DEPARTMENT OF STATISTICS,

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SUBJECT TITLE : **STATISTICAL QUALITY CONTROL**

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Unit IV

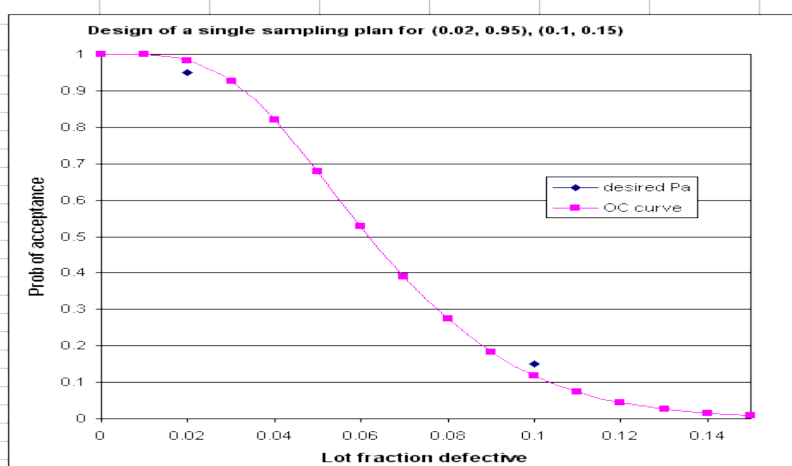
DESIGNING A SAMPLING PLAN

Suppose we want to design a single attribute sampling plan so that

1. a good lot with a defective rate of 2% will be accepted 95% of the time $(\alpha = 0.05)$, and
2. a bad lot with a defective rate of 10% will be accepted 15% of the time $(\beta = 0.15)$.

In other words, we want to find an attribute sampling plan whose *OC* curve passes through the two points (0.02, 0.95) and (0.1, 0.15). The acceptance number has a much greater effect on the P_a and hence the shape of the *OC* curve than the sample number.

Single sampling plan		
	n	c
	75	4
Lot fraction defective	Desired Pa	Bin Pa
0		1
0.01		0.9990343
0.02	0.95	0.982601
0.03		0.9249983
0.04		0.8187513
0.05		0.6788516
0.06		0.5291795
0.07		0.3902097
0.08		0.2738979
0.09		0.1840401
0.1	0.15	0.1189414
0.11		0.0742265
0.12		0.0448738
0.13		0.0263502
0.14		0.0150617
0.15		0.0083953



after experimenting with values of *n* and *c*, you should find that the sampling plan which has an *OC* curve approximately passing through the two points (0.02, 0.95) and (0.1, 0.15) is $n = 61$, $c = 3$.

Double sampling plans

Conditions for application:

- The sample units are selected from a finite lot and production is continuous.
- Production is steady, so that results of past, present and future lots are broadly indicative of a continuous process.
- Lots are submitted sequentially in the order of their production.
- Inspection is by attributes, with the lot quality defined as the proportion defective.

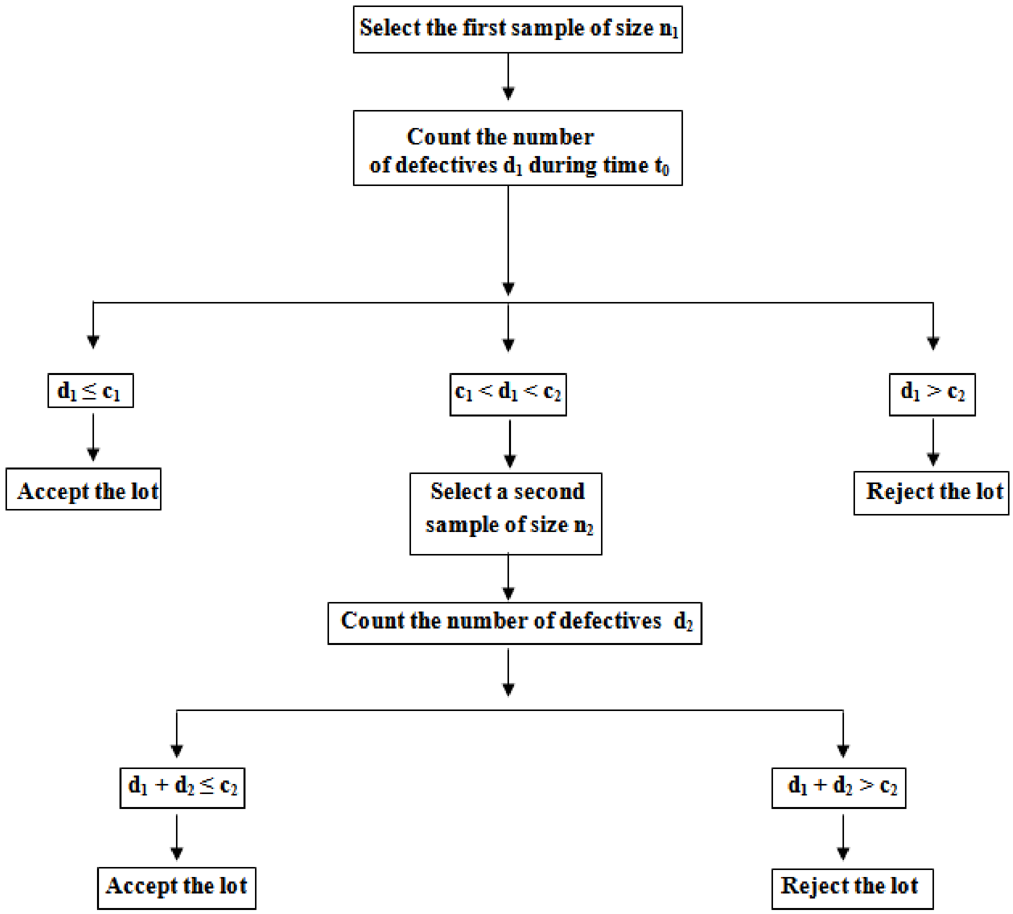
Operating procedure

Double and multiple sampling plans were invented to give a questionable lot another chance. For example, if in double sampling the results of the first sample are not conclusive with regard to accepting or rejecting, a second sample is taken. Application of double sampling requires that a first sample of size n_1 is taken at random from the (large) lot. The number of defectives is then counted and compared to the first sample's acceptance number a_1 and rejection number r_1. Denote the number of defectives in sample 1 by d_1 and in sample 2 by d_2, then:

If $d_1 \leq a_1$, the lot is accepted.
If $d_1 \geq r_1$, the lot is rejected.
If $a_1 < d_1 < r_1$, a second sample is taken.

If a second sample of size n_2 is taken, the number of defectives, d_2, is counted. The total number of defectives is $D_2 = d_1 + d_2$. Now this is compared to the acceptance number a_2 and the rejection number r_2 of sample 2. In double sampling, $r_2 = a_2 + 1$ to ensure a decision on the sample.

If $D_2 \leq a_2$, the lot is accepted.
If $D_2 \geq r_2$, the lot is rejected.



The parameters required to construct the OC curve are similar to the single sample case. The two points of interest are $(p_1, 1-\alpha)$ and (p_2, β), where p_1 is the lot fraction defective for plan 1 and p_2 is the lot fraction defective for plan 2. As far as the respective sample sizes are concerned, the second sample size must be equal to, or an even multiple of, the first sample size.

There exist a variety of tables that assist the user in constructing double and multiple sampling plans. The index to these tables is the p_2/p_1 ratio, where $p_2 > p_1$. One set of tables, taken from the Army Chemical Corps Engineering Agency for $\alpha=0.05$ and $\beta=0.10$, is given below:

Tables for n1=n2				
	accept		approximation	values
R=	numbers		of pn1	for
p2/p1	c1	c2	P=0.95	P=0.10
11.90	0	1	0.21	2.50
7.54	1	2	0.52	3.92
6.79	0	2	0.43	2.96
5.39	1	3	0.76	4.11
4.65	2	4	1.16	5.39
4.25	1	4	1.04	4.42
3.88	2	5	1.43	5.55
3.63	3	6	1.87	6.78
3.38	2	6	1.72	5.82
3.21	3	7	2.15	6.91
3.09	4	8	2.62	8.10
2.85	4	9	2.90	8.26
2.60	5	11	3.68	9.56
2.44	5	12	4.00	9.77
2.32	5	13	4.35	10.08
2.22	5	14	4.70	10.45
2.12	5	16	5.39	11.41

Tables for n2=2n1				
	accept		approximation	values
R=	numbers		of pn1	for
p2/p1	c1	c2	P=0.95	P=0.10
14.50	0	1	0.16	2.32
8.07	0	2	0.30	2.42
6.48	1	3	0.60	3.89
5.39	0	3	0.49	2.64
5.09	0	4	0.77	3.92

Tables for n2=2n1				
	accept		approximation	values
R=	numbers		of pn1	for
p2/p1	c1	c2	P=0.95	P=0.10
4.31	1	4	0.68	2.93
4.19	0	5	0.96	4.02
3.60	1	6	1.16	4.17
3.26	1	8	1.68	5.47
2.96	2	10	2.27	6.72
2.77	3	11	2.46	6.82
2.62	4	13	3.07	8.05
2.46	4	14	3.29	8.11
2.21	3	15	3.41	7.55
1.97	4	20	4.75	9.35
1.74	6	30	7.45	12.96

For example

We wish to construct a double sampling plan according to

p_1=0.01 p_2=0.05 β=0.10 $n_1=n_2$.

The plans in the corresponding table are indexed on the ratio

$R=p_2/p_1=5$.

We find the row whose R is closet to 5. This is the 5th row (R=4.65). This gives c_1=2 and c_2=4. The value of n1 is determined from either of the two columns labeled pn1.

The left holds α constant at 0.05 ($P=0.95=1-\alpha$) and the right holds β constant at 0.10 (P=0.10). Then holding α constant, we find pn_1=1.16, so n1=1.16/p_1=116. And, holding β constant, we find pn_1=5.39, so n_1=5.39/p_2=108. Thus the desired sampling plan is

n_1=108 c_1=2 n_2=108 c_2=4.

If we opt for $n_2=2n_1$, and follow the same procedure using the appropriate table, the plan is:

$n_1=77$ $c_1=1$ $n_2=154$ $c_2=4$.

The first plan needs less samples if the number of defectives in sample 1 is greater than 2, while the second plan needs less samples if the number of defectives in sample 1 is less than 2.

OC CURVE For Double sampling plan

O.C. curves quantifies manufacturer's (producer's) risk and consumer's (purchaser's) risk. This is a **graph** of the percentage defective in a lot versus the probability that the sampling plan will accept a lot.

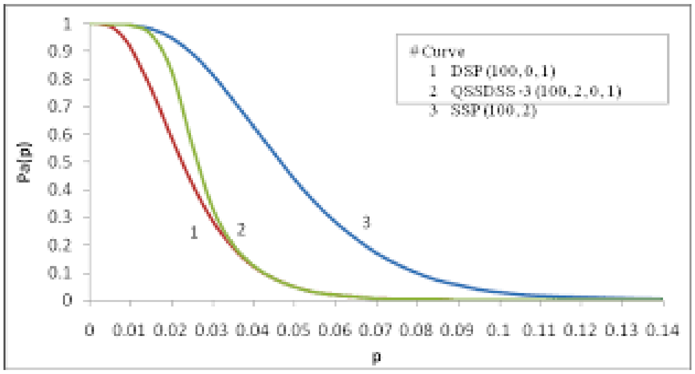
If the submitted lot quality is p, N the lot size, n_1 the size of the first sample, c_1 the acceptance number of the first sample. For the rejected lot there are two choices, either to take a second sample or reject the lot on the basis of the first sample. n_2 is the size of the second sample and c_2 is the acceptance number of the second sample. Let us denote $p_{2.}$ as the probability of getting D defectives in the first sample and $p_{.d}$ the probability of getting p defective in the second sample.

$P(A_1)$ denote the probability of acceptance on the first sample $P(A_2)$ the probability of acceptance on the basis of second sample. $P(R_1)$ denote the probability of rejection on the basis of first sample $P(R_2)$ denote the probability of rejection based on the second sample.

We can use the Poisson approximation to obtain the probability of acceptance P_a for values of the submitted lot quality.

Under Poisson model, the OC function of the Double sampling plan as given by Dodge (1959) is,

$$P_a(p) = \sum_{r=0}^{c_1} \frac{e^{-n_1 p}(n_1 p)^r}{r!} + \left[\sum_{k=c_1+1}^{c_2} \frac{e^{-n_1 p}(n_1 p)^k}{k!} \left\{ \sum_{r=0}^{c_2-k} \frac{e^{-n_2 p}(n_2 p)^r}{r!} \right\} \right]$$



ASN Curve for a Double Sampling Plan

Since when using a double sampling plan the sample size depends on whether or not a second sample is required, an important consideration for this kind of sampling is the Average Sample Number (ASN) curve. This curve plots the ASN versus p′, the true fraction defective in an incoming lot.

We will illustrate how to calculate the ASN curve with an example. Consider a double-sampling plan $n_1=50$, $c_1=2$, $n_2=100$, $c_2=6$, where n_1 is the sample size for plan 1, with accept number c_1, and n_2,c_2 are the sample size and accept number, respectively, for plan 2.

Let p′=0.06. Then the probability of acceptance on the first sample, which is the chance of getting two or less defectives, is 0.416 (using binomial tables). The probability of rejection on the second sample, which is the chance of getting more than six defectives, is (1-0.971) = 0.029. The probability of making a decision on the first sample is 0.445, equal to the sum of 0.416 and 0.029. With complete inspection of the second sample, the average size sample is equal to the size of the first sample times the probability that there will be only one sample plus the size of the combined samples times the probability that a second sample will be necessary. For the sampling plan under consideration, the ASN with complete inspection of the second sample for a p′=0.06 is

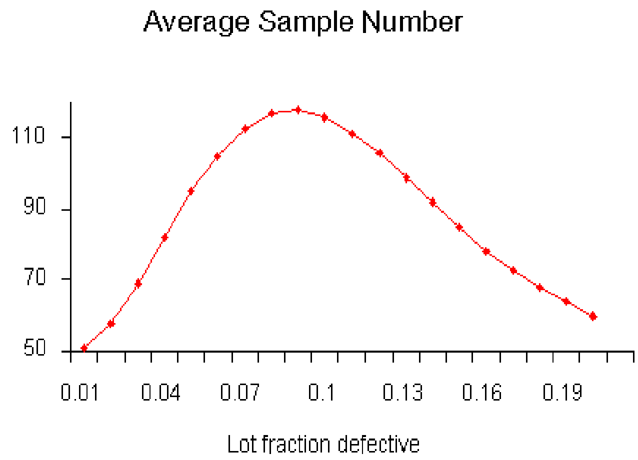
$$50(0.445)+150(0.555)=106.$$

The general formula for an average sample number curve of a double-sampling plan with complete inspection of the second sample is

$$ASN=n_1P_1+(n_1+n_2)(1-P_1)=n_1+n_2(1-P_1).$$

where P_1 is the probability of a decision on the first sample. The graph below shows a plot of the ASN versus p′.

The ASN curve for a double sampling plan



Average outgoing quality (AOQ) of Double sampling Plan

i) If the lot of size N is accepted based on first sample of size n1, $(N – n_1)$ units remain uninspected. If the incoming quality of the lot is p we expect that $p(N – n_1)$ defective units are left in the lot after the inspection on the first sample is P_{a1}. Therefore, the expected no. of defective unit per lot in the outgoing stage is $p(N - n)P_{a1}$ ------------(1)

ii) If the lot is rejected based on the first sample the lot is rejected and all the units go in for 100% inspection and defective units found are replaced by non defective units, So there is no defective unit in the outgoing stage. The probability the lot will be rejected based on the first sample is $(1 – P_{a1})$. The expected no. of defective units per lot at the outgoing stage is
$0 \times (1 – P_{a1}) = 0$ -------------(2)

iii) If the lot is accepted based on the second sample of size n_2, $(N – n_1 -n_2)$ defective units are left in the lot after inspection of the second sample. The probability the lot will be accepted based on the second sample is P_{a2}. Therefore, the expected no. of defective units per lot in the outgoing stage is $p (N – n_1 -n_2) P_{a2}$. ---------------(3)

iv) If the lot is rejected based on the second sample all the units of the lot go in for 100% inspection and all defective units found are replaced by non-defective units, so there is no significant defective units in the outgoing stage. The probability that the lot will be rejected in the second stage is $(1 - P_{a2})$. The expected no. of defective units per lot at the outgoing stage is

$0 \times (1 - P_{a2}) = 0$ -------------(4)

Therefore, the Average Outgoing Quality is

$$AOQ = \left(\frac{p}{N}\right)\left[(N - n_1)P_{a1} + (N - n_1 - n_2)P_{a2}\right]$$

where $P_{a1} = \sum_{r=0}^{c_1} \frac{e^{-n_1 p}(n_1 p)^r}{r!}$,

$$P_{a2} = \sum_{k=c_1+1}^{c_2} \frac{e^{-n_1 p}(n_1 p)^k}{k!} \left\{ \sum_{r=0}^{c_2-k} \frac{e^{-n_2 p}(n_2 p)^r}{r!} \right\}$$

Average Total Inspection

Average Amount of Total Inspection (ATI) The expected value of the sample size required for coming to a decision in an acceptance rectification sampling inspection plan calling for 100% inspection of the rejected lots is called average amount of total inspection. It is a function of the incoming quality. The curve obtained on plotting ATI against p is called the ATI Curve.

$$ATI = n_1 P_{a1} + (n_1 + n_2)(P_a - P_{a1}) + N(1 - P_a)$$

$$= n_1 + n_2(1 - P_{a1}) + (N - n_1 - n_2)(1 - P_a)$$

as only n_1 units will be inspected if the lot is accepted based on the first sample or $(n_1 + n_2)$ units will be inspected if the lot is accepted based on the second sample or all the N units will be inspected if the lot is rejected.

Advantages of double sampling plans

1. One of the possible advantages in the double or sequential sampling procedures is that, if a lot is very bad or if it is very good, it can be either rejected or accepted with small initial sample/samples
2. A double sampling plan may give similar levels of the consumer's and the producer's risk but require less sampling in the long run than a single sampling plan.

Determination of the Parameters of SSP. (i.e, n & c).

The lot size N is invariably known. Thus, the two unknown quantities that need to be determined in the sampling plan are 'n' and 'c'.

In a lot of incoming quality p, the number of defectives pieces is Np and non-defective pieces is $N - Np = N(1-p)$. The probability of getting exactly x defectives in a sample of size n from this lot is given by

$$g(x,p) = \left[\binom{Np}{x} \times \binom{N-Np}{n-x}\right] \Big/ \binom{N}{n}. \quad [\text{hypergeometric dist.}]$$

Probability of accepting a lot of quality p is

$$P_a(p) = \sum_{x=0}^{c} g(x,p) = \sum_{x=0}^{c} \left[\binom{Np}{x} \times \binom{N-Np}{n-x} \Big/ \binom{N}{n}\right] \quad \text{— (1)}$$

Hence, the consumer's risk is given by

$$P_c = P[\text{Accepting a lot of quality } p_t]$$
$$= \sum_{x=0}^{c} g(x, p_t) = \sum_{x=0}^{c} \left[\binom{Np_t}{x} \times \binom{N-Np_t}{n-x} \Big/ \binom{N}{n}\right] \quad \text{— (2)}$$

To protect himself against poor quality, the consumer usually demands a small value of P_c for given p.

The producer's risk is given by:

$$P_p = P[\text{Rejecting a lot of quality } \bar{p})$$
$$= 1 - P[\text{Accepting a lot of quality } \bar{p})$$
$$= 1 - \sum_{x=0}^{c} g(x, \bar{p})$$
$$= 1 - \sum_{x=0}^{c} \left[\binom{N\bar{p}}{x} \times \binom{N-N\bar{p}}{n-x} \Big/ \binom{N}{n}\right] \quad \text{—— (3)}$$

If the process average fraction defective is $\bar{p}$ as claimed by the producer, then the average amount of Total Inspection per lot is $ATI = n + (N-n)P_p$. —— (4)

Since n items have to be inspected in each case and the remaining $(N-n)$ items will be inspected only if $d > c$, i.e., if the lot is rejected when the lot quality is $\bar{p}$, and the probability for this is P_p.

The computation of hypergeometric probabilities is extremely difficult and the Binomial approximation to hypergeometric distribution is used.

$$P_c = \sum_{x=0}^{c} \binom{Np_i}{x} \left(\frac{n}{N}\right)^x \left(1 - \frac{n}{N}\right)^{Np_i - x} \quad \text{—— (5)}$$

$$P_p = 1 - \sum_{x=0}^{c} \binom{n}{x} (\bar{p})^x (1-\bar{p})^{n-x} \quad \text{—— (6)}$$

In most of the practical problems, $\bar{p}$ is likely to be less than 0.10 and n is likely to be sufficiently large to use Poisson approximation to Binomial distribution. Thus

$$P_p = 1 - \sum_{x=0}^{c} \left[\frac{(n\bar{p})^x e^{-(n\bar{p})}}{x!} \right]. \quad \text{—— (7)}$$

and consequently,

$$ATI = n + (N-n)\left[1 - \sum_{x=0}^{c} \left\{ \frac{e^{-(n\bar{p})} (n\bar{p})^x}{x!} \right\} \right] \text{— (8)}$$

Consumer's requirement fixes the values of P_c and p_i, Consumer's requirement fixes the values of P_c and p_i the N is always fixed. For given values of P_c and p_i the N is always fixed. For given values of P_c and p_i the unknowns n and c equation (2) which involves two unknowns n and c. is satisfied by a large number of pairs of n and c. To safegaurd producer's interest, out of these possible pairs one involving the minimum amount of inspection as given in (3) is chosen. Dodge and Romig, by applying numerical methods of solution of equations, have prepared extensive tables, for minimising values of n and c for $P_c = 0.10$ and different values of $\bar{p}$.