UNIT –I

Introduction to game theory

Game theory seeks to analyze competing situations which arise out of conflicts of interest. Abraham Maslow's hierarchical model of human needs lays emphasis on fulfilling the basic needs such as food, water, clothes, shelter, air, safety and security. There is conflict of interest between animals and plants in the consumption of natural resources. Animals compete among themselves for securing food. Man competes with animals to earn his food. A man also competes with another man. In the past, nations waged wars to expand the territory of their rule. In the present day world, business organizations compete with each other in getting the market share. The conflicts of interests of human beings are not confined to the basic needs alone. Again considering Abraham Maslow's model of human needs, one can realize that conflicts also arise due to the higher levels of human needs such as love, affection, affiliation, recognition, status, dominance, power, esteem, ego, self-respect, etc. Sometimes one witnesses clashes of ideas of intellectuals also. Every intelligent and rational participant in a conflict wants to be a winner but not all participants can be the winners at a time. The situations of conflict gave birth to Darwin's theory of the 'survival of the fittest'.

DEFINITION OF GAME THEORY

In the perception of Robert Mockler, "Game theory is a mathematical technique helpful in making decisions in situations of conflicts, where the success of one part depends at the expense of others, and where the individual decision maker is not in complete control of the factors influencing the outcome".

According to Edwin Mansfield, "A game is a competitive situation where two or more persons pursue their own interests and no person can dictate the outcome. Each player, an entity with the same interests, make his own decisions. A player can be an individual or a group".

Assumptions for a Competitive Game

Game theory helps in finding out the best course of action for a firm in view of the anticipated countermoves from the competing organizations. A competitive situation is a competitive game if the following properties hold:

1. The number of competitors is finite, say N.
2. A finite set of possible courses of action is available to each of the N competitors.
3. A play of the game results when each competitor selects a course of action from the set of courses available to him. In game theory we make an important assumption that al the players select their courses of action simultaneously. As a result, no competitor will be in a position to know the choices of his competitors.

4. The outcome of a play consists of the particular courses of action chosen by the individual players. Each outcome leads to a set of payments, one to each player, which may be either positive, or negative, or zero.

Managerial Applications of the Theory of Games

The techniques of game theory can be effectively applied to various managerial problems as detailed below:

1) Analysis of the market strategies of a business organization in the long run.
2) Evaluation of the responses of the consumers to a new product.
3) Resolving the conflict between two groups in a business organization.
4) Decision making on the techniques to increase market share.
5) Material procurement process.
6) Decision making for transportation problem.
7) Evaluation of the distribution system.
8) Evaluation of the location of the facilities.
9) Examination of new business ventures and
10) Competitive economic environment.

Definitions used in the Theory of Games

Players:

The competitors or decision makers in a game are called the players of the game.

Strategies:

The alternative courses of action available to a player are referred to as his strategies.

Pay off:

The outcome of playing a game is called the pay off to the concerned player.

Optimal Strategy:

A strategy by which a player can achieve the best pay off is called the optimal strategy for him.

Zero-sum game:

A game in which the total payoffs to all the players at the end of the game is zero is referred to as a zero-sum game.

Non-zero sum game:

Games with "less than complete conflict of interest" are called non-zero sum games. The problems faced by a large number of business organizations come under this category. In such games, the gain of one player in terms of his success need not be completely at the expense of the other player.

Payoff matrix:

The tabular display of the payoffs to players under various alternatives is called the payoff matrix of the game.

Pure strategy:

If the game is such that each player can identify one and only one strategy as the optimal strategy in each play of the game, then that strategy is referred to as the best strategy for that player and the game is referred to as a game of pure strategy or a pure game.

Mixed strategy:

If there is no one specific strategy as the 'best strategy' for any player in a game, then the game is referred to as a game of mixed strategy or a mixed game. In such a game, each player has to choose different alternative courses of action from time to time.

N-person game:

A game in which N-players take part is called an N-person game.

Maximin-Minimax Principle:

The maximum of the minimum gains is called the maximin value of the game and the corresponding strategy is called the maximin strategy. Similarly the minimum of the maximum losses is called the minimax value of the game and the corresponding strategy is called the minimax strategy. If both the values are equal, then that would guarantee the best of the worst results.

Saddle point:

A saddle point of a game is that place in the payoff matrix where the maximum of the row minima is equal to the minimum of the column maxima. The payoff at the saddle point is called **the value of the game** and the corresponding strategies are called the **pure strategies. Dominance**:

One of the strategies of either player may be inferior to at least one of the remaining ones. The superior strategies are said to dominate the inferior ones.

Types of Games:

There are several classifications of a game. The classification may be based on various factors such as the number of participants, the gain or loss to each participant, the number of strategies available to each participant, etc. Some of the important types of games are enumerated below.

Two person games and n-person games:

In two person games, there are exactly two players and each competitor will have a finite number of strategies. If the number of players in a game exceeds two, then we refer to the game as n-person game.

Zero sum game and non-zero sum game:

If the sum of the payments to all the players in a game is zero for every possible outcome of the game, then we refer to the game as a zero sum game. If the sum of the payoffs from any play of the game is either positive or negative but not zero, then the game is called a non-zero sum game

2x2 two person game and 2xn and mx2 games:

When the number of players in a game is two and each player has exactly two strategies, the game is referred to as 2x2 two person game.

A game in which the first player has precisely two strategies and the second player has three or more strategies is called an 2xn game.

A game in which the first player has three or more strategies and the second player has exactly two strategies is called an mx2 game.

3x3 and large games:

When the number of players in a game is two and each player has exactly three strategies, we call it a 3x3 two person game.Two-person zero sum games are said to be larger if each of the two players has 3 or more choices.The examination of 3x3 and larger games is involves difficulties. For such games, the technique of linear programming can be used as a method of solution to identify the optimum strategies for the two players.

Two-person zero sum game

A game with only two players, say player A and player B, is called a two-person zero sum game if the gain of the player A is equal to the loss of the player B, so that the total sum is zero.

Assumptions for two-person zero sum game:

For building any model, certain reasonable assumptions are quite necessary. Some assumptions for building a model of two-person zero sum game are listed below.

a) Each player has available to him a finite number of possible courses of action. Sometimes the set of courses of action may be the same for each player. Or, certain courses of action may be available to both players while each player may have certain specific courses of action which are not available to the other player.
b) Player A attempts to maximize gains to himself. Player B tries to minimize losses to himself.
c) The decisions of both players are made individually prior to the play with no communication between them.
d) The decisions are made and announced simultaneously so that neither player has an advantage resulting from direct knowledge of the other player's decision.
e) Both players know the possible payoffs of themselves and their opponents.

Minimax and Maximin Principles

The selection of an optimal strategy by each player without the knowledge of the competitor's strategy is the basic problem of playing games.

The objective of game theory is to know how these players must select their respective strategies, so that they may optimize their payoffs. Such a criterion of decision making is referred to as minimax-maximin principle. This principle in games of pure strategies leads to the best possible selection of a strategy for both players.

For example, if player A chooses his ith strategy, then he gains at least the payoff min a_{ij}, which is minimum of the ith row elements in the payoff matrix. Since his objective is to maximize his payoff, he can choose strategy i so as to make his payoff as large as possible.
i.e., a payoff which is not less than $\max\limits_{1\le i\le m} \min\limits_{1\le j\le n} a_{ij}$.

Similarly player B can choose jth column elements so as to make his loss not greater than $\min\limits_{1\le j\le n} \max\limits_{1\le i\le m} a_{ij}$.

If the maximin value for a player is equal to the minimax value for another player, i.e.

$$\max_{1\le i\le m} \min_{1\le j\le n} a_{ij} = V = \min_{1\le j\le n} \max_{1\le i\le m} a_{ij}$$

then the game is said to have a saddle point (equilibrium point) and the corresponding strategies are called optimal strategies. If there are two or more saddle points, they must be equal.

The amount of payoff, i.e., V at an equilibrium point is known as the **value of the game**.

The optimal strategies can be identified by the players in the long run.

Fair game:

The game is said to be fair if the value of the game $V = 0$.

Problem 1:

Solve the game with the following pay-off matrix.

$$
\begin{array}{cc}
 & \textbf{Player B} \\
 & \text{Strategies} \\
 & \begin{array}{c} \begin{array}{ccccc} I & II & III & IV & V \end{array} \end{array} \\
\text{Player A Strategies} \quad \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \left[\begin{array}{ccccc} -2 & 5 & -3 & 6 & 7 \\ 4 & 6 & 8 & -1 & 6 \\ 8 & 2 & 3 & 5 & 4 \\ 15 & 14 & 18 & 12 & 20 \end{array}\right]
\end{array}
$$

Solution:

First consider the minimum of each row.

Row	Minimum Value
1	-3
2	-1
3	2
4	12

Maximum of {-3, -1, 2, 12} = 12

Next consider the maximum of each column.

Column	Maximum Value
1	15
2	14
3	18
4	12
5	20

Minimum of {15, 14, 18, 12, 20}= 12

We see that the maximum of row minima = the minimum of the column maxima. So the game has a saddle point. The common value is 12. Therefore the value V of the game = 12.

Interpretation:

In the long run, the following best strategies will be identified by the two players:

The best strategy for player A is strategy 4.

The best strategy for player B is strategy IV.

The game is favourable to player A.

Problem 2:

Solve the game with the following pay-off matrix

$$\begin{array}{cc} & \textbf{Player Y} \\ & \text{Strategies} \\ & \begin{array}{ccccc} I & II & III & IV & V \end{array} \\ \text{Player X Strategies} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{array}{|ccccc|} \hline 9 & 12 & 7 & 14 & 26 \\ 25 & 35 & 20 & 28 & 30 \\ 7 & 6 & -8 & 3 & 2 \\ 8 & 11 & 13 & -2 & 1 \\ \hline \end{array} \end{array}$$

Solution:

First consider the minimum of each row.

Row	Minimum Value
1	7
2	20
3	-8
4	-2

Maximum of {7, 20, –8, -2} = 20

Next consider the maximum of each column.

Column	Maximum Value
1	25
2	35
3	20
4	28
5	30

Minimum of {25, 35, 20, 28, 30}= 20

It is observed that the maximum of row minima and the minimum of the column maxima are equal. Hence the given the game has a saddle point. The common value is 20. This indicates that the value V of the game is 20.

Interpretation.

The best strategy for player X is strategy 2.

The best strategy for player Y is strategy III.

The game is favourable to player A.

Problem 3:

Solve the following game:

Player B

Strategies

		I	*II*	*III*	*IV*
Player A Strategies	1	1	–6	8	4
	2	3	–7	2	–8
	3	5	–5	–1	0
	4	3	–4	5	7

Solution

First consider the minimum of each row.

Row	Minimum Value
1	-6
2	-8
3	-5
4	-4

Maximum of {-6, -8, -5, -4} = -4

Next consider the maximum of each column.

Column	Maximum Value
1	5
2	-4
3	8
4	7

Minimum of {5, -4, 8, 7}= - 4

Since the max {row minima} = min {column maxima}, the game under consideration has a saddle point. The common value is –4. Hence the value of the game is –4.

Interpretation.

The best strategy for player A is strategy 4.

The best strategy for player B is strategy II.

Since the value of the game is negative, it is concluded that the game is favorable to player B.

Games with Mixed Strategies

Games with Mixed Strategies

In certain cases, no pure strategy solutions exist for the game. In other words, saddle point does not exist. In all such game, both players may adopt an optimal blend of the strategies called **Mixed Strategy** to find a saddle point. The optimal mix for each player may be determined by assigning each strategy a probability of it being chosen. Thus these mixed strategies are probabilistic combinations of available better strategies and these games hence called **Probabilistic games**.

The probabilistic mixed strategy games without saddle points are commonly solved by any of the following methods

Sl. No.	Method	Applicable to
1	Analytical Method	2x2 games
2	Graphical Method	2x2, mx2 and 2xn games

Analytical Method

A 2 x 2 payoff matrix where there is no saddle point can be solved by analytical method. Given the matrix

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

Value of the game is

$$V = \frac{a_{11}\,a_{22} - a_{21}\,a_{12}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$$

With the coordinates

$$x_1 = \frac{a_{22} - a_{21}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}, \quad x_2 = \frac{a_{11} - a_{12}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$$

$$y_1 = \frac{a_{22} - a_{12}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}, \quad y_2 = \frac{a_{11} - a_{21}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$$

1. Solve

$$\begin{array}{cc} & B \\ A & \begin{bmatrix} 5 & 1 \\ 3 & 4 \end{bmatrix} \end{array}$$

Solution

It is a 2 x 2 matrix and no saddle point exists. We can solve by analytical method

$$\begin{array}{cc} & B \\ A & \begin{bmatrix} 5 & 1 \\ 3 & 4 \end{bmatrix} \end{array}$$

$$V = \frac{a_{11}\, a_{22} - a_{21}\, a_{12}}{(a_{11} + a_{22}) - (a_{12} + a_{21})} = \frac{20 - 3}{9 - 4}$$

V = 17 / 5

$$x_1 = \frac{a_{22} - a_{21}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}, \quad x_2 = \frac{a_{11} - a_{12}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$$

$S_A = (P_1, P_2) = (1/5, 4/5)$

$S_B = (q_1, q_2) = (3/5, 2/5)$

$$y_1 = \frac{a_{22} - a_{12}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}, \quad y_2 = \frac{a_{11} - a_{21}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$$

2. Solve the given matrix

$$\begin{array}{cc} & B \\ A & \begin{bmatrix} 2 & -1 \\ -1 & 0 \end{bmatrix} \end{array}$$

Solution

$$\begin{array}{cc} & B \\ A & \begin{bmatrix} 2 & -1 \\ -1 & 0 \end{bmatrix} \end{array}$$

$$V = \frac{a_{11}\, a_{22} - a_{21}\, a_{12}}{(a_{11} + a_{22}) - (a_{12} + a_{21})} = \frac{0 - 1}{2 + 2}$$

V = - 1 / 4

$S_A = (P_1, P_2) = (1/4, 3/4)$

$S_B = (q_1, q_2) = (1/4, 3/4)$

Graphical method

The graphical method is used to solve the games whose payoff matrix has

- Two rows and n columns (2 x n)
- m rows and two columns (m x 2)

Algorithm for solving 2 x n matrix games

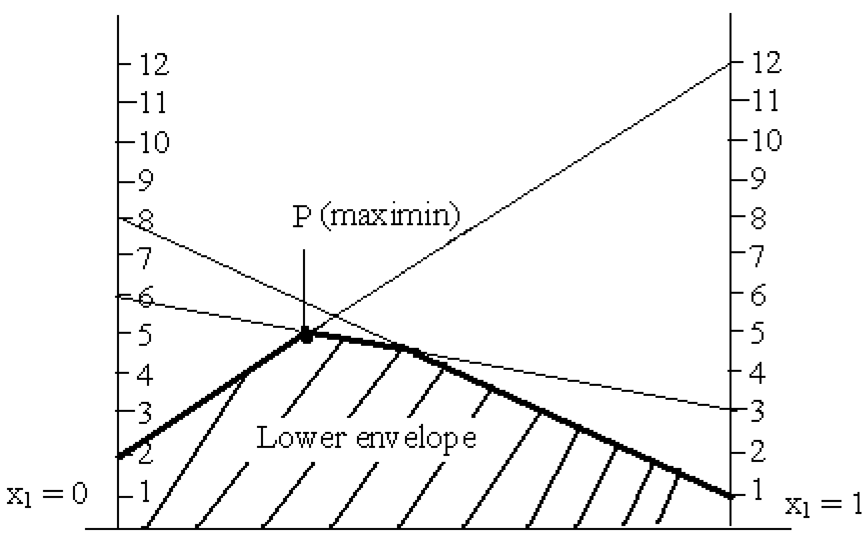
- Draw two vertical axes 1 unit apart. The two lines are $x_1 = 0$, $x_1 = 1$
- Take the points of the first row in the payoff matrix on the vertical line $x_1 = 1$ and the points of the second row in the payoff matrix on the vertical line $x_1 = 0$.
- The point a_{1j} on axis $x_1 = 1$ is then joined to the point a_{2j} on the axis $x_1 = 0$ to give a straight line. Draw 'n' straight lines for j=1, 2… n and determine the highest point of the lower envelope obtained. This will be the **maximini point**.
- The two or more lines passing through the maximini point determines the required 2 x 2 payoff matrix. This in turn gives the optimum solution by making use of analytical method.

Example 1

Solve by graphical method

$$\begin{array}{c c} & \begin{array}{ccc} B1 & B2 & B3 \end{array} \\ \begin{array}{c} A1 \\ A2 \end{array} & \begin{bmatrix} 1 & 3 & 12 \\ 8 & 6 & 2 \end{bmatrix} \end{array}$$

Solution



$$\begin{array}{c c} & \begin{array}{cc} B2 & B3 \end{array} \\ \begin{array}{c} A1 \\ A2 \end{array} & \begin{bmatrix} 3 & 12 \\ 6 & 2 \end{bmatrix} \end{array}$$

$$V = \frac{a_{11}\,a_{22} - a_{21}\,a_{12}}{(a_{11} + a_{22}) - (a_{12} + a_{21})} = \frac{6 - 72}{5 - 18}$$

V = 66/13

S_A = (4/13, 9 /13)

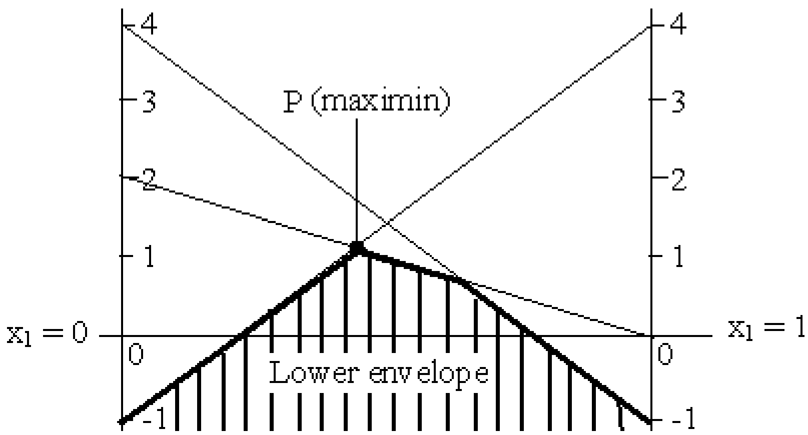
S_B = (0, 10/13, 3 /13)

Example 2

Solve by graphical method

$$\begin{array}{c|ccc} & B1 & B2 & B3 \\ \hline A1 & 4 & -1 & 0 \\ A2 & -1 & 4 & 2 \end{array}$$

Solution



$$\begin{array}{c|cc} & B1 & B3 \\ \hline A1 & 4 & 0 \\ A2 & -1 & 2 \end{array}$$

$$V = \frac{a_{11}\,a_{22} - a_{21}\,a_{12}}{(a_{11} + a_{22}) - (a_{12} + a_{21})} = \frac{8-0}{6+1}$$

V = 8/7

S_A = (3/7, 4 /7)

S_B = (2/7, 0, 5 /7)

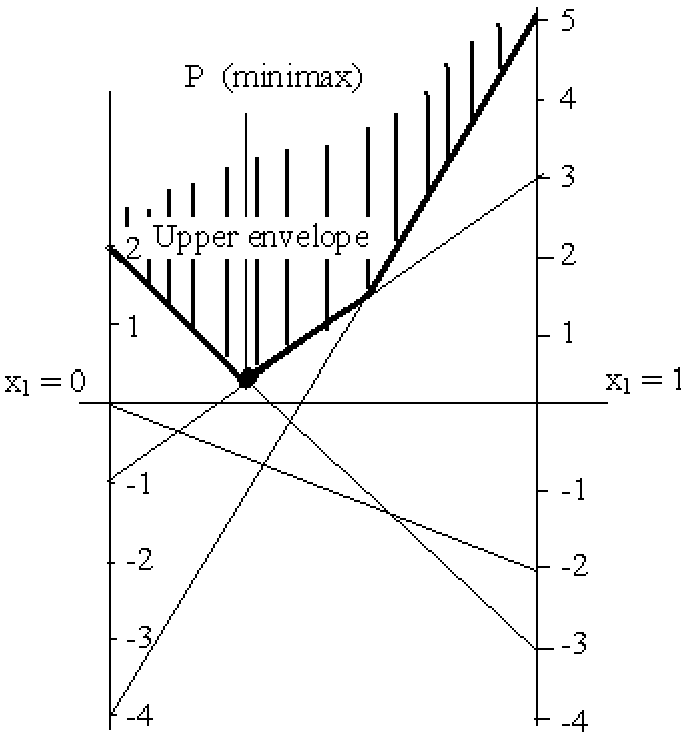
<u>Algorithm for solving m x 2 matrix games</u>

- Draw two vertical axes 1 unit apart. The two lines are x_1 =0, x_1 = 1
- Take the points of the first row in the payoff matrix on the vertical line x_1 = 1 and the points of the second row in the payoff matrix on the vertical line x_1 = 0.
- The point a_{1j} on axis x_1 = 1 is then joined to the point a_{2j} on the axis x_1 = 0 to give a straight line. Draw 'n' straight lines for j=1, 2… n and determine the lowest point of the upper envelope obtained. This will be the **minimax point**.
- The two or more lines passing through the minimax point determines the required 2 x 2 payoff matrix. This in turn gives the optimum solution by making use of analytical method.

Example 1
Solve by graphical method

$$\begin{array}{c|cc} & B1 & B2 \\ \hline A1 & -2 & 0 \\ A2 & 3 & -1 \\ A3 & -3 & 2 \\ A4 & 5 & -4 \end{array}$$

Solution



$$\begin{array}{c|cc} & B1 & B2 \\ \hline A2 & 3 & -1 \\ A3 & -3 & 2 \end{array}$$

$$V = \frac{a_{11}\,a_{22} - a_{21}\,a_{12}}{(a_{11} + a_{22}) - (a_{12} + a_{21})} = \frac{6-3}{5+4}$$

V = 3/9 = 1/3

S_A = (0, 5 /9, 4/9, 0)

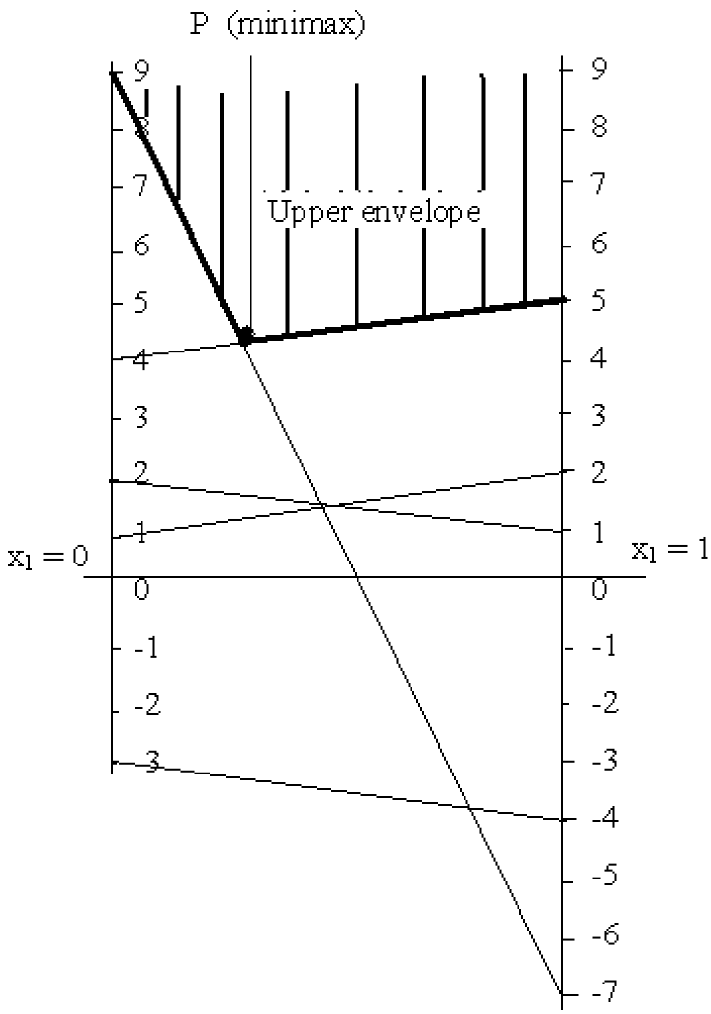
S_B = (3/9, 6 /9)

Example 2

Solve by graphical method

$$\begin{array}{c|cc} & B1 & B2 \\ \hline A1 & 1 & 2 \\ A2 & 5 & 4 \\ A3 & -7 & 9 \\ A4 & -4 & -3 \\ A5 & 2 & 1 \end{array}$$

Solution



$$\begin{array}{c|cc} & B1 & B2 \\ \hline A2 & 5 & 4 \\ A3 & -7 & 9 \end{array}$$

$$V = \frac{a_{11}\, a_{22} - a_{21}\, a_{12}}{(a_{11} + a_{22}) - (a_{12} + a_{21})} = \frac{45 + 28}{14 + 3}$$

V = 73/17
S_A = (0, 16/17, 1/17, 0, 0)
S_B = (5/17, 12 /17)

DOMINANCE PROPERTY

Principle of dominance is applicable to both pure strategies and mixed strategies. Sometimes, it is observed that one of the pure strategies of either players is always inferior to at least one of the remaining strategies. The superior strategies are said to dominate the inferior ones. The player would have no incentive to use inferior strategies which are dominated by the superior ones. In such cases of dominance, the size of the pay-off matrix by deleting those strategies which are dominated by the others.

The dominance properties are:

1. If all the elements of a row say K^{th}, are less than or equal to the corresponding elements of any other row (say r^{th} row), then k^{th} row is dominated by the r^{th} row.
2. If all the elements of column, say k^{th}, are greater than or equal to the corresponding elements of any other column, say r^{th} then K^{th} column is dominated by the r^{th} row.
3. A pure strategy may be dominated, if it is inferior to average of two or more other pure strategies.

Problem: *Solve the following game*

Player B

Player A		1	2	3	4
	1	3	2	4	0
	2	3	4	2	4
	3	4	2	4	0
	4	0	4	0	8

Solution: Since all the elements of 3^{rd} row are greater than or equal to the corresponding elements of 1^{st} row, third (3^{rd}) row is dominating 1^{st} row and hence row 1 can be eliminated.

Player B

Player A		1	2	3	4
	1	3	4	2	4
	3	4	2	4	0
	4	0	4	0	8

Again, all the elements of the 1^{st} column are greater than or equal to the corresponding elements of the 3^{rd} column.

$\therefore$ 3^{rd} column is dominating the first column. Eliminating 1^{st} column, the matrix is reduced to

$$\begin{array}{cc|c|c|c|}
 & & \multicolumn{3}{c}{\text{Player } B} \\
 & & 2 & 3 & 4 \\
\hline
 & 2 & 4 & 2 & 4 \\
\hline
\text{Player } A & 3 & 2 & 4 & 0 \\
\hline
 & 4 & 4 & 0 & 8 \\
\hline
\end{array}$$

Here, the linear combination of 2^{nd} and 4^{th} column dominates the 2^{nd}, because

$$4 > \frac{2+4}{2},\ 2 = \frac{4+0}{2} \quad \text{and } 4 = \frac{0+8}{2}$$

Eliminating 1st column, the reduced matrix becomes,

		Player B 3	4
	2	2	4
Player A	3	4	0
	4	0	8

Here again, the convex linear combination of 3 and 4 of player '*A*' dominate 2nd because

$$2 = \frac{4+0}{2}, \quad 4 = \frac{0+8}{2}$$

$\therefore$ Eliminating 1st row, the reduced matrix becomes

		Player B 3	4
Player A	3	4	0
	4	0	8

the probabilities of mixed strategies for player "A" and '*B*'.

Thus, optimum strategies are:

For player *A*, [0, 0, 2/3, 1/3];

For player *B*, [0, 0, 2/3, 1/3] and

The value of the game is

$$\frac{a_{11}a_{22} - a_{21}a_{12}}{a_{11} + a_{22} - (a_{12} + a_{21}} = \frac{4 \times 8 - 0 \times 0}{4 + 8 - (0 \times 0)} = \frac{32}{12} = \boxed{\frac{8}{3}}$$

Problem: *Solve the following game by using the principle of dominance*

$$\begin{array}{ccc} & & \text{Player } B \\ & & \begin{array}{cccccc} B_1 & B_2 & B_3 & B_4 & B_5 & B_6 \end{array} \\ \text{Player } A & \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array} & \left[\begin{array}{cccccc} 4 & 2 & 0 & 2 & 1 & 1 \\ 4 & 3 & 1 & 3 & 2 & 2 \\ 4 & 3 & 7 & -5 & 1 & 2 \\ 4 & 3 & 4 & -1 & 2 & 2 \\ 4 & 3 & 3 & -2 & 2 & 2 \end{array}\right] \end{array}$$

Solution: The pay-off matrix has no saddle point.

All the elements of row A_1 are dominated by row A_2 and row A_5 is dominated by row A_4. Hence row A_1 and A_5 can be eliminated.

Hence, the pay-off matrix is reduced to

$$\begin{array}{ccc} & & \text{Player } B \\ & & \begin{array}{cccccc} B_1 & B_2 & B_3 & B_4 & B_5 & B_6 \end{array} \\ \text{Player } A & \begin{array}{c} A_2 \\ A_3 \\ A_4 \end{array} & \left[\begin{array}{cccccc} 4 & 3 & 1 & 3 & 2 & 2 \\ 4 & 3 & 7 & -5 & 1 & 2 \\ 4 & 3 & 4 & -1 & 2 & 2 \end{array}\right] \end{array}$$

From player B's point of view, column B_1 and B_2 are dominated by columns B_4, B_4 and B_6 and column B_6 is dominated by column B_5. Hence strategies B_1, B_2 and B_6 are eliminated. The modified pay-off matrix is

$$\begin{array}{ccc} & & \text{Player } B \\ & & \begin{array}{ccc} B_3 & B_4 & B_5 \end{array} \\ \text{Player } A & \begin{array}{c} A_2 \\ A_3 \\ A_4 \end{array} & \left[\begin{array}{ccc} 1 & 3 & 3 \\ 7 & -5 & 1 \\ 4 & -1 & 2 \end{array}\right] \end{array}$$

Now, none of the single row or column dominates another row or column *i.e.* none of the pure strategies of A and B is inferior to any of the other strategies.

This strategy of B is superior to strategy B_5 because the B_5 strategy will result him in greater loss. So, the strategy B_5 can be eliminated. The modified matrix is

$$\begin{array}{ccc} & & \text{Player } B \\ & & \begin{array}{cc} B_3 & B_4 \end{array} \\ \text{Player } A & \begin{array}{c} A_2 \\ A_3 \\ A_4 \end{array} & \left[\begin{array}{cc} 1 & 3 \\ 7 & -5 \\ 4 & -1 \end{array}\right] \end{array}$$

Thus the resulting matrix (2 × 2) is given by

		Player B	
		B_3	B_4
	A_2	1	3
Player A	A_3	7	–5

Solving this (2 × 2) game is

Optimal strategy for A – [0, 6/7, 1/7, 0, 0]

Optimal strategy for B [0, 0, 4/7, 3/7, 0, 0]

The value of the game to player A is $\frac{13}{7}$.