DEPARTMENT OF STATISTICS,

GOVERNMENT ARTS COLLEGE, COIMBATORE – 641018

SUBJECT TITLE : STATISTICAL QUALITY CONTROL

SUBJECT CODE : 18 BST 63C

PREPARED BY : Dr. P. VASANTHAMANI

MOBILE NUMBER : 9994575462

Unit III

Product Control:

Product control is another method of **statistical** quality **control** in which the quality of a **product** is controlled while the **product** is ready to dispatch or sell to the customers. **Product control** used the technique of acceptance sampling to detect defects and **control** the quality of a **product**.

Acceptance sampling uses statistical **sampling** to determine whether to accept or reject a production lot of material. It has been a common quality control technique used in industry. It is usually done as products leaves the factory, or in some cases even within the factory.

Meaning of Acceptance Sampling or Sampling Inspection:

One method of controlling the quality of a product is 100% inspection which requires huge expenditure in terms of time, money and labour. Moreover, due to boredom and fatigue involved in repetitive inspection process, there exists a possibility to overlook and some defective products may pass the inspection point. Also when the quality of a product is tested by destructive testing (e.g., life of a candle or testing of electrical fuses) then 100% inspection shall destroy all the products.

The alternative is statistical sampling inspection methods. Here from the whole lot of products/items to be inspected, some items are selected for inspection.

If that sample of items conforms to be desired quality requirements then the whole lot is accepted, if it does not, the whole lot is rejected. Thus the sample items are considered to be the representative of the whole lot. This method of acceptance or rejection of a sample is called Acceptance Sampling.

In general acceptance sampling method proves to be economical and is used under the assumption when the quality characteristics of the item are under control and relatively homogeneous.

Acceptance sampling is an inspection procedure used to determine whether to accept or reject a specific quantity of material. As more firms initiate total quality management (TQM) programs and work closely with suppliers to ensure high levels of quality, the need for acceptance sampling will decrease. The TQM concept is that no defects should be passed from a producer to a customer, whether the customer is an external or internal customer. However, in reality, many firms must still rely on checking their materials inputs.

The basic procedure is straightforward.

1. A random sample is taken from a large quantity of items and tested or measured relative to the quality characteristic of interest.
2. If the sample passes the test, the entire quantity of items is accepted.
3. If the sample fails the test, either (a) the entire quantity of items is subjected to 100 percent inspection and all defective items repaired or replaced or (b) the entire quantity is returned to the supplier.

Industrial Uses of Acceptance sampling

1. To determine the quality and acceptability of incoming raw materials, component parts, products etc.
2. To decide the acceptability of semi-finished products for further processing as it undergoes the operations from machine to machine or section to section within the factory.
3. To determine the quality of outgoing products. 4. For improving maintaining and controlling the quality of the products manufactured. 3.3. Basic in Acceptance Sampling In any production process, the producer gets his lot checked at various stages or the customer is anxious to satisfy himself about the quality of goods. An ideal way of doing this, seems to inspect each and every item presented for acceptance.

Terms Used in Acceptance Sampling:
Following terms are generally used in acceptance sampling:

PRODUCER’S AND CONSUMER’S RISK

Acceptance sampling involves both the producer (or supplier) of materials and the consumer (or buyer). Consumers need acceptance sampling to limit the risk of rejecting good-quality materials or accepting bad-quality materials. Consequently, the consumer, sometimes in conjunction with the producer through contractual agreements, specifies the parameters of the plan. Any company can be both a producer of goods purchased by another company and a consumer of goods or raw materials supplied by another company.

Consumer's risk:

Any sampling scheme would involve certain risk on the part of the consumer in the sense that he has to accept certain % of bad lots. i.e., lots of quality $p_{t.}$ or greater fraction defective.
Consumers' Risk is the probability of wrongly accepting a lot that is not of acceptable quality. It is a point on the OC curve corresponding to a predetermined and usually low probability of acceptance. This probability is then called the 'consumer's risk' and the corresponding lot quality is called the Limiting Quality (LQ or LQL).
Consumer's risk is the probability of accepting a lot with fraction defective $p_{t.}$ and is denoted by β. i.e., β = P[accepting a lot of quality $p_{t.}$]. it is usually taken as 0.10 or 10%.

Producer's risk:
The producer has also to face the situation that some good lots will be rejected. Producer's risk is the prob. of rejecting a good lot denoted by α.
α = P [rejecting a lot of quality $\bar{p}$] where $\bar{p}$ is process average fraction defective.
Producer's Risk is the probability of wrongly rejecting a compliant lot, with level of non-conformance at or below the acceptance quality limit (or proportion of non-conforming units for lots consisting of discrete items). Generally, the quality of an acceptable lot is expressed as the Acceptance Quality Limit. It is a point on the OC curve corresponding to some predetermined and usually low probability of rejection. This probability of rejection is called the 'producer's risk'.

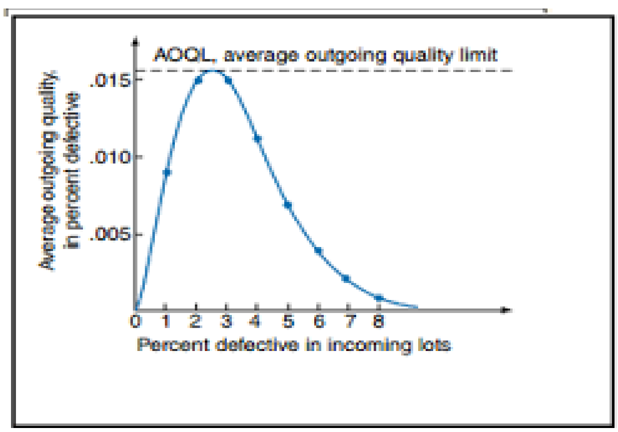
Acceptable Quality Level (AQL):
The producer wishes to maintain the standard of quality at a specified level agreed to by the consumer.
If α is the producer's risk then the quality which results in 100 x (1 – α)% acceptance of the good lots submitted for inspection is called AQL.
Usually P[rejecting a lot of quality $p_{1.}$] = 0.05
P_α = P[accepting a lot of quality $p_{1.}$] = 0.95.
p_1 is the AQL And the lot of this quality is considered satisfactory

It is the desired quality level at which probability of an acceptance is high. It represents maximum proportion of defectives which the consumer finds acceptable, or it is the maximum percent defectives that for the purpose of sampling inspection can be considered satisfactory.

Lot Tolerance Percent Defective (LTPD) or Rejectable Quality Level (RQL):
The lot tolerance proportion defective usually denoted by p_t is a lot quantity which is considered bad by the consumer. This is a maximum% or proportion defective that a consumer can tolerate.
The consumer may specify a proportion defective p_t he will
It is the quality level at which the probability of acceptance is low and below this level the lots are rejected. This prescribes the dividing line between good and bad lots. Lots at this quality level are considered to be poor.

Average outgoing Quality (A.O.Q):
Acceptance sampling plans provides the assurance that the average quality level or percent defectives actually going to consumers will not exceed certain limit.
The AOQ curve indicates that as the actual percent defectives in a production process increases, initially the effect is for the lots to be passed for acceptance even though the number of defectives has gone up and the percent defectives going to the consumer increases.

The AOQ curve starts from the point (0,0) as a submitted lot having zero defectives assured to have zero defectives in the accepted lot also. The curve raises to a maximum and false on the other side reaching very near zero, for very bad lots since many of these bad lots would be screened and the screening inspection would be large and hence AOQ will be small. This AOQ curve has a maximum and this maximum is the AOQL of the sampling plan.



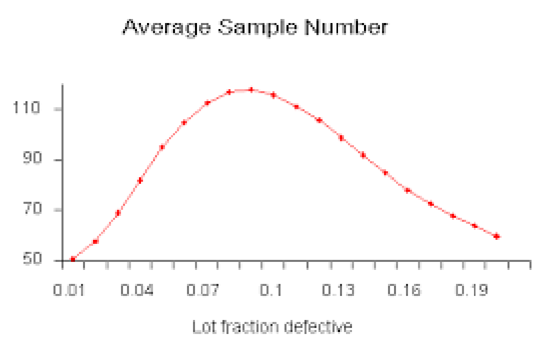
Process average fraction defective
Process average for any manufactured product is obtained by finding the % of defectives. In the product over long time this is denoted by $\bar{p}$.

Rectifying Inspection Plans
Lot by lot sampling plans in which a specified quality objective is attained through corrective inspection of rejected lots is rectifying inspection plans. The inspection of the rejected lots and replacing the defective pieces found in the rejected lots by good ones eliminates the number of defectives in the lot to a great extent thus improving the lot quality.
Most of the rectified inspection plans for lot by lot sampling calls for 100% inspection of the rejected pieces found by good ones.
The two important points related to rectifying inspection pans are

i) The average quality of the product after sampling and 100% inspection of rejected lots called the average outgoing quality (OQ) and
ii) The average amount of inspection required for the rectified inspection plan called the (ATI).

Average Total Inspection (ATI)
It relates to rectification inspection plans. Average number of items inspected per inspection from the lot under a particular sampling plan is known as ATI.

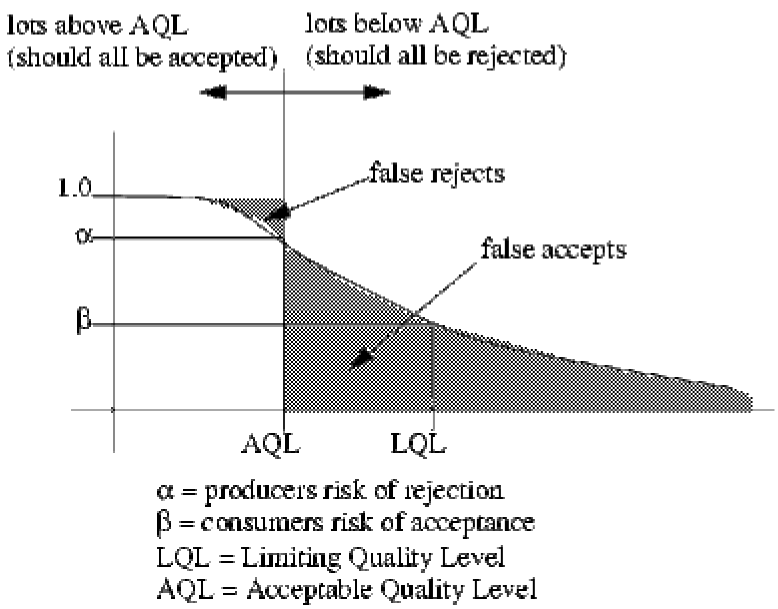
Average Sample Number (ASN)
Average sample number is the expected size of the sample required to arrive at a decision about the acceptance or rejection of the lot under sampling inspection plan. This depends on 'p', the actual proportion of defectives percent in the lot. The graph of the ASN $E_p{}^{(n)}$ against proportion 'p' of defectives is known as ASN curve.



A sampling plan resulting in to two lowest ASN curve is considered better than any other sampling plan, all other factors kept constant.

Indifference Quality Level(IQL)
IQL is somewhere between AQL and RQL (LTPD). This is frequently defined as quality level having probability of acceptance of 0.5 for a sampling plan.

Limiting Quality Level(LQL)
It is the proportion of non-conforming items associated with the consumer's risk. It can be regarded as minimum quality that the consumer would not want to accept even for a single batch. The AQL is the minimum long-term average.
LQL is a level of quality for a product measured by inspection below which the customer is absolutely not able to accept the product. It is determined by identifying the proportion of defective output from the process that cause the customer to reject entire batches.



Operating Characteristic Curves (OC curve)
OC curve is a sampling plan is a graphic representation of a relation between the probability of acceptance $P_a(p)$ for variations in the incoming lot quality p.
The OC curve depicts or shows the discriminating power of an acceptance sampling plan. The OC plots the probabilities of accepting a lot against the fraction defective p'
For example: If the actual percentage defective p is 10% there is probability of 0.097 of accepting this lot and 0.903 probability of rejecting it. With no defective the probability of acceptance will be 1.

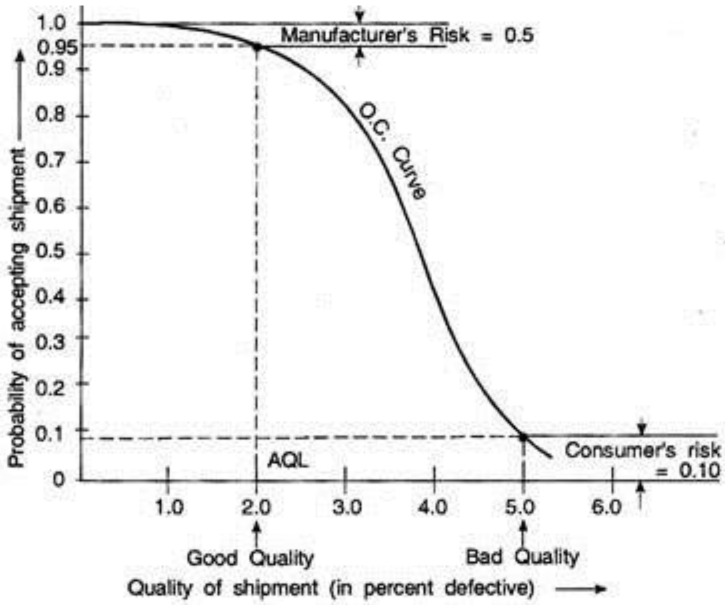


Fig. 60.1. O.C. Curve indicating sampling risks.

If the curve is steeper, it indicates a better sampling plan and this can be got by increasing the sample size.

Types of OC curves:
There are two types of OC curves Type A and Type B.
Type A OC curve gives the probability of acceptance for an individual lots coming from finite population,
Type B OC curve gives the probability of acceptance from lots coming from continuous process.

Sampling Inspection plan for attributes

The commonly used sampling inspection plans for attributes are
1.Sinle Sampling Plans
2. Double sampling plans and
3. Sequential sampling Plans

Single Sampling Plans
If the decision of accepting or rejecting the lot is made on the basis of one sample only the acceptance sampling plan is described as Single sampling plan.
It is based on three parameters N, n and c. where
N is the lot size,
n is the sample size and

c is the acceptance number i.e., maximum number of allowable defectives in the sample.
The single sampling plan can be described as follows.
Step 1 Select a random sample of size n from a lot of size N.
Step 2 Inspect all the items from the sample. Let d be the number of defectives in the sample.
Step 3 if d ≤ c, accept the lot by replacing the defectives found in the sample by standard ones.
Step 4 If d > c, reject the lot. In this case we inspect the entire lot and replace all the defectives by standard ones.
Single sampling plan is very simple to understand and carry out. The basic problem is the choice of n and c which has to be determined in advance. The most economical single sampling plan is obtained by minimizing the Average total inspection.

There are three ways to calculate the probability of lot acceptance. Suppose that the sampling plan is (*N*, *n*, *c*) and the quality level of the lot is *p*. The first method is an exact one. The number *X* of defectives found in a sample will follow a Hypergeometric distribution, so the probability of lot acceptance is

$$Pa = P(X \le c) = \sum_{i=0}^{c} \frac{\binom{Np}{i}\binom{N-Np}{n-i}}{\binom{N}{n}}, \quad c = 0, 1, 2, \ldots, \min(n, Np).$$

Note that *Np* is an integer in this formula.

The second method is approximate. When the ratio $\frac{n}{N}$ is small, the Hypergeometric distribution can be approximated well by the Binomial distribution. Thus the probability of acceptance can be approximated by

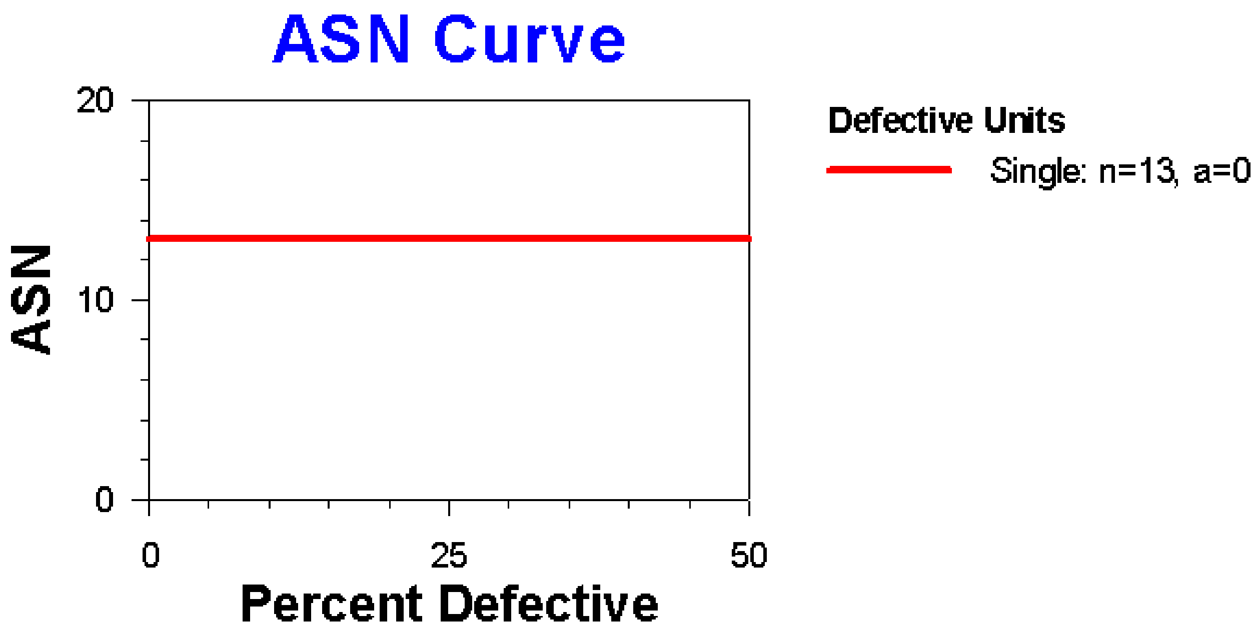
$$Pa = P(X \le c) = \sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i}.$$

The third method is by using Poisson distribution. Here n →α, p →0 and np is a constant.

$$Pa = P(X \le c) = \sum_{x=0}^{c} \frac{e^{-\lambda}\lambda^x}{x!} \qquad \text{Where } \lambda = np$$

Average Sample number ASN

ASN for single sampling plan is the sample size n. ASN = E(n) = n



The ASN curve shows the average number of units inspected (y-axis) for different incoming qualities (bottom axis). For single sampling plans the ASN is a constant so the ASN curve is a straight across line.

Average Outgoing Quality (AOQ) we can also calculate the AOQ for a (n,c) sampling plan, provided rejected lots are 100 % inspected and defectives are replaced with good parts.

Assume all lots come in with exactly a p0 proportion of defectives. After screening a rejected lot, the final fraction defectives will be zero for that lot. However, accepted lots have fraction defective p0. Therefore, the outgoing lots from the inspection stations are a mixture of lots with fractions defective p0 and 0. Assuming the lot size is N, we have.

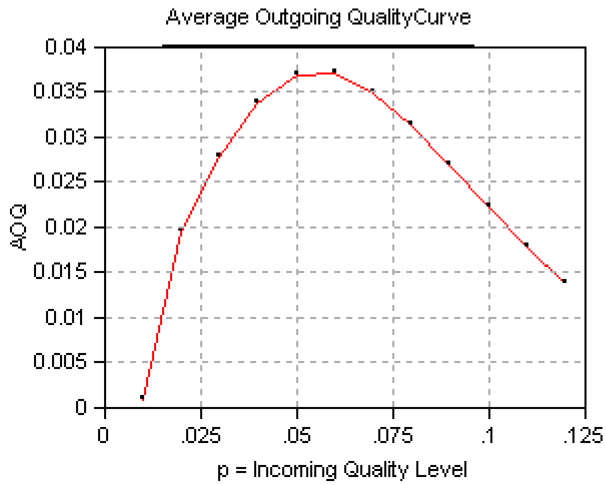
$$AOQ=Pap(N-n)N.$$

For example, let N=10000, n=52, c=3, and p, the quality of incoming lots, equals 0.03. Now at p=0.03, we glean from the OC curve table that pa=0.930 and

$AOQ=(0.930)(0.03)(10000-52)10000=0.02775.$

Setting p=0.01,0.02,…,0.12, we can generate the following table.

AOQ	p
0.0010	0.01
0.0196	0.02
0.0278	0.03
0.0338	0.04
0.0369	0.05
0.0372	0.06
0.0351	0.07
0.0315	0.08
0.0270	0.09
0.0223	0.10
0.0178	0.11
0.0138	0.12



The Average Total Inspection (ATI)

If all lots contain zero defectives, no lot will be rejected.

If all items are defective, all lots will be inspected, and the amount to be inspected is N.

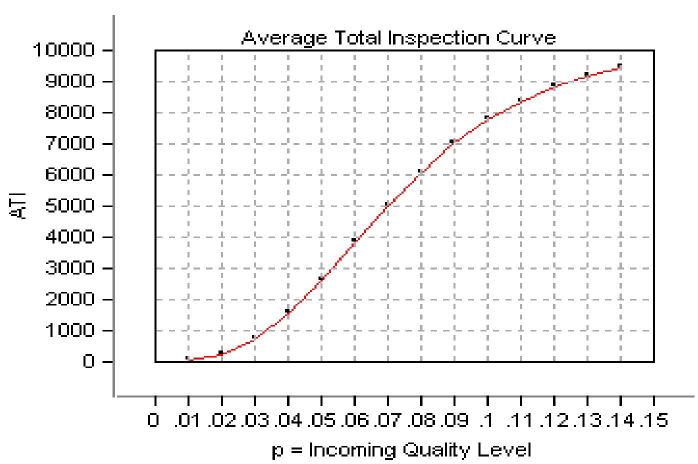
Finally, if the lot quality is $0 < p < 1$, the average amount of inspection per lot will vary between the sample size n, and the lot size N.

Let the quality of the lot be p and the probability of lot acceptance be Pa, then the ATI per lot is ATI = n + (1−Pa)(N−n).

For example, let N=10000, n=52, c=3, and p=0.03. We know from the OC table that Pa=0.93. Then ATI = 52+(1−0.930)(10000−52)=753. (Note that while 0.930 was rounded to three decimal places, 753 was obtained using more decimal places.)

Setting p=0.01,0.02,…,0.14 generates the following table.

ATI	p
70	0.01
253	0.02
753	0.03
1584	0.04
2655	0.05
3836	0.06
5007	0.07
6083	0.08
7012	0.09
7779	0.10
8388	0.11
8854	0.12
9201	0.13
9453	0.14



OC Curve

The OC curve depicts or shows the discriminating power of an acceptance sampling plan. The OC plots the probabilities of accepting a lot against the fraction defective p’

We assume that the lot size N is very large, as compared to the sample size n, so that removing the sample doesn't significantly change the remainder of the lot, no matter how many defects are in the sample. Then the distribution of the number of defectives, d, in a random sample of n items is approximately binomial with parameters n and p, where p is the fraction of defectives per lot.

The probability of observing exactly d defectives is given by

$$Pa = P(X \le c) = \sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i}$$

Using this formula with n=52, c=3, and p=0.01,0.02,…,0.12, we find:

Pa	Pd
0.998	0.01
0.980	0.02
0.930	0.03
0.845	0.04
0.739	0.05
0.620	0.06
0.502	0.07
0.394	0.08
0.300	0.09
0.223	0.10
0.162	0.11
0.115	0.12