Title of the paper
Mathematics for Economists

Unit –I

Matrix-types- matrix operations – determinants- properties – solving equations using matrix-inverse of matrix- Cramer's rule (problems only- not exceeding 3x3 matrix.)

DEFINITION

Any rectangular arrangement of numbers (real or complex) (or of real valued or complex valued expressions) is called a **matrix**. If a matrix has m rows and n columns then the **order** of matrix is said to be m by n (denoted as m × n).

The general m × n matrix is $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots a_{1j} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots a_{2j} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_{i1} & a_{i2} & a_{i3} & \dots a_{ij} & \dots & a_{in} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_{mi} & a_{m2} & a_{m2} & \dots a_{mj} & \dots & a_{mn} \end{bmatrix}$

where a_{ij} denote the element of i^{th} row & j^{th} column. The above matrix is usually denoted as $[a_{ij}]_{m \times n}$.

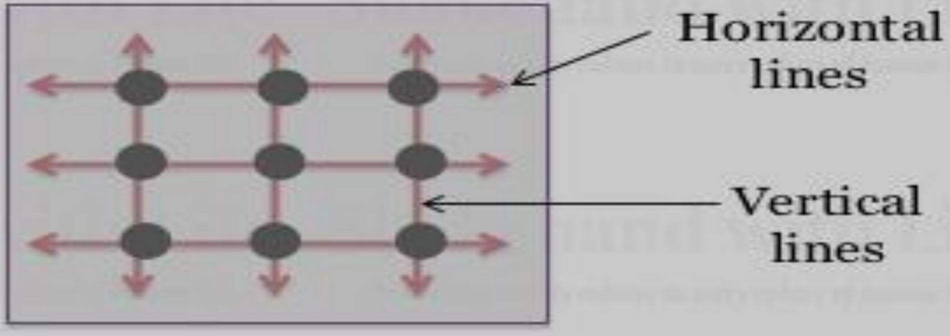
Note :

i. The elements a_{11}, a_{22}, a_{33},........ are called as **diagonal elements**. Their sum is called as **trace of A** denoted as $T_r(A)$.

ii. Capital letters of English alphabets are used to denote a matrix.

A matrix is a rectangular array of numbers. In other words, a set of 'm', 'n' numbers arranged in the form of rectangular array of m rows and n column is called $m \times n$ matrix read as 'm' by 'n' matrix.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \dots\dots\dots \\ a_{21} & a_{22} & a_{23} \dots\dots\dots \\ a_{31} & a_{32} & a_{33} \dots\dots\dots \end{bmatrix}_{m \times n}$$



We can understand matrix by the fig. shown above, which shows a mess of vertical and horizontal lines. The crossing points of vertical and horizontal lines are the position of elements of matrix. In above fig there are 9 crossing points which can be find out by multiplying no. of horizontal and vertical lines i.e. $3 \times 3 = 9$.

The numbers a_{11}, a_{12} ....etc are called elements of the matrix A. 'm' is the numbers of rows and 'n' is number of column.

e.g. $$A=\begin{bmatrix} 2 & 3 & 4 \\ 5 & 4 & 6 \\ 2 & 4 & 3 \end{bmatrix}_{3\times 3}$$

Notation of Matrices

- Matrices are denoted by capital letters A, B, C or X , Y , Z etc.
- Its elements are denoted by small letters a, b, c ..... etc.
- The elements of the matrix are enclosed by any of the brackets i.e. [ ], ( ), { }.
- The position of the elements of a Matrix is indicated by the subscripts attached to the element. e.g. a_{13} indicates that element 'a' lies in first row and third column i.e. first subscript denotes row and second subscript denote column.

Order of Matrices

- The number of rows and columns of a matrix determines the order of the matrix.
- Hence , a matrix , having m rows and n columns is said to be of the order $m \times n$ (read as 'm' by 'n').
- In particular, a matrix having 3 rows and 4 columns is of the order 3×4 and it is called a 3×4 matrix e.g.

- $A = \begin{bmatrix} 2 & 3 & 5 \\ 4 & 6 & 7 \end{bmatrix}$ is a matrix of order 2×3 since there are two rows and three columns.

- $A = \begin{bmatrix} 3 & 5 & 7 \end{bmatrix}$ is a matrix of order 1×3 since there are one row and three columns.

- $A = \begin{bmatrix} 5 \\ 6 \\ 8 \end{bmatrix}$ is a matrix of order 3×1 since there are three rows and one column.

Rectangular Matrix : A matrix in which the number of rows and columns are not equal is called a rectangular matrix e.g. ,

$$A=\begin{bmatrix} 2 & 4 & 5 \\ 4 & 6 & 7 \end{bmatrix} 2 \times 3 \text{ is rectangular matrix of order } 2\times3$$

Square Matrix : A matrix in which the number of rows is equal to the number of columns is called a square matrix e.g.

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 2 & 5 \\ 7 & 8 & 5 \end{bmatrix}$$

Principal Diagonal

Note : The elements 2, 2 , 5 in the above matrix are called diagonal elements and the line along which they lie is called the principal diagonal

- **Diagonal Matrix :** A square matrix in which all diagonal elements are non- zero and all non-diagonal elements are zeros is called a diagonal matrix e.g.

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{bmatrix} \text{ is a diagonal matrix of } 3 \times 3$$

- **Scalar Matrix :** A diagonal matrix in which diagonal elements are equal (but not equal to 1), is called a scalar matrix e.g.

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \text{ is a scalar matrix of } 3 \times 3$$

- **Identity (or Unit) Matrix :** A square matrix whose each diagonal element is unity and all other elements are zero is called and Identity (or Unit) Matrix. An Identity matrix of order 3 is denoted by I_3 or simply by I.

e.g.

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ is a unit matrix of order } 3$$

- **Null (Zero) Matrix :** A matrix of any order (rectangular or square) whose each of its element is zero is called a null matrix (or a Zero matrix) and is denoted by O. e.g.

$O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ and $O = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ are null matrices of order 2×2 and 2×3 respectively.

- **Row Matrix :** A matrix having only one row and any number of columns is called a row matrix (or a row vector) e.g.

$$A = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \text{ is a row matrix of order } 1 \times 3$$

- **<u>Column Matrix :</u>** A matrix having only one column and any number of rows is called a column matrix (or a column vector) e.g.

$$A = \begin{bmatrix} 1 \\ 3 \\ 6 \end{bmatrix} \text{ is a column matrix or order } 3 \times 1$$

- **<u>Upper Triangular and Lower Triangular Matrix</u>:** A square matrix is called an upper triangular matrix if all the elements below the principal diagonal are zero and it is said to be lower triangular matrix if all the elements above the principal diagonal are zero e.g.

UTM $A = \begin{bmatrix} 2 & 3 & 4 \\ 0 & 1 & 5 \\ 0 & 0 & 7 \end{bmatrix}$, LTM $B = \begin{bmatrix} 5 & 0 & 0 \\ 8 & 7 & 0 \\ 4 & 3 & 1 \end{bmatrix}$

- **<u>Sub Matrix :</u>** A matrix obtained by deleting some rows or column or both of a given matrix is called its sub matrix. e.g.

$A = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 1 & 6 \\ 3 & 9 & 7 \end{bmatrix}$, Now $\begin{bmatrix} 2 & 3 \\ 3 & 9 \end{bmatrix}$ is a sub matrix of given matrix A. The sub matrix obtained by deleting 2^{nd} row and 3^{rd} column of matrix A.

Trace of Matrix

The sum of all the elements on the principal diagonal of a square matrix is called the trace of the matrix. It is denoted by **tr. A** .

If $A=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ then

trace of A = tr.A = $a_{11} + a_{22} + a_{33}$

e.g. $A = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 1 & 5 \\ 3 & 9 & 7 \end{bmatrix}$, then

trace of A = tr. A =2+1+7=10

Equality of Two Matrices

Two matrices 'A' and 'B' are said to be equal if only if they are of the same order and each element of 'A' is equal to the corresponding element of 'B'. Given

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13}\ldots\ldots \\ a_{21} & a_{22} & a_{23}\ldots\ldots \\ a_{31} & a_{32} & a_{33}\ldots\ldots \end{bmatrix} m\times n \text{ and } B = \begin{bmatrix} b_{11} & b_{12} & b_{13}\ldots\ldots \\ b_{21} & b_{22} & b_{23}\ldots\ldots \\ b_{31} & b_{32} & b_{33}\ldots\ldots \end{bmatrix} x\times y$$

The above two matrices will be equal if and only if

- ✓ No. of rows and column of A and B are equal i.e. m=x and n=y.
- ✓ Each element of A is equal to the corresponding element of B i.e. $a_{11}=b_{11}$, $a_{12} = b_{12}$, $a_{13} = b_{13}$ ……… and so on

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$, according to the condition of equality of matrices both matrices have same order i.e. 2×2 but the 2nd necessary condition is a=3, b=4, c=5 , d= 6

TYPES

Types of matrices

- Row matrix- a matrix of order having only one row and n columns.

  For ex-   [1 2 3 6]

- Column matrix-a matrix of order having only one column and m rows.

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Types cont.

- Square matrix- If the numbers of rows of a matrix is equal to the numbers of its columns.

$$\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$$

- Transpose matrix- Let A be an m x n matrix. The transpose of A, denoted by A^T or A’ is an n x m matrix, which is obtained by interchanging rows and columns of A.

TYPES CONT.

- Scalar matrix- A diagonal matrix in which all the diagonal elements are equal is called scalar matrix.

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

- Diagonal matrix- In a square matrix in which all the non-diagonal elements are zero is called diagonal matrix

$$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

types

Types of Matrices

- Triangular Matrix: If every element above or below the diagonal is zero, the matrix is said to be a triangular matrix.

$$\begin{bmatrix} 1 & 4 & 3 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix} \quad \text{Upper Triangular Matrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 3 & 2 & 0 \\ 5 & -6 & 3 \end{bmatrix} \quad \text{Lower Triangular Matrix}$$

TYPES CONT.

- Singular matrix – a square matrix A is said to be a singular matrix if its determinant is equal to zero . $|A| = 0$

- Non-singular matrix- a square matrix A is said to be a non- singular matrix if its determinant is not equal to zero.$|A| \neq 0$

TYPES CONT..

- Zero matrix or null matrix- if all the elements of a matrix are zero is called a zero matrix.

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

- Identity matrix- a square matrix in which all the diagonal elements are one and all the non-diagonal elements are zero.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

TYPES CONT.

- Equality of matrices- two matrices A and B are said to be equal, if they are of the same order and their corresponding elements are equal.

- Sub matrix- in a given matrix, by deleting a few rows and columns, we a new matrix called the sub-matrix.

Equality of Matrices

- Two matrices A & B are said to be equal iff:
  i. A and B are of the same order
  ii. All the elements of A are equal as that of corresponding elements of B
- Two matrices A = [aij] & B = [bij] of the same order are said to be equal if aij = bij

If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $\quad B = \begin{bmatrix} x & y \\ z & w \end{bmatrix}$

If A & B are equal, then

$x=1, y=2, z=3, w=4$

Trace of a Matrix

- In a square matrix A, the sum of all the diagonal elements is called the trace of A. It is denoted by tr A.
  - Ex: If $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 7 & 6 & 1 \end{bmatrix}$ tr A = 1+4+1 = 6
  - Ex: If $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ tr B = 1+4 = 5

Operations on Matrices

- Addition/Subtraction
- Scalar Multiplication
- Matrix Multiplication

Addition and Subtraction

- Two matrices may be added (or subtracted) iff they are the same order.

- Simply add (or subtract) the corresponding elements. So, **A + B = C**

Addition / Subtraction (Problems for Practice)

Q1: If $A = \begin{bmatrix} 1 & 3 & 4 \\ -2 & 4 & 8 \\ 3 & -2 & -1 \end{bmatrix}$ and B $\begin{bmatrix} 4 & 1 & 0 \\ 1 & 3 & 5 \\ 0 & 1 & 6 \end{bmatrix}$

find A+B, A-B.

Q2: If $A = \begin{bmatrix} 3 & 8 & 11 \\ 6 & -3 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -6 & 15 \\ 3 & 8 & 17 \end{bmatrix}$

find A+B, A-B.

Scalar Multiplication

- To multiply a scalar times a matrix, simply multiply each element of the matrix by the scalar quantity

$$k\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} ka_{11} & ka_{12} \\ ka_{21} & ka_{22} \end{bmatrix}$$

- Ex: If $A = \begin{bmatrix} 3 & 8 & 11 \\ 6 & -3 & 8 \end{bmatrix}$, then

$$10A = \begin{bmatrix} 30 & 80 & 110 \\ 60 & -30 & 80 \end{bmatrix}$$

Matrix Multiplication

Example: $A=\begin{bmatrix}1 & 2 & 3\\ 0 & 1 & 4\end{bmatrix}$, $B=\begin{bmatrix}-1 & 2\\ 2 & 3\\ 5 & 0\end{bmatrix}$, Evaluate $C = AB$.

$$\begin{bmatrix}1 & 2 & 3\\ 0 & 1 & 4\end{bmatrix}\begin{bmatrix}-1 & 2\\ 2 & 3\\ 5 & 0\end{bmatrix} \Rightarrow \begin{cases} c_{11} = 1\times(-1)+2\times 2+3\times 5 = 18\\ c_{12} = 1\times 2+2\times 3+3\times 0 = 8\\ c_{21} = 0\times(-1)+1\times 2+4\times 5 = 22\\ c_{22} = 0\times 2+1\times 3+4\times 0 = 3\end{cases}$$

$$C = AB = \begin{bmatrix}1 & 2 & 3\\ 0 & 1 & 4\end{bmatrix}\begin{bmatrix}-1 & 2\\ 2 & 3\\ 5 & 0\end{bmatrix} = \begin{bmatrix}18 & 8\\ 22 & 3\end{bmatrix}$$

Transpose of a Matrix

- The matrix obtained by interchanging the rows and columns of a matrix A is called the transpose of A (written as A^T *or* $A`$).

Example: $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

The transpose of A is $A^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$

- For a matrix $A = [a_{ij}]$, its transpose $A^T = [b_{ij}]$, where $b_{ij} = a_{ji}$.

1.5 Determinants

Determinant of order 2

Consider a 2×2 matrix: $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

- Determinant of A, denoted $|A|$, is a number and can be evaluated by

$$|A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

1.5 Determinants

Determinant of order 2

- easy to remember (for order 2 only)..

$$|A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = +a_{11}a_{22} - a_{12}a_{21}$$

- +

Example: Evaluate the determinant: $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$

$$\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = 1 \times 4 - 2 \times 3 = -2$$

1.5 Determinants of order 3

Consider an example: $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

Its determinant can be obtained by:

$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = 3\begin{vmatrix} 4 & 5 \\ 7 & 8 \end{vmatrix} - 6\begin{vmatrix} 1 & 2 \\ 7 & 8 \end{vmatrix} + 9\begin{vmatrix} 1 & 2 \\ 4 & 5 \end{vmatrix}$$

$$= 3(-3) - 6(-6) + 9(-3) = 0$$

You are encouraged to find the determinant by using other rows or columns

1.5 Determinants

The following properties are true for determinants of *any* order.

1. If every element of a row (column) is zero, e.g., $\begin{vmatrix} 1 & 2 \\ 0 & 0 \end{vmatrix} = 1\times 0 - 2\times 0 = 0$, then $|A| = 0$.

2. $|A^T| = |A|$ ← determinant of a matrix = that of its transpose

3. $|AB| = |A||B|$

Example - Applying Cramer's Rule on a System of Two Equations

Solve the system:

- 8x+5y= 2
- 2x-4y= -10

The coefficient matrix is: $\begin{bmatrix} 8 & 5 \\ 2 & -4 \end{bmatrix}$ and $\begin{vmatrix} 8 & 5 \\ 2 & -4 \end{vmatrix} = (-32) - (10) = -42$

So:

$$x = \frac{\begin{vmatrix} 2 & 5 \\ -10 & -4 \end{vmatrix}}{-42}$$ and $$y = \frac{\begin{vmatrix} 8 & 2 \\ 2 & -10 \end{vmatrix}}{-42}$$

$$x = \frac{\begin{vmatrix} 2 & 5 \\ -10 & -4 \end{vmatrix}}{-42} = \frac{-8-(-50)}{-42} = \frac{42}{-42} = -1$$

$$y = \frac{\begin{vmatrix} 8 & 2 \\ 2 & -10 \end{vmatrix}}{-42} = \frac{-80-4}{-42} = \frac{-84}{-42} = 2$$

Solution: (-1,2)

Applying Cramer's Rule on a System of Two Equations

$$\begin{cases} ax+by=e \\ cx+dy=f \end{cases}$$

$$D=\begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

$$D_x=\begin{vmatrix} e & b \\ f & d \end{vmatrix}$$

$$D_y=\begin{vmatrix} a & e \\ c & f \end{vmatrix}$$

$$x=\frac{D_x}{D} \quad y=\frac{D_y}{D}$$

$$\begin{cases} 2x-3y=-16 \\ 3x+5y=14 \end{cases}$$

$$D=\begin{vmatrix} 2 & -3 \\ 3 & 5 \end{vmatrix}=(2)(5)-(-3)(3)=10+9=19$$

$$D_x=\begin{vmatrix} -16 & -3 \\ 14 & 5 \end{vmatrix}=(-16)(5)-(-3)(14)=-80+42=-38$$

$$D_y=\begin{vmatrix} 2 & -16 \\ 3 & 14 \end{vmatrix}=(2)(14)-(3)(-16)=28+48=76$$

$$x=\frac{D_x}{D}=\frac{-38}{19}=-2 \quad y=\frac{D_y}{D}=\frac{76}{19}=4$$

EVALUATING A 3X3 DETERMINANT
(EXPANDING ALONG THE TOP ROW)

- Expanding by Minors (little 2x2 determinants)

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1\begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1\begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 3 & -2 \\ 2 & 0 & 3 \\ 1 & 2 & 3 \end{vmatrix} = (1)\begin{vmatrix} 0 & 3 \\ 2 & 3 \end{vmatrix} - (3)\begin{vmatrix} 2 & 3 \\ 1 & 3 \end{vmatrix} + (-2)\begin{vmatrix} 2 & 0 \\ 1 & 2 \end{vmatrix}$$

$$= (1)(-6) \ - (3)(3) \ + (-2)(4)$$

$$= -6 \ -9 \ -8 = -23$$

Example 1

Consider the following equations:

$$2x_1 - 4x_2 + 5x_3 = 36$$

$$-3x_1 + 5x_2 + 7x_3 = 7$$

$$5x_1 + 3x_2 - 8x_3 = -31$$

$$[A][x] = [B]$$

where

$$[A] = \begin{bmatrix} 2 & -4 & 5 \\ -3 & 5 & 7 \\ 5 & 3 & -8 \end{bmatrix}$$

Example 1

$$[x] = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \; and \; [B] = \begin{bmatrix} 36 \\ 7 \\ -31 \end{bmatrix}$$

$$D = \begin{vmatrix} 2 & -4 & 5 \\ -3 & 5 & 7 \\ 5 & 3 & -8 \end{vmatrix} = -336$$

$$D_1 = \begin{vmatrix} 36 & -4 & 5 \\ 7 & 5 & 7 \\ -31 & 3 & -8 \end{vmatrix} = -672$$

Example 1

$$D_2 = \begin{vmatrix} 2 & 36 & 5 \\ -3 & 7 & 7 \\ 5 & -31 & -8 \end{vmatrix} = 1008$$

$$D_3 = \begin{vmatrix} 2 & -4 & 36 \\ -3 & 5 & 7 \\ 5 & 3 & -31 \end{vmatrix} = -1344$$

$$x_1 = \frac{D_1}{D} = \frac{-672}{-336} = 2$$

$$x_2 = \frac{D_2}{D} = \frac{1008}{-336} = -3$$

$$x_3 = \frac{D_3}{D} = \frac{-1344}{-336} = 4$$

Example 1

- Solve the system : $3x - 2y + z = 9$
- $x + 2y - 2z = -5$

$x + y - 4z = -2$

$$x = \frac{\begin{vmatrix} 9 & -2 & 1 \\ -5 & 2 & -2 \\ -2 & 1 & -4 \end{vmatrix}}{\begin{vmatrix} 3 & -2 & 1 \\ 1 & 2 & -2 \\ 1 & 1 & -4 \end{vmatrix}} = \frac{-23}{-23} = 1$$

$$y = \frac{\begin{vmatrix} 3 & 9 & 1 \\ 1 & -5 & -2 \\ 1 & -2 & -4 \end{vmatrix}}{\begin{vmatrix} 3 & -2 & 1 \\ 1 & 2 & -2 \\ 1 & 1 & -4 \end{vmatrix}} = \frac{69}{-23} = -3$$

Example 1

$$z = \frac{\begin{vmatrix} 3 & -2 & 9 \\ 1 & 2 & -5 \\ 1 & 1 & -2 \end{vmatrix}}{\begin{vmatrix} 3 & -2 & 1 \\ 1 & 2 & -2 \\ 1 & 1 & -4 \end{vmatrix}} = \frac{0}{-23} = 0$$

The solution is

(1, -3, 0)

Using Equal Matrices

Equal matrices can be used to solve for variables.

$$\begin{bmatrix} 3 & 4 \\ z^2 & 3 \\ -7 & 4 \end{bmatrix} = \begin{bmatrix} x & y+2 \\ 9 & 3 \\ -7 & 4 \end{bmatrix}$$

Set up each equation separately with corresponding entries

$3 = x$

$4 = y + 2$ ($y = 2$)

$z^2 = 9$ ($z = +3$ or -3)

Try these

$$\begin{bmatrix} 0 & 6y \\ -5x+5 & w^2 \end{bmatrix} = \begin{bmatrix} x & 3 \\ z & 49 \end{bmatrix}$$

$$\begin{bmatrix} p+2 \\ r^2+4 \\ q \end{bmatrix} = \begin{bmatrix} -5 \\ 29 \\ 2+p \end{bmatrix}$$

$$\begin{bmatrix} 0 & 6y \\ -5x+5 & w^2 \end{bmatrix} = \begin{bmatrix} x & 3 \\ z & 49 \end{bmatrix}$$

$$x = 0;\ y = \frac{1}{2};\ z = 5;\ w = 7 \text{ or } w = -7$$

$$\begin{bmatrix} p+2 \\ r^2+4 \\ q \end{bmatrix} = \begin{bmatrix} -5 \\ 29 \\ 2+p \end{bmatrix}$$

$$p = -7;\ r = 5 \text{ or } r = -5;\ q = -3$$

Matrices - Operations

Properties of transposed matrices:

1. $(\mathbf{A}+\mathbf{B})^T = \mathbf{A}^T + \mathbf{B}^T$
2. $(\mathbf{AB})^T = \mathbf{B}^T \mathbf{A}^T$
3. $(k\mathbf{A})^T = k\mathbf{A}^T$
4. $(\mathbf{A}^T)^T = \mathbf{A}$

Matrices - Operations

1. $(\mathbf{A}+\mathbf{B})^T = \mathbf{A}^T + \mathbf{B}^T$

$$\begin{bmatrix} 7 & 3 & -1 \\ 2 & -5 & 6 \end{bmatrix} + \begin{bmatrix} 1 & 5 & 6 \\ -4 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 8 & 8 & 5 \\ -2 & -7 & 9 \end{bmatrix} \longrightarrow \begin{bmatrix} 8 & -2 \\ 8 & -7 \\ 5 & 9 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 2 \\ 3 & -5 \\ -1 & 6 \end{bmatrix} + \begin{bmatrix} 1 & -4 \\ 5 & -2 \\ 6 & 3 \end{bmatrix} = \begin{bmatrix} 8 & -2 \\ 8 & -7 \\ 5 & 9 \end{bmatrix}$$

References

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