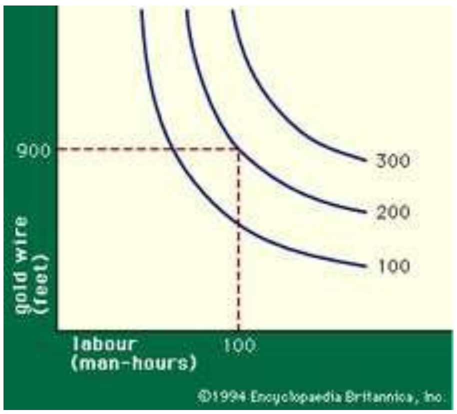
UNIT – III

Production function

Production function, in economics, equation that expresses the relationship between the quantities of productive factors (such as labour and capital) used and the amount of product obtained. It states the amount of product that can be obtained from every combination of factors, assuming that the most efficient available methods of production are used.



The production function can thus answer a variety of questions. It can, for example, measure the marginal productivity of a particular factor of production (*i.e.*, the change in output from one additional unit of that factor). It can also be used to determine the cheapest combination of productive factors that can be used to produce a given output.

Linear Homogeneous Production Function

Definition: The **Linear Homogeneous Production Function** implies that with the proportionate change in all the factors of production, the output also increases in the same proportion. Such as, if the input factors are doubled the output also gets doubled. This is also known as **constant returns to a scale.**

The production function is said to be homogeneous when the elasticity of substitution is equal to one. The linear homogeneous production function can be used in the empirical studies because it can be handled wisely. That is why it is widely used in linear programming and input-output analysis. This production function can be shown symbolically:

nP = f(nK, nL)

Where, n = number of times
nP = number of times the output is increased
nK= number of times the capital is increased
nL = number of times the labor is increased

Thus, with the increase in labor and capital by “n” times the output also increases in the same proportion. The concept of linear homogeneous production function can be further comprehended through the illustration given below:

In the case of a linear homogeneous production function, the expansion is always a straight line through the origin, as shown in the figure. This means that the proportions between the factors used will always be the same irrespective of the output levels, provided the factor prices remains constant.

COBB – DOUGLAS PRODUCTION FUNCTION

Development of this production function started in the 1920s when Paul Douglas calculated estimates for **production factors** for **labor** (workers) and **capital** (here in a broad sense: money, buildings, machines). He wanted to show how they relate to each other and he wished to express this relation as a mathematical function. Charles Cobb suggested using an existing production function equation proposed by Kurt Wicksell as a base, which Douglas and Cobb improved and expanded upon. The results they got very closely reflected American macroeconomic data at the time. Although accurate, economists criticized the results for using sparse data. Even when conducting small scale research you need a proper sample size to make your results statistically significant. It is even more important when you want to try to estimate industry-wide macroeconomic theories. Over the years, the theory was improved and expanded using US census data, and proved accurate for other countries as well. Paul Douglas formally presented the results in 1947.

The Cobb-Douglas production function is known for being the first time a proper aggregate production function was estimated and developed to be accurately used to analyze whole branches of industry. It was a cornerstone for macroeconomics, and has been widely used, adopted and improved since its inception. The importance of Cobb-Douglas production function to macroeconomics can be compared to the importance of the Pythagorean theorem to math.

Production function formula (Cobb-Douglas)

The Cobb-Douglas production function formula for a single good with two factors of production is expressed as following:

$Y = A * L^{\beta} * K^{\alpha}$,

this production function equation is the basis of our Cobb-Douglas production function calculator, where:

- Y is the **total production** or **output** of goods.
- A is the **total factor productivity**. It is a positive constant, and is used to show the change in output that is not the result of main **production factors**.
- L is the **labor** input which indicates the total number of labor that went into production.
- K is the **capital** input which shows the quantity of capital that was used during production.
- α is the **output elasticity** of capital.
- β is the **output elasticity** of labor.

Output elasticity is the responsiveness of total production quantities to changes in quantities of a production factor. It is a percentage change in total production resulting from a percentage change in a factor. The more capital or labor we use, the more of a good we are going to get, but it is not a one to one conversion. It means that a 1 percent change in either factor would not result in a 1% change in total production, but is rather dependent on the level of output elasticity associated with the factor. Each of these values is a **positive constant** no bigger than 1 and is dependent on the level of available technology ($0 <= \alpha <= 1, 0 <= \beta <= 1$). In practice they have to be smaller than 1 because a perfect production process does not exist - inefficiencies in labor and capital occur. Output elasticities can be found using historic production data for an industry. Suppose that output elasticity for labor - β is equal to 0.3. A 1% increase in labor would equal to approximately a 0.3% increase in total production in that case.

Cobb-Douglas production function characteristics

Now that you know a little more about Cobb-Douglas production function, its history and the main components, it is time to move on to the Cobb-Douglas production function characteristics:

- **output elasticity**, as mentioned above, is constant. It means that for a given Cobb-Douglas production function for a specific industry the value of α (output elasticity of capital), and β (output elasticity of labor) should not change.

- **marginal product** represents additional quantities of **output** we get by increasing the amount of a production factor used by a unit. In case of Cobb-Douglas production function, the marginal product is **positive** and **decreasing**. It happens because output elasticity is positive. It is, however, smaller than one, so the Cobb-Douglas production function has **diminishing marginal returns**. It means that while increases in capital or labor will result in increased **total production**, each time the increase will be a bit smaller than before.
- **returns to scale** represent the proportional change in **output** when the proportional change is the same in all factors. For Cobb-Douglas production function returns to scale are equal to output elasticities of both labor and capital: $\alpha + \beta$.

If $\alpha + \beta = 1$ you can say that the returns to scale are **constant**. It means that doubling the amount of both capital and labor would result in double the output. With the United States industry data available, this is what Paul Douglas observed when he was first establishing the function.

Optimal Combination of Resources

The optimal combination of resources is achieved through cost-minimization and profit-maximization.

The Least-Cost Rule

A firm is producing a specific output with the least-cost combination of resources when the last dollar spent on each resource yields the same marginal product.

Marginal product Marginal product
of labor (MPL) = of capital (MPc)
Price of Labor (PL) Price of Capital (Pc)

The Profit-Maximizing Rule

In competitive markets, a firm will achieve its profit-maximizing combination of resources when each resource is employed to the point at which its marginal revenue product equals its resource price.
MRPL/PL = MRPC/PC = 1 (*ratios must equal 1)

* The profit maximizing equation hold the premise that the firm is also using the least cost combination. However, a firm operating at least cost combination may not be operating at the output that maximizes its profits.

Cost Minimisation for a Given Output and Output-Maximisation for a Given Cost

Cost Minimisation for a Given Output and Output-Maximisation for a Given Cost!

Cost Minimisation for a Given Output:

In the theory of production, the profit maximisation firm is in equilibrium when, given the cost- price function, it maximises its profits on the basis of the least cost combination of factors. For this, it will choose that combination which minimises its cost of production for a given output. This will be the optimal combination for it.

Assumptions:

This analysis is based on the following assumptions:

1. There are two factors, labour and capital.

2. All units of labour and capital are homogeneous.

3. The prices of units of labour (w) and that of capital (r) are given and constant.

4. The cost outlay is given.

5. The firm produces a single product.

6. The price of the product is given and constant.

7. The firm aims at profit maximisation.

8. There is perfect competition in the factor market.

Explanation:

Given these assumptions, the point of least-cost combination of factors for a given level of output is where the isoquant curve is tangent to an isocost line. In Figure 15, the isocost line GH is tangent to the isoquant 200 at point M. The firm employs the combination of OC of capital and OL of labour to produce 200 units of output at point M with the given cost-outlay GH.

At this point, the firm is minimising its cost for producing 200 units. Any other combination on the isoquant 200, such as R or T, is on the higher isocost line KP which shows higher cost of production. The isocost line EF shows lower cost but output 200 cannot be attained with it. Therefore, the firm will choose the minimum cost point M which is the least-cost factor combination for producing 200 units of output. M is thus the optimal combination for the firm.

The point of tangency between the isocost line and the isoquant is an important first order condition but not a necessary condition for the producer's equilibrium.

There are two essential К or second order conditions for the equilibrium of the firm:

1. The first condition is that the slope of the isocost line must equal the slope of the isoquant curve. The slope of the isocost line is equal to the ratio of the price of labour (w) and the price of capital (r). The slope of the isoquant curve is equal to the marginal rate of technical substitution of labour and capital ($MRTS_{LK}$) which is, in turn, equal to the ratio of the marginal product of labour to the marginal product of capital (MP_L/MP_K' condition for optimality can be written as.

w/r $MP_L/MP_K = MRTS_{LK}$

The second condition is that at the point of tangency, the isoquant curve must be convex to the origin. In other words, the marginal rate of technical substitution of labour for capital ($MRTS_{LK}$) must be diminishing at the point of tangency for equilibrium to be stable. In Figure 16, S cannot be the point of equilibrium for the isoquant IQ_1 is concave where it is tangent to the isocost line GH. At point S, the marginal rate of technical substitution between the two factors increases if move to the right or left on the curve IQ_1.

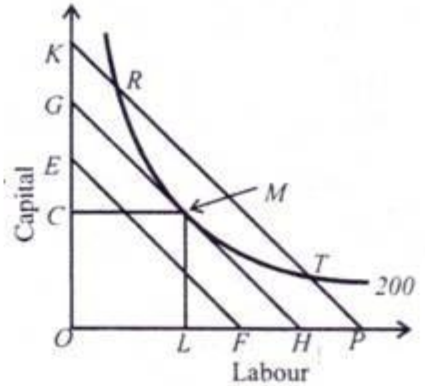


Fig. 15

Moreover, the same output level can be produced at a lower cost AB or EF and there will be a comer solution either at C or F. If it decides to produce at EF cost, it can produce the entire output with only OF labour. If, on the other hand, it decides to produce at a still lower cost CD, the entire output can be produced with only OC capital.

Both the situations are impossibilities because nothing can be produced either with only labour or only capital. Therefore, the firm can produce the same level of output at point M, where the isoquant curve IQ is convex to the origin and is tangent to the isocost line GH. The analysis assumes that both the isoquants represent equal level of output, $IQ = IQ_1$.

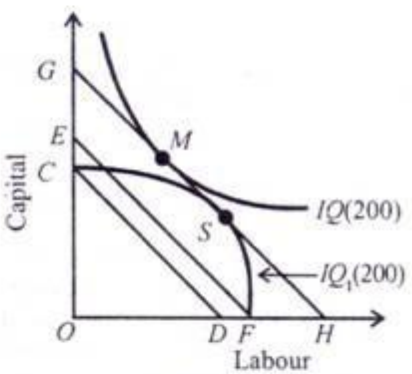


Fig. 16

Output-Maximisation for a Given Cost:

The firm also maximises its profits by maximising its output, given its cost outlay and the prices of the two factors. This analysis is based on the same assumptions, as given above. The conditions for the equilibrium of the firm are the same, as discussed above.

1. The firm is in equilibrium at point P where the isoquant curve 200 is tangent to the isocost line CL in Figure 17. At this point, the firm is maximising its output level of 200 units by employing the optimal combination of OM of capital and ON of labour, given its cost outlay CL.

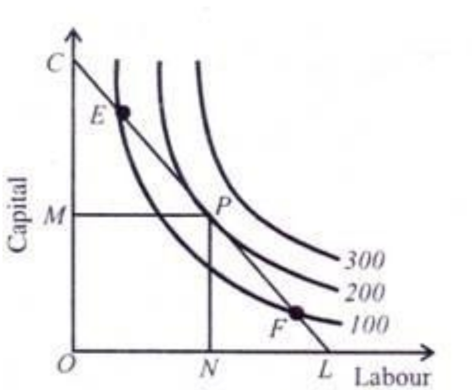


Fig. 17

But it cannot be at points E or F on the isocost line CL, since both points give a smaller quantity of output, being on the isoquant 100, than on the isoquant 200. The firm can reach the optimal factor combination level of maximum output by moving along the isocost line CL from either point E or F to point P.

This movement involves no extra cost because the firm remains on the same isocost line. The firm cannot attain a higher level of output such as isoquant 300 because of the cost constraint. Thus the equilibrium point has to be P with optimal factor combination OM+ ON. At point P, the slope of the isoquant curve 200 is equal to the slope of the isocost line CL. It implies $w/r = MP_L/MP_K = MRTS_{LK}$.

2. The second condition is that the isoquant curve must be convex to the origin at the point of tangency with the isocost line, as explained above in terms of Figure 16.

EXPANSION PATH THEORY OF PRODUCTION

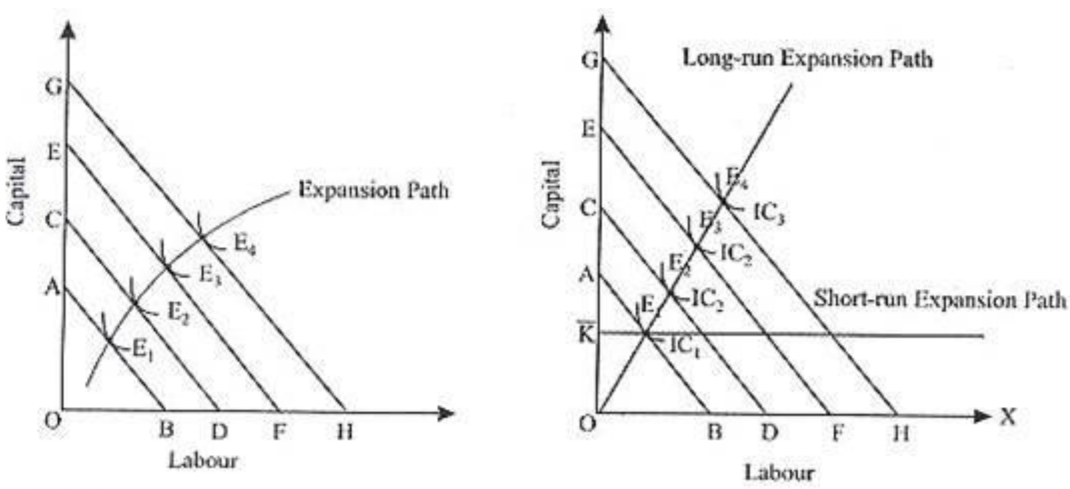
So far we have assumed away the expansion of financial resources of the firm. As the producer becomes financially well-off, he has to change the factor combinations with the expansion of his output, given the factor prices.

In Fig. 7.12, AB, CD, EF and GH are the four iso- cost lines representing different levels of total cost or outlay. All iso-cost lines are parallel to one-another indicating that prices of the two factors remain the same". E_1, E_2, E_3 and E_4 are the points of producer’s equilibrium corresponding to the point of tangencies of the above four iso- cost lines with the highest possible isoquant in each case.

The line joining the least cost combinations like E_1, E_2, E_3 and E_4 is called the expansion path. It is so called, because, it shows how for the given relative prices of the two inputs (the slope of factor price line), the optimal factor combinations with which the producer plans it output will alter as he expands the volume of output.

Expansion path may be defined as the locus of efficient combinations of the factors (the points of tangency between the isoquants and the iso-cost lines). It is the curve along which output or expenditure changes, when factor prices remain constant.

Hence, the optimal proportion of the inputs will remain unchanged. It is also known as scale-line, as it shows how the producer will change the quantities of the two factors, when it raises the scale of production.



(a) Linear Production Function

(b) Linear Homogenous Production Function

Fig. 7.12: Expansion Path

The expansion path may have different shapes and slopes depending upon the relative prices of the factors used and shape of the isoquant. In case of constant returns to scale (homogenous production function), the expansion path will be a straight line through the origin, indicating constancy of the optimal proportion of the inputs of the firm, even with changes in the size of the firm’s input budget. (Fig. 7.12 (b)). In short-run, however, the expansion path will be parallel to X-axis (when capital is hold constant at K shown in Fig. 7.12 (b)).

As expansion path depicts least cost combinations for different levels of output, it shows the cheapest way of producing each output, given the relative prices of the factors. It is difficult to tell precisely the particular point of expansion path at which the producer in fact be producing, unless one knows the output which he wants to produce or the size of the cost or outlay it wants to incur.

But, this much is certain that though for a given isoquant map, there can be different expansion paths for different relative prices of the factors. Yet, when prices of the variable factors are given, a rational producer will always try to produce at one or the other point of the expansion path.

Production Function of a Multiproduct Firm

A. The Production-Possibility Curve of the Firm:

Each product is assumed to be produced by two factors, L and K. For each product we have a production function

$x = f_1(L, K)$

$y = f_2(L, K)$

Each production function may be presented by a set of isoquants with the usual properties. We may now obtain the production-possibility curve of the firm by using the device of the Edge-worth box. We assume that the firm has total quantities of factors 0L and 0K (figure 3.45) measured along the sides of the Edge-worth box. Any point of the Edge- worth box shows a certain combination of quantities of x and y produced by the available factors of production.

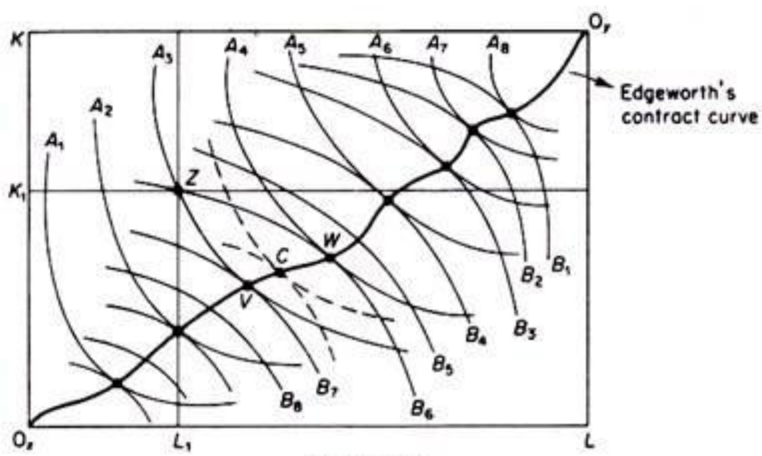


Figure 3.45

The production function for commodity x is represented by the set of isoquants denoted by A which are convex to the origin 0_X. The production function for commodity y is represented by the set of isoquants denoted by B which are convex to the origin 0_y. The further down an isoquant B lies, the higher the quantity of y it represents. The two sets of isoquants have points of tangency, which form the contract curve.

Only points lying on the contract curve are efficient, in the sense that any other point shows the use of all resources for producing a combination of outputs which includes less quantity of at least one commodity. For example, assume that initially the firm produces at point Z, at which the quantity of x is A_3 and the quantity of y is B_6. The production of the level A_3 of x absorbs 0_xL_1 of labour and 0_XK_1 of capital. The remaining resources, L_1L and K_1K, are used in the production of commodity y.

It can be shown that the firm can produce more of either x or y or of both commodities by reallocating its resources so as to move to any point between V and W on the contract curve. If the firm moves to W it will be producing the same level of y(B_6), but a higher level of x(A_4). If the firm chooses to produce at V, it will produce the same quantity of x(/l_3), but more of y(B_7).

Finally, if the firm produces at any intermediate point between V and W, for example at point C, it will attain higher levels of production of both x and y. Thus points on the contract curve are efficient in that any other point off this curve implies a smaller level of output of at least one product. The choice of the actual point on the contract curve depends on the ratio of the prices of the two commodities (see below).

To determine the choice of levels of x and y we need to derive the production-possibility curve (or product-transformation curve) of the firm. This shows the locus of points of levels of x and y which use up all the available resources of the firm. The production- possibility curve is derived from the contract curve. Each point of tangency between isoquants, that is, any one point of the contract curve, defines a combination of x and y levels of output which lies on the production-possibility curve. For example, point V, representing the output pair A_3 from x and B_7 from y, is point V on figure 3.46. Similarly point W of the contract curve is point W' on the production-possibility curve.

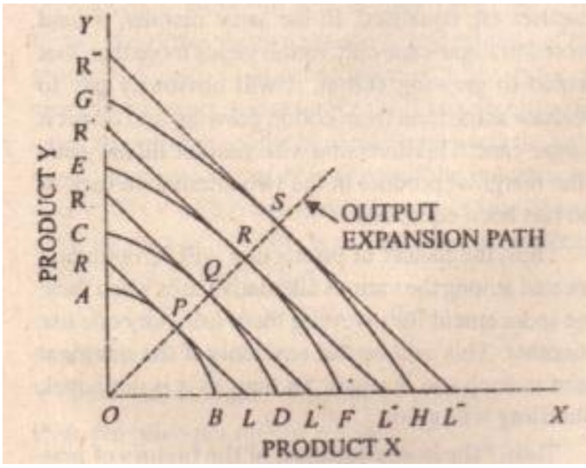
ISO – REVENUE CURVE

We know that the cortices,both human and material, at the disposal of the community are strictly limited and they are ca of alternative uses, whereas we want to price innumerable commodities, i.e., the ends are unlimited. We have, therefore, to choose the most desirable assortment of goods that we can produce with the resources that we command and a given state of technical knowledge. Had the resources at our disposal been unlimited. there ·Gould have been no problem, and we would have produced more of everything to satisfy our wants. If some resources were lying idle, then also it would have been possible to increase the production of all goods. But, in an economy characterized by full employment, some good can be produced only by foregoing the production of some other good. This is in keeping with the opportunity cost principle.

increasing quantities of Y have to be sacrificed. Hence, the marginal rate of transformation increases as more of X is produced and less of Y. This makes the production possibility curve concave in the origin. The marginal rate of transformation (MRT) at any point on the production possibility curve is given by the slope of the curve at that point.

Iso-Revenue Line

We have seen that the production possibility curve shows the various combinations of the two goods which “can be produced with given resources. The question remains as to which of these various combinations the firm will decide to produce. Which is considered the most desirable? Surely. the firm will have to decide which combination out of the so many available will be most profitable. In order to hit on the most desirable combination. we shall introduce the price factor or the revenue factor (Price paid by the purchaser



is revenue for the seller). The producer must maximizes revenue. We shall. therefore. draw the lso-Revenue line yield the same revenue. Output Expansion Path. In Fig. 17.2 RL. R'L', R' 'L"and R" 'L" 'arc the iso rrevenuer lines each showing that every point on the line represents the same revenue from the sale of the products X and Y. All is our production possibility curve at which RL is the tangent touching it at P. Similarly, R' L' touches CD a higher production possibility curve OItQ and R"touches still higher curve at Rand R' L iso-venue line touches the higher curve at S. Joining P, Q. Rand S. we get Pullout E~collusion Path. The Screenonline lire.

If the resources at the disposal of the firm are represented by the production possibility curve All. then it will choose for production the combination of X and Y represented by the point P. At this point. its revenue will be maximum. Here the marginal rate of transformation Roxy on the given production possibility curve will he equal to the peke ratio of the Q. Rand S. P. Q. R, S expansion path is the locus of all the revenue counterproductive combinations with the varying amount of resources.

where $MRTS^x_{L,K}$ = marginal rate of technical substitution of the factors K and L in the production of commodity x.

Similarly, the slope of isoquant B is

$$-\frac{\partial K}{\partial L}=\frac{MP_{L,y}}{MP_{K,y}}=MRTS^y_{L,K}$$

where $MRTS^y_{L,K}$ = marginal rate of technical substitution of the factors K and L in the production of commodity y.

At the points of tangency of isoquants on the contract curve the two slopes are equal

$$-\frac{\partial K}{\partial L}=\frac{MP_{L,x}}{MP_{K,x}}=\frac{MP_{L,y}}{MP_{K,y}}$$

The slope of the contract curve, say from V to W, can be represented also by the slope of the production-possibility curve PP, from V' to W' (in discrete terms).

The slope of the production-possibility curve is

$$-\frac{\partial y}{\partial x}=MRPT_{x,y}$$

where $MRPT_{x,y}$ = marginal rate of product transformation.

A reduction in the level of y releases factors of production

$$\partial L_y(MP_{L,y})+\partial K_y(MP_{K,y})$$

An increase in the level of x requires additional factors

$$\partial L_x(MP_{L,x})+\partial K_x(MP_{K,x})$$

If the factors are to be fully employed, then the quantities released from the decrease in y must be equal to the quantities used in increasing x. Thus

$$-\partial L_y=+\partial L_x$$
$$-\partial K_y=+\partial K_x$$

Now the total differential of the production-possibility curve, its slope, is

$$-\frac{\partial y}{\partial x}=\frac{\partial L_y(MP_{L,y})+\partial K_y(MP_{K,y})}{\partial L_x(MP_{L,x})+\partial K_x(MP_{K,x})} \tag{3.10}$$

For efficient production the firm must stay *on* the curve, not inside it. This implies that

$$\begin{bmatrix}\text{Slope of}\\ \text{isoquant } A\end{bmatrix}=\frac{MP_{L,x}}{MP_{K,x}}=\frac{MP_{L,y}}{MP_{K,y}}=\begin{bmatrix}\text{Slope of}\\ \text{isoquant } B\end{bmatrix} \tag{3.11}$$

This yields

$$MP_{L,x}=MP_{K,x}\left(\frac{MP_{L,y}}{MP_{K,y}}\right) \tag{3.12}$$

and

$$MP_{L,y}=MP_{K,y}\left(\frac{MP_{L,x}}{MP_{K,x}}\right) \tag{3.13}$$

Dividing the total differential by $\partial L_y(=-\partial L_x)$ we find

$$-\frac{\partial y}{\partial x}=\frac{MP_{L,y}+MP_{K,y}\left(\dfrac{\partial K_y}{\partial L_y}\right)}{-MP_{L,x}-MP_{K,x}\left(\dfrac{\partial K_x}{\partial L_y}\right)} \tag{3.14}$$

Substituting expressions 3.12 and 3.13 in expression 3.14 we obtain

$$-\frac{\partial y}{\partial x}=\frac{MP_{K,y}\left(\frac{MP_{L,x}}{MP_{K,x}}+\frac{\partial K_y}{\partial L_y}\right)}{-MP_{K,x}\left(\frac{MP_{L,y}}{MP_{K,y}}+\frac{\partial K_x}{\partial L_y}\right)}$$

The first terms in the brackets are equal by expression (3.11). The second terms in the brackets are also equal by the condition that $\partial K_y = \partial K_x$.
Consequently the bracketed terms cancel out and we have

$$[\text{slope of production-possibility curve}] = -\frac{\partial y}{\partial x}=\frac{MP_{K,y}}{MP_{K,x}}$$

Similarly, we may derive

$$-\frac{\partial y}{\partial x}=\frac{MP_{L,y}}{MP_{L,x}}$$

Thus the slope of the production-possibility curve (or product-transformation curve) is

$$-\frac{\partial y}{\partial x}=\frac{MP_{L,y}}{MP_{L,x}}=\frac{MP_{K,y}}{MP_{K,x}}$$

The optimal combination of the output pair is the one which yields the highest revenue, given the production-possibility curve, that is, given the total quantities of factors which define this curve. To find the equilibrium of the firm we need an additional tool, the iso-revenue curve.

B. The Iso-revenue Curve of the Multiproduct Firm:

An iso-revenue curve is the locus of points of various combinations of quantities of y and x whose sale yields the same revenue to the firm (figure 3.47).

The slope of the iso-revenue curve is equal to the ratio of the prices of the commodities:

$$\begin{bmatrix}\text{slope of}\\ \text{isorevenue}\end{bmatrix}=\frac{0A}{0B}=\frac{P_x}{P_y}$$

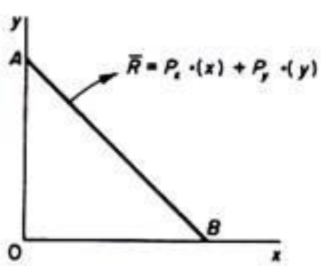


Figure 3.47

Proof:

Assume that we want an iso-revenue depicting R total revenue.

(a) If we sell only y the total revenue is (0A). Py = R, that is, the quantity of y which yields R is (0A) = R/Py.

(*b*) Similarly the quantity of x that yields $\bar{R}$ is $0B = \bar{R}/Px$.
Dividing $0A$ by $0B$ we obtain

$$\begin{bmatrix}\text{Slope of}\\ \text{isorevenue}\end{bmatrix} = \frac{0A}{0B} = \frac{\frac{\bar{R}}{Py}}{\frac{\bar{R}}{Px}} = \frac{Px}{Py}$$

Formally the $\bar{R}$ isorevenue curve may be obtained from the equation

$$\bar{R} = P_x \cdot (x) + P_y \cdot (y)$$

Solving for y we obtain

$$y = \frac{\bar{R}}{P_y} - \frac{P_x}{P_y} \cdot (x)$$

Given the prices of the two commodities and any value for $\bar{R}$, we may compute the points of the $\bar{R}$ line by assigning successive values to $x(x = 0, 1, 2, \ldots)$. For example:

if	$x = 0,$	$y = \frac{\bar{R}}{P_y} = 0A$
if	$x = 1,$	$y = \frac{\bar{R}}{P_y} - \frac{P_x}{P_y}$
if	$x = 2,$	$y = \frac{\bar{R}}{P_y} - 2\frac{P_x}{P_y}$
$\vdots$	$\vdots$	$\vdots$
if	$x = 0B = \frac{\bar{R}}{P_x},$	$y = \frac{\bar{R}}{P_y} - \frac{P_x}{P_y}\frac{\bar{R}}{P_x} = 0$

We may in the same way define the whole set of isorevenue curves by assigning to R various values. The further away from the origin an isorevenue curve is, the larger the revenue of the firm will be.

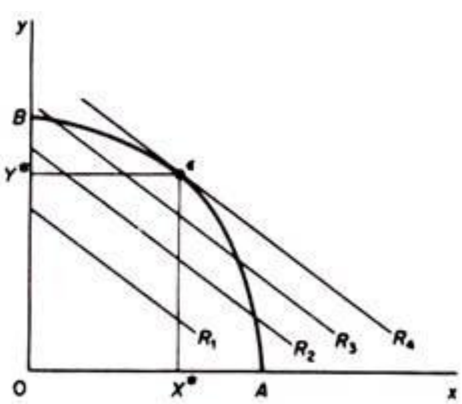


Figure 3.48

C. Equilibrium of the Multiproduct Firm:

The firm wants to maximize its profit given:

(i) The constraint set by the factors of production,

(ii) The transformation curve, and

(iii) The prices of the commodities (P_x, P_y) and of the factors of production (w, r).

Assuming that the quantity of the factors and their prices are given, then maximization of n is achieved by maximizing the revenue, R. Graphically the equilibrium of the firm is defined by the point of tangency of the given product-transformation curve and the highest iso-revenue curve (figure 3.48). At the point of tangency the slopes of the iso-revenue and the product-transformation curves are equal.

Thus the condition for equilibrium is that these slopes be equal:

$$-\frac{dy}{dx}=\frac{MP_{L,y}}{MP_{L,x}}=\frac{MP_{K,y}}{MP_{K,x}}=\frac{P_x}{P_y}$$

BOOKS REFERRED

Advanced Economic Theory – H.L. Ahuja

Modern Micro Economics – A. Koutsoyiannis

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