for a given trial.

Normal Distribution

- The normal distribution is very important in the statistical analysis due to the central limit theorem.
- The theorem states that any distribution becomes normally distributed when the number of variables is sufficiently large.
- For instance, the binomial distribution tends to change into the normal distribution with mean and variance.

Normal Distribution :

Normal Distribution Formula

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

$\mu =$ mean of x

$\sigma =$ standard deviation of x

$\pi \approx 3.14159 \ldots$

$e \approx 2.71828 \ldots$

Properties of Normal Distribution :

- Its shape is symmetric.
- The mean and median are the same and lie in the middle of the distribution
- Its standard deviation measures the distance on the distribution from the mean to the inflection point (the place where the curve changes from an “upside-down-bowl” shape to a “right-side-up-bowl” shape).
- Because of its unique bell shape, probabilities for the normal distribution follow the Empirical Rule, which says the following:
- About 68 percent of its values lie within one standard deviation of the mean. To find this range, take the value of the standard deviation, then find the mean plus this amount, and the mean minus this amount.
- About 95 percent of its values lie within two standard deviations of the mean.
- Almost all of its values lie within three standard deviations of the mean.

References

- https://www.slideshare.net/silashah12/probability-49638034