UNIT – II

MODERN UTILITY ANALYSIS

The modern utility analysis is the outcome of the failure of the indifference curve technique to explain consumer behaviour among risky or uncertain choices. The traditional utility analysis is also concerned with consumer behaviour among riskless choices. Such choices are certain, based as they are on the principle of diminishing marginal utility and on the proportionality rule.

The consumer is certain about his income, tastes and the goods he purchases and maximises his satisfaction by choosing that combination which gives him the highest total utility. But in reality, many goods and services involve risk or uncertainty, such as investments in shares of stock, insurance and gambling.

It was Neumann and Morgenstem who in their Theory of Games and Economic Behaviour studied the behaviour of an individual in risky situations. Their theory was refined by Friedman and Savage and by Markowitz. The solution to the problem of risky situations was provided by Daniel Bernoulli who tried to solve St. Petersburg Paradox. We explain these different views on choices involving risk or uncertainty.

The Bernoulli Hypothesis:

The neo-classical theory assumes that the consumer is a rational being who does not indulge in gambling or even in fair bet with 50-50 odds. The reason why people were unwilling to stake even at fair bets was provided by Daniel Bernoulli, the 18th century Swiss mathematician.

Staying in St. Petersburg in 1732 for some time, Bernoulli found that Russians were unwilling to make bets even at better than 50- 50 odds knowing fully that their mathematical expectations of winning money in a particular kind of gamble were greater the more money they bet. This contradiction is known as St. Petersburg Paradox. To explain it, Bernoulli composed the following game.

A coin is tossed and a payment is made to the player, depending upon which toss of the com first comes up 'heads'. If heads occurs on the first toss, the player receives £ 2 and the game stops. If it comes up in the second throw, £2^2 = £4 is paid and the game stops. If heads appears for the first time after n tosses, £2^n is paid to the player. How much would a rational person be willing to pay to take part in this game? Or, what is the expected monetary value of the pay-off to such a game? The expected monetary value of the game is infinite. The probability that heads will occur on the first toss of the coin is 1/2. The probability of obtaining heads for the first time on the nth toss is $(1/2)^n$. Since there is no finite number of throws within which guarantee can be given that a head will occur, the expected pay-off of the game or the expected monetary value of the game,

$$EMV = (_{1/2})2 + (_{1/2})^2 2^2 + (_{1/2})^3 2^3 + \ldots\ldots\ldots\ldots.. + (_{1/2})^{n.2n}$$

cc

$$= \Sigma^{\infty}_{n=1} (_{1/2})^n\, 2^n = 1 + 1 + 1 + \ldots. + 1\ldots$$

= infinity.

As the EMV is infinity, a person whose objective is to maximise expected monetary value would be willing to pay everything he has to play the game. Bernoulli resolved the St. Petersburg Paradox by suggesting that the reason why people would not be prepared to pay their entire income to play such a game is that the marginal utility of money diminishes as income rises.

A person who stakes Rs. 100 at even odds of winning or losing Rs. 10 will not play the game if he is a rational being. For if he wins, he will have Rs. 110, which are equal to the gain of utility from Rs. 10 won added to Rs. 100. If he loses, he will have Rs. 90 which is equal to the loss of utility from Rs. 10 lost subtracted from Rs. 100.

Though the monetary gain or loss is equal, the loss in utility is greater than the gain in utility in this game. Thus in Bernoulli's view, rational decisions in the case of risky choices would be made on the basis of expectations of total utility rather than the mathematical expectations of monetary value. This is illustrated in Figure 1.

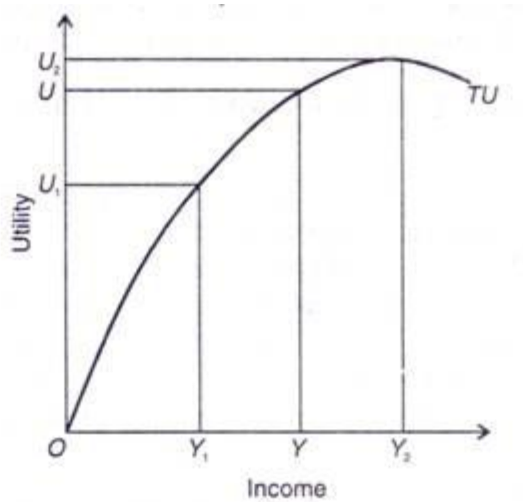

Fig. 1

Where TU is the total utility curve which becomes less and less steep at higher levels of income, indicating diminishing marginal utility of income. Suppose the person is at the income level OY (Rs. 100 in our example) which gives him utility OU. He is considering whether or not to accept a fair bet with a 50-50 probability of either increasing his income to OY_2 (Rs. 110) or reducing it to OY_1 (Rs. 90) by an equal amount.

He will consider its effect on his utility. If his income increases to OY_2 his utility rises to OU_2 and if his income decreases to OY_1his utility falls to OU_1. As is clear from the figure, the loss in utility by UU_1 is greater than the gain in utility by UU_2 .The loss or gain in total utility refers to marginal utility. Since the expectation of loss in utility is greater than the gain in utility, this person will not accept a fair bet.

Bernoulli's solution to the St. Petersburg Paradox in terms of expected utility instead of expected monetary value of the game led Neumann and Morgenstem to construct their utility index under risky choices.

The Neumann-Morgenstern Method of Measuring Utility:

J. Von Neumann and O. Morgenstem in their book Theory' of Games and Economic Behaviour evolved the method of cardinal measurement of expected utility from risky choices which are found in gambling, lottery tickets, etc. For this, they constructed a utility index which is called the N-M utility index.

Assumptions:

The N-M utility index is based on the following assumptions:

(1) The individual behaves in risky situations in order to maximise expected utility.

(2) His choices are transitive: if he prefers A prize (win) to B prize and B to C, then he prefers A to C.

(3) There is probability P which lies between 0 and 1 (0< P< 1) such that the individual is indifferent between prize A which is certain and the lottery tickets offering prizes C and B with probability P and 1 – P respectively.

(4) If two lottery tickets offer the same prizes, the individual prefers the lottery ticket with the higher probability of winning.

(5) The individual can completely order probability combinations of uncertain choices.

(6) Uncertainty or risk does not possess utility or disutility of its own.

The N-M Utility Index:

Neumann and Morgenstern have suggested the following method of measuring the utility index. "Consider three events, C, A, B, for which the order of individual's preferences is the one stated. Let a be a real number between 0 and 1, such that A is exactly equally desirable with the combined event consisting of a change of probability 1- a for B and the remaining chance of probability a for C. Then we suggest the use of a as a numerical estimate for the ratio of the preference of A over B to that of C over B."

Their formula becomes A=B (1- a + aC). Substituting P for a probability, we have A = B (1 -P) + P.C.

Given the assumptions, it is possible to derive a cardinal utility index based on the above formula.

Suppose there are the three events (lotteries) C, A, B. Out of these, event (lottery) A is certain, C has probability P, and B probability (1-P), and if their respective utilities are U_a ,U_b and U_c then U_a =PU_c (1-P)U_b

Since the consumer is expected to maximize utility, the utility of A with certainty must be equal to some value P, the expected utility of the events (lotteries) C and B.

In order to construct a utility index based on the N-M equation, we have to assign utility values C and B. These utility values are arbitrary except for the fact that higher value should be assigned to a preferred event (lottery). Suppose we assign the following arbitrary utility values: U_c = 100 utils, U_b =0 util, and P= 4/5 or 0.8, then

U_a = (4/5) 100 + (1-4/5) (0)

= 80 + (1/5) (0) = 80

Thus the utility index in this situation is

Situation U_a U_b U_c

1 80 0 100

Proceeding this way, one can derive utility values for U_a, U_b, U_c, etc. and construct a complete N-M utility index for all possible combinations starting from two arbitrary situations involving probabilities of risk.

It's Appraisal:

The N-M utility index provides conceptual measurement of cardinal utility under risky choices. It is meant to be used for making predictions about two or more alternatives relating to gambling, lottery tickets, etc. and out of them which one a person may prefer.

The N-M index is based on the expected values of utilities. It provides a method to measure cardinally the marginal utility of money. But it does not refer to whether the marginal utility of money diminishes or increases. In this sense, this method of measuring utility is incomplete.

But the N-M cardinal utility is different from the neo-classical cardinal utility. It is not like measures of length or weight. Nor does it measure the intensity of introspective satisfaction or pleasure from goods and services, as is the case with the neo-classical utility'. The N-M method of measuring utility analyses the actions of a person making risky choices.

Despite the fact that there is arbitrariness in computing the N-M utility index, it is measurable upto a linear transformation. It does not involve additively but permits ordinal measurement of relative preferences of risky choices.

The Friedman-Savage Hypothesis:

The Neumann-Morgenstern method is based on the expected values of utilities and therefore, does not refer to whether the marginal utility of money diminishes or increases. In this respect, this method of measuring utility is incomplete. When a person gets an insurance policy, he pays to escape or avoid risk. But when he buys a lottery ticket, he gets a small chance of a large gain.

Thus he assumes risk. Some people indulge both in buying insurance and gambling and thus they both avoid and choose risks. Why'? The answer has been provided by the Freedman-Savage Hypothesis as an extension of the N-M method.

It states that marginal utility of money diminishes for incomes below some level, it increases for incomes between that level and some higher level of income, and again diminishes for all incomes above that higher level. This is illustrated in Figure 2 in terms of the total utility curve TU where utility is plotted on the vertical axis and income on the horizontal axis.

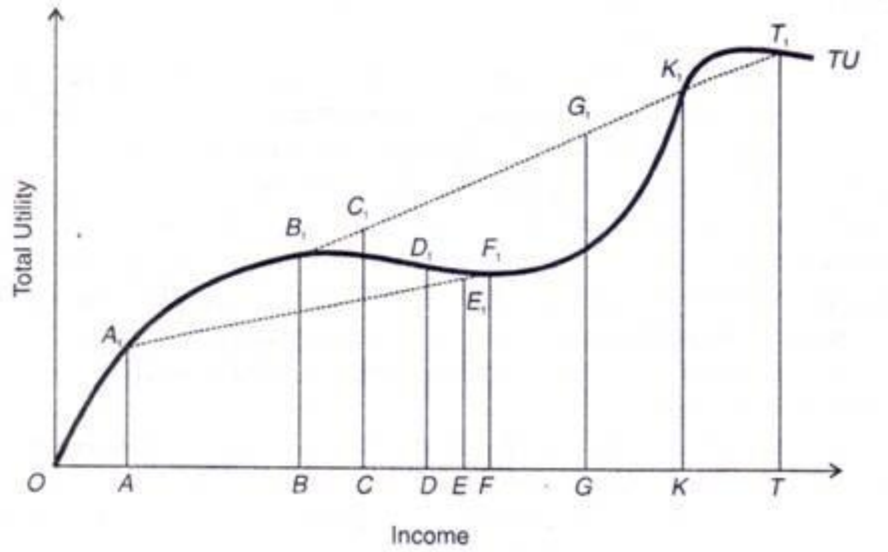

Fig. 2

Suppose a person buys insurance for his house against the small chance of a heavy loss from fire and also buys a lottery ticket which offers a small chance of a large win. Such a conflicting behaviour of a person who buys insurance and also gambles has been shown by Friedman and Savage with a total utility curve. Such a curve first rises at a diminishing rate so that the marginal utility of money declines and then it rises at an increasing rate so that the marginal utility of income increases.

The curve TU in the figure first rises facing downward up to point $F_1$and then facing upward up to point K_1 .Suppose the person's income from his house is OF with FF_1 utility without a fire. Now he buys insurance to avoid risk from a fire. If the house is burnt down by fire, his income is reduced to OA with AA utility. By joining points A_1 and F_1, we get utility points between these two uncertain income situations. If the probability of no fire is P, then the expected income of this person on the basis of the N-M utility index is

Y = P (OF) + (1 -P) (OA).

Let the expected income (Y) of the person be OE, then its utility is EE_1 on the dashed line A_t F_r Now assume that the cost of insurance, (insurance premium) is FD. Thus the person's assured income with insurance is OD (= OF-FD) which gives him greater utility DD_1 than $EE_1$from expected income OE with probability of no fire. Therefore, the person will buy insurance to avoid risk and have the assured income OD by paying FD premium in case his house is burnt down by fire.

With OD income left with the person after buying insurance of the house against fire, he decides to purchase a lottery ticket which costs DB. If he does not win, his income would fall to OB with utility BB_1. If he wins, his income would increase to OK with utility KK_1 Thus his expected income with probability P’ of not winning the lottery is

Y_1 = P'(OB) + (1 -P’) (OK)

Let the expected income F, of the person be OC, then its utility is CC_1 on the dashed line B_1K_1which gives him greater utility (CC_1) by purchasing the lottery ticket than DD_1 if he had not bought it. Thus the person will also buy the ticket along with insurance for the house against fire.

Let us take OG expected income in the rising portion F_1K_1 of the TU curve when the marginal utility of income is increasing. In this case, the utility of buying the lottery ticket is GG_1which is greater than DD_1 if he were not to buy the lottery. Thus he will stake his money on the lottery.

In the last stage when the expected income of the person is more than OK in the region $K_1T_1$of the TU curve, the marginal utility of income is declining and consequently, he is not willing to undertake risks in buying lottery tickets or in other risky investments except at favorable odds. This region explains St. Petersburg Paradox.

Friedman and Savage believe that the TU curve describes the attitudes of people towards risks in different socio-economic groups. However, they recognise many differences between persons even in the same socio-economic group. Some are habitual gamblers while others avoid risks. Still, Friedman and Savage believe that the curve describes the propensities of the main groups.

According to them, people in the middle income group with increasing marginal utility of income are those who are willing to take risks to improve their lot. If they succeed in their efforts in having more money by taking risks, they lift themselves up into the next higher socio-economic group. They do not want just more consumer goods. Rather, they want to rise in the social scale and to change their patterns of life. That is why, the marginal utility of income increases for them.

The Markowitz Hypothesis:

Prof. Markowitz found the Friedman-Savage hypothesis contrary to common observations. According to him, it is not correct to say that the poor and the rich are unwilling to gamble and take risks except at favourable odds. Rather, both purchase lotteries and gamble on horse races. They also play the games at casinos and gamble alike in the stock market.

Thus Friedman and Savage failed to observe the actual behaviour of the poor and the rich because they assume that the marginal utility of income depends on the absolute level of income. Markowitz has modified it by relating the marginal utility of income to changes in the level of present income.

According to Markowitz, when income increases by a small increment, it leads to increasing marginal utility of income. But large increases in income lead to diminishing marginal utility of income. That is why at higher levels of income people are reluctant to indulge in gambling even at fair bets and people in slowly rising income groups indulge in gambling to improve their position.

On the other hand, when there are small decreases in income, the marginal utility of income rises. But large decreases in income lead to diminishing marginal utility of income. That is why people insure against small losses but indulge in gambling where large losses are involved.

This is called the Markowitz hypothesis which is explained in Figure 3 where Markowitz takes three inflexion points M, N and P in the upper portion of the diagram with present income at the middle point N on the TU curve of income.

The marginal utility of income curve MU is derived in the lower portion of the diagram where the present income level is OB. With a small increase in the income of a person from OB to OC, the marginal utility of income increases from point S to T on the MU curve. But large increases in income beyond OC lead to diminishing marginal utility of income from point T onwards along the MU curve.

On the other hand, small decreases in income from OB to O A lead to increasing marginal utility of income from S to R on the MU curve. But large decreases in income to the left of A lead to diminishing marginal utility of income from point R towards O along the MU curve.

The Markowitz hypothesis is an improvement over the Friedman-Savage hypothesis. Instead of the absolute level of income, it takes the present level of income of a person. It suggests that a person's behaviour towards insurance and gambling is the same whether he is poor or rich. The emphasis is on small or large increases or decreases in the present income of a person that determines his behaviour towards insurance and gambling.

Critical Appraisal of Modern Utility Analysis:

In the modem utility analysis of risk or uncertainty, the Neumann and Morgenstem hypothesis implies measurable utility up to a linear transformation thereby reintroducing diminishing or increasing marginal utility. The Friedman-Savage hypothesis contains an added element.

It attempts to explain the shape of the curve of total utility of income. These hypotheses are thus attempts to rehabilitate the measurement of utility. But the N-M theory of risky choices along with its variants like the Friedman- Savage hypothesis and Markowitz hypothesis ate still a subject of controversy on two counts; firstly, from the practical standpoint, and secondly, whether it is a cardinal or an ordinal method.

Firstly, it is doubtful if risk is measurable when Neumann and Morgenstem assume that the risk does not possess any utility or disutility of its own, they ignore the pleasures or pains of uncertainty- bearing.

Secondly, in the majority of individual choices the element of uncertainty is very little.

Thirdly, individual choices are of an infinite variety. Guaranteed that they are uncertain, it is possible to measure them with the N-M method? Lastly, it does not measure the ‘strength of feelings’ of individuals towards goods and services under uncertain choices.

The question whether the N-M method measures utility cardinally or ordinally, there is great confusion among economists. Robertson in his Utility and All That uses it in the cardinal sense, while Profs. Baumol, Fellner and others are of the view that the ranking of utility makes it ordinal. According to Baumol, the N-M theory has nothing in common with the neo-classical theory regarding cardinality.

In the neo-classical theory the word “cardinal” is used to denote introspective absolute marginal measurement of utility while in this theory it is used operationally. In the N-M theory, utility numbers are assigned to lottery tickets according to a person’s ranking of the prizes and the prediction is made numerically as to which of the two tickets will be chosen. Though the N-M formula is used to derive the utility index, yet it says nothing about diminishing marginal utility. Thus the N-M utility is not the neoclassical cardinal utility.

The refinements made by Friedman-Savage and Markowitz have tendered to drop the neo-classical assumption that the marginal utility of income diminishes for all ranges of income. Thus the theory of measurement of utility under risky choices is superior to the neo-classical introspective cardinalism of certain choices.

Economists like Dorfman, Samuelson and Solow have derived the Paretian indices of utility from the N-M formula. And when the N-M index based on individual ranking is constructed, it conveys information about his preferences.

Baumol uses further the N-M measurement in the ordinal sense when he equates the N-M marginal utility with the marginal rate of substitution. He writes: “The N-M marginal utility X of ends up as no more than the marginal rate of substitution between and the probability of winning the pre-specified prize (E) of the standard lottery ticket. This is surely not cardinal measurement in the classical sense.”