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UNIT II

CRITERIA FOR DEDUCTING LACK OF CONTROL using control charts

A control chart is a graph mainly derived from a normal distribution curve. The y-axis denotes a quality characteristic or a particular characteristic of the product or process, which is controlled and is marked in units, in which the test value is expressed. The x-axis consists of time intervals or sample number. There is a center line, which is the average of the value of the studied matter or may also indicate the nominal value. The upper boundary characterizes the upper control limit (UCL), while the lower designates the lower control limit (LCL), respectively. The gathered data are plotted in sequence, and then, the pattern occurring on the chart is interpreted. A sample of a control chart is given in Figure 1.

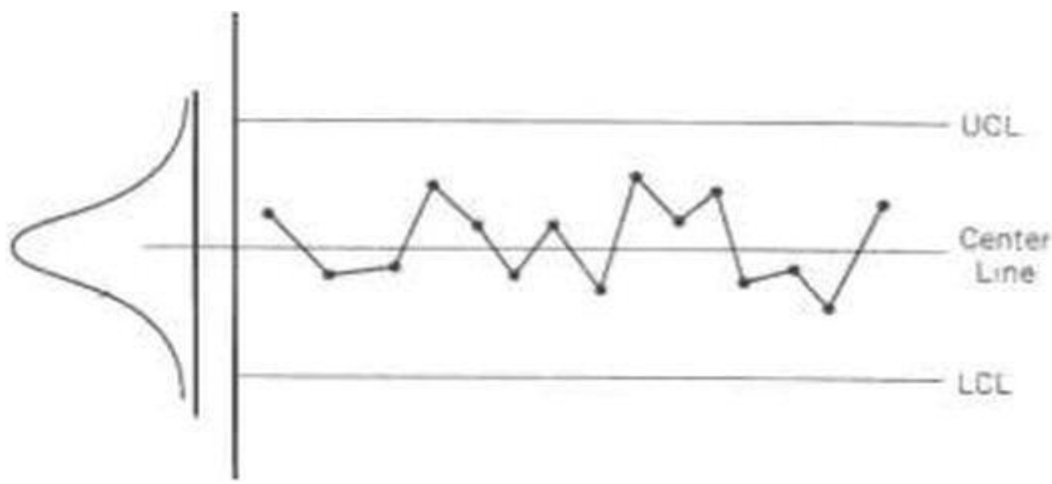
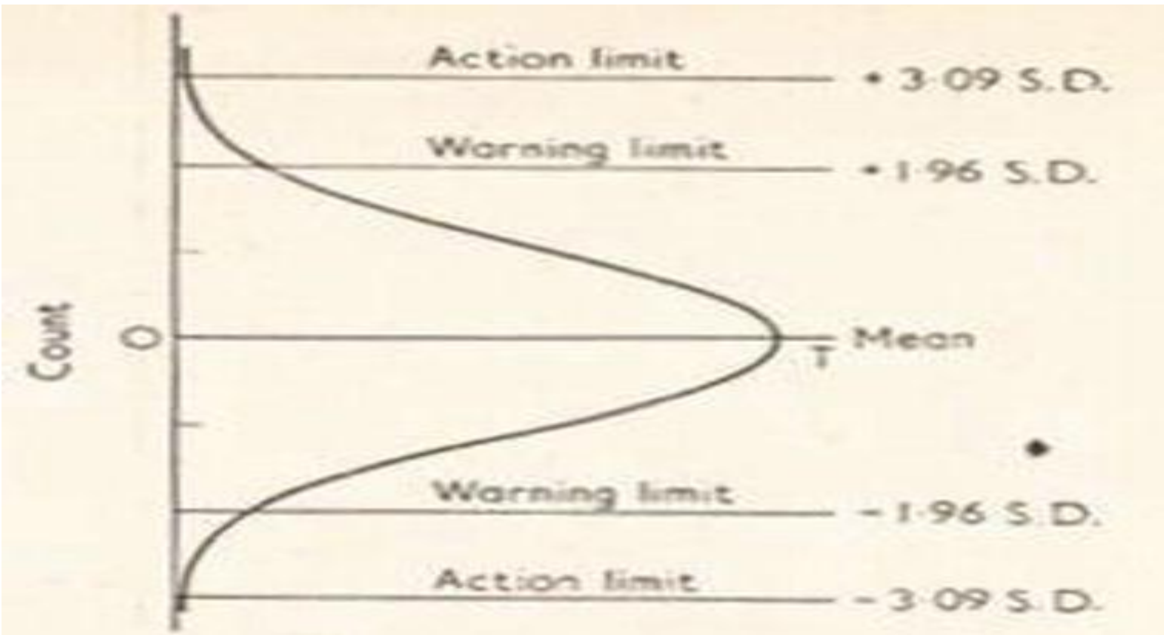


Figure 1.

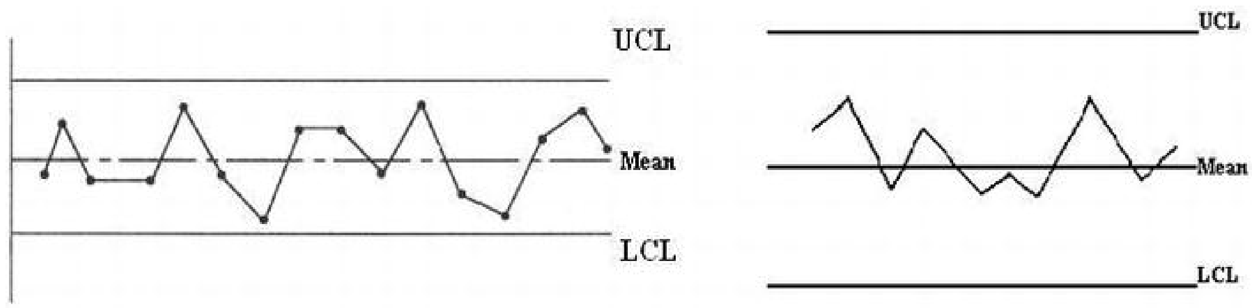
Sample of a control chart.

As can be seen from Figure 1, there is a close relationship between the normal distribution curve and the control chart. Control charts are constructed on the basis of expanding the sigma limits above and below of the mean. By taking a deeper look, it can be expressed that expansion of 1.962 σ from the mean may be regarded as the "warning limit," and the expansion of 3.09 σ from he mean is the "action limit" for large samples (Figure 2). Similarly, $2\sigma/\sqrt{n}$ and $3\sigma/\sqrt{n}$ are the same limits, respectively, for small samples (Figure 3)



The distribution of the points on a control chart is important, and the patterns occurring on the control chart must be examined and interpreted. Since the values distribute at a distance around the mean value and support visually the variation in the spread of the test results, they provide useful information about the process to make modifications in order to reduce variability. For interpreting the control charts, the principles of the control charts must be known, and their users must be familiar with the process. It is the author's view that during the interpretation of control charts, not only statistics but also experience and common sense must be combined with it. If there is a run toward the warning limit, this may suggest that a change must be made. On the other hand, a similar run would also mean that a change in time may prevent the next item from lying outside the limits. This must be evaluated for every occasion on its own.

Two examples of a typical control chart where production is under control or a normal behavior is noticed.



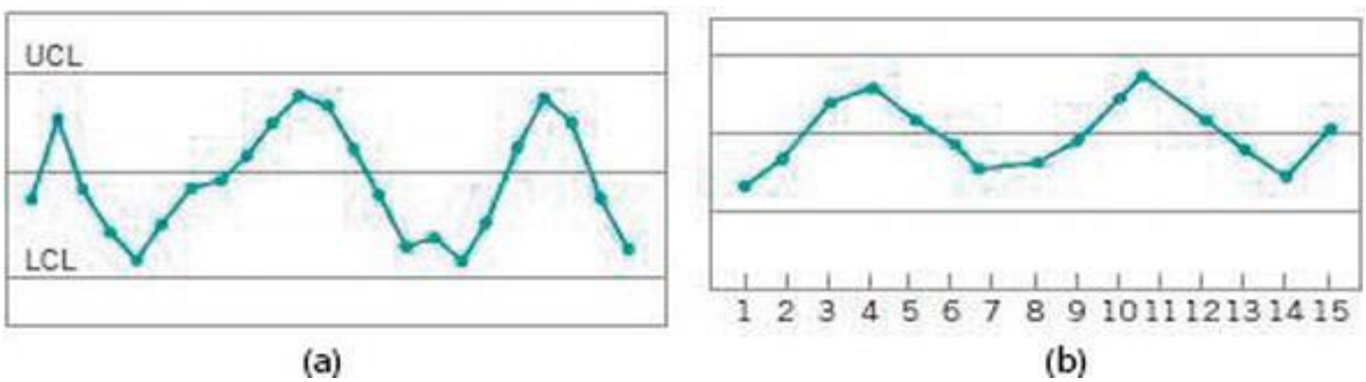
Two examples of a typical control chart.

The main interpretation of control charts is that all the points should lie in between the UCL and LCL. If sample points fall in between the control limits in a continued production, then the process is in control, and as such, no action must be taken. If a point falls out of them, then the process is out-of-control, and further investigative and corrective action ought to be taken. If, however, points get close to the UCL and LCL's, one must search for the root of the problem and solve it without stopping production. If on the other hand, points cross the UCL and LCL's, production must be stopped, and the problem must be investigated and solved. Faulty production is worse than no production.

On the other hand, even if none of the points lie out of the control limits, this does not mean that the chance factor had played a role. All the points on the control chart may lie in between the UCL and LCL's like a typical chart but this does not mean that production is under control. Incidentally, they may well be out-of-control soon. The reason for this is the pattern occurring on the control chart. Patterns give information about the condition of the process, and their early identification may trigger the alarm for the user to investigate their causes and to prevent any faults before they occur. Patterns having deviations from normal behavior are indicators of raw material, machine (setting, adjustment, tool abrasion, and systematic causes of deterioration) or measuring method, human, and environmental factors starting to change the quality characteristic of the product. To interpret control charts, every cause must be studied one by one and investigated and corrective action ought to be taken.

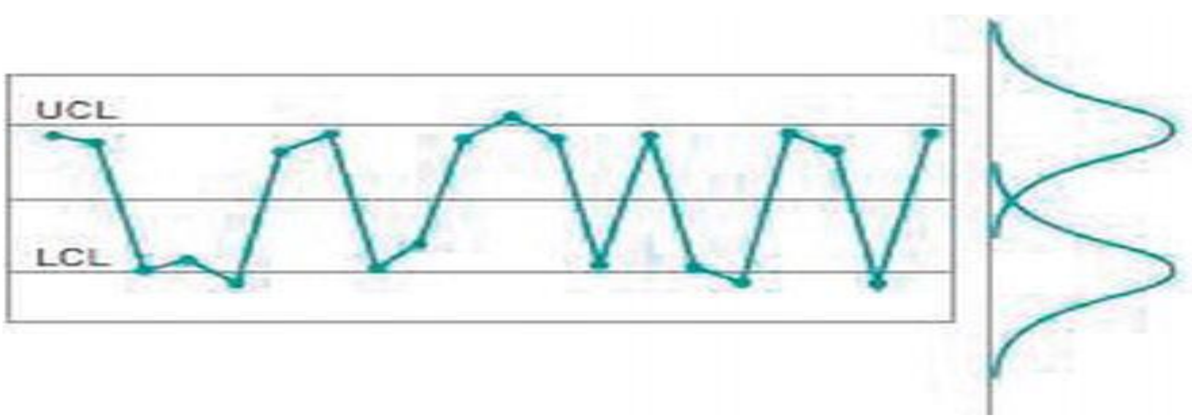
$\bar{X}$ and R charts are interpreted together. If the underlying distribution is normal, then the two charts are statistically independent, and their joint consideration gives the user more information about the process. If there is an assignable cause in the process, it will show itself in both. If the underlying distribution is not normal, this nonnormality effects the $\bar{X}$ and R charts, leading corrective action not to be taken on time. As such, normality tests must be done at the beginning. $\bar{X}$ and s charts are also interpreted together.

Patterns occurring on control charts **Cyclic patterns:** Two examples of control charts showing a cyclic pattern are given in Figure . An $\bar{X}$ control chart having a cyclic pattern between the UCL and LCL may result from systematic environmental changes, such as temperature or heat or stress buildup, raw material deliveries, operator fatigue, regular rotation of operators and/or machines, and fluctuation in voltage or pressure. An R-control chart having a cyclic pattern may result from maintenance schedules, fatigue, or tool wear. The process is not out-of-control, but elimination or reduction of the source of variability will improve the product.



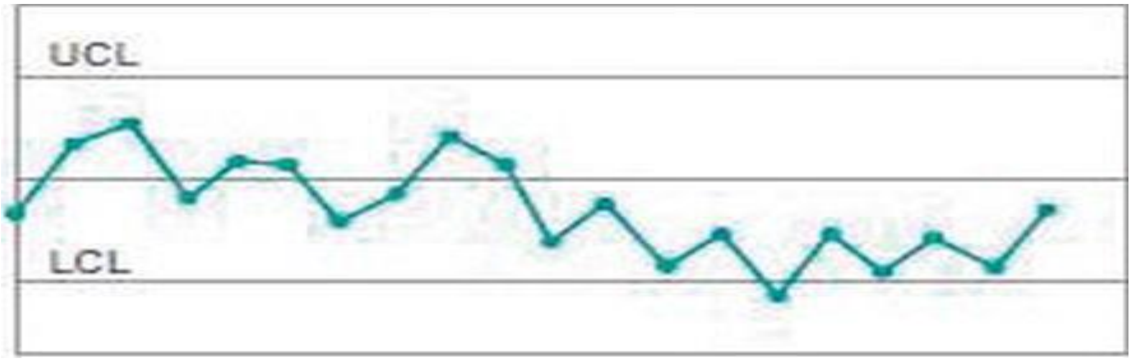
Two examples of control charts showing a cyclic pattern.

Mixture: An example of a control chart showing a mixture pattern is given in Figure .In a mixture pattern, the plotted points gather around the UCL and LCL, but few points fall near the center line. In this outline, there are two or more overlapping distributions generating the process output. An $\bar{X}$ control chart having a mixture pattern may be the result of "over-control," where process adjustments are done too often, or if many machines do the same production, but are adjusted wrongly.



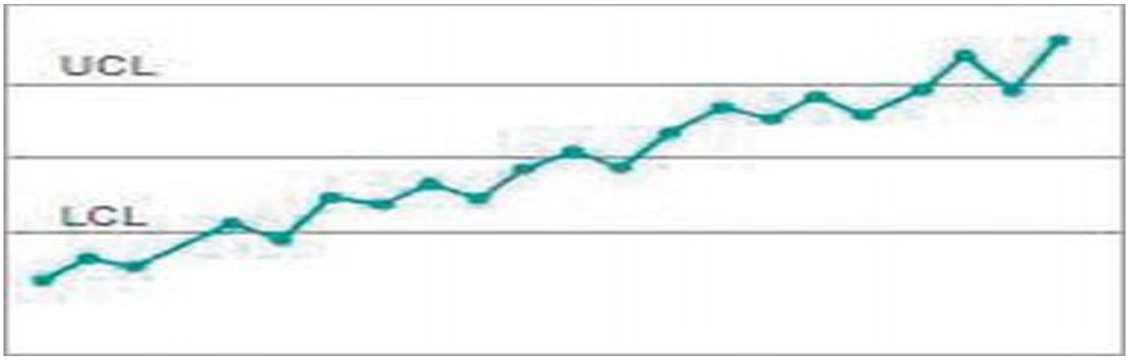
Example of a control chart showing a mixture pattern.

Shift in process level: An example of a control chart showing a shift in process level pattern is given in Figure . An $\bar{X}$ control chart having a shift in process level pattern may result from introduction of new workers, methods, raw material, machine, change in the inspection method or standards, change in the either skill, attentiveness, or motivation of the operators.



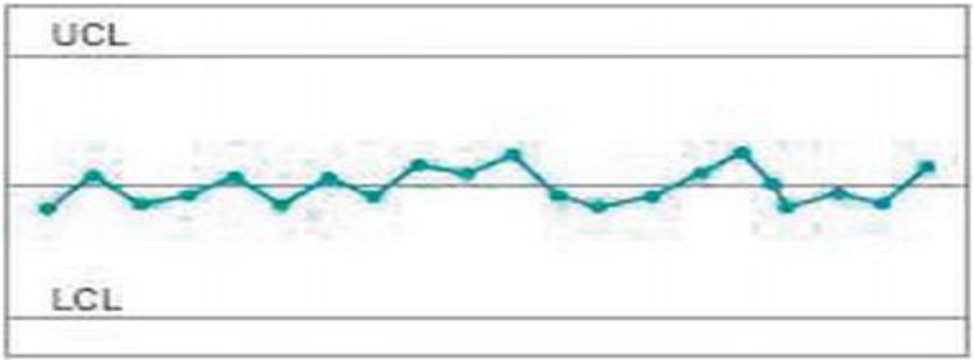
Example of a control chart showing a shift in process level pattern.

Trend: An example of a control chart showing a trend pattern is given in Figure . In a trend pattern, the plotted points continuously move in one direction. An $\bar{X}$control chart having a trend pattern may result from gradual wearing or deterioration of a tool or component, human causes, such as operator fatigue or the presence of supervision, and seasonal influences like temperature.



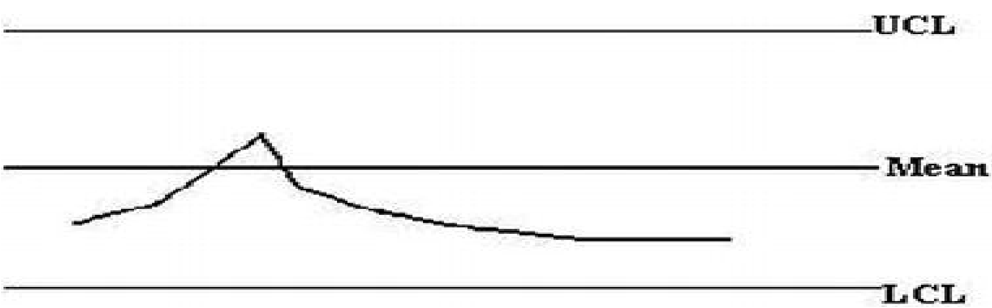
Example of a control chart showing a trend pattern.

Stratification: An example of a control chart showing a stratification pattern is given in Figure . In a stratification pattern, the plotted points tend to cluster around the center line, and there is a lack of natural variability in the pattern. An $\bar{X}$control chart having a stratification pattern may result from incorrect calculation of the control limits, or if there are subgroups, several different underlying distributions might be collected in the sampling process.



Example of a control chart showing a stratification pattern.

Approaching LCL: An example of a control chart showing an approach to LCL pattern is given in Figure. A p-control chart having an approach to LCL pattern may represent a real improvement in process quality. But, downward shifts are not always attributable to improved quality. This is due to the fact that errors in the inspection process may be resulting from inadequately trained or inexperienced inspectors or from improperly calibrated test and inspection equipment during that particular shift. Besides, inspection may pass nonconforming units owing to a lack in training. The same interpretation is valid for np-control charts also.



Example of a control chart showing an approach to LCL pattern.

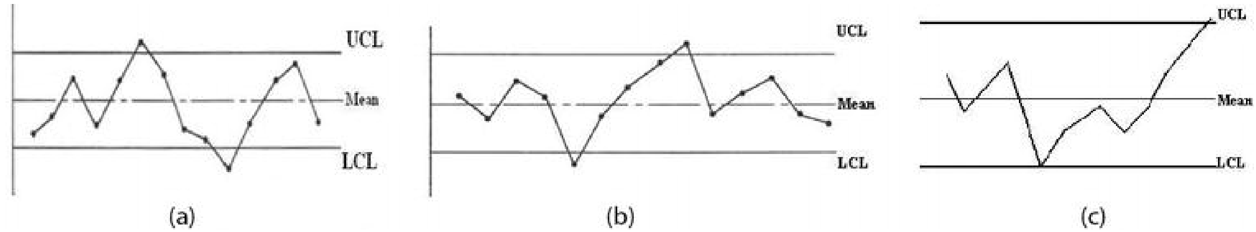
Approaching UCL or LCL: An example of a control chart showing an approach to UCL or LCL pattern is given in Figure. A c-control chart having approached the UCL line may be because of temperature control and an approach to the LCL may be due to inspection error.

Example of a control chart showing an approach to UCL or LCL pattern.

Categorical guidelines other than patterns

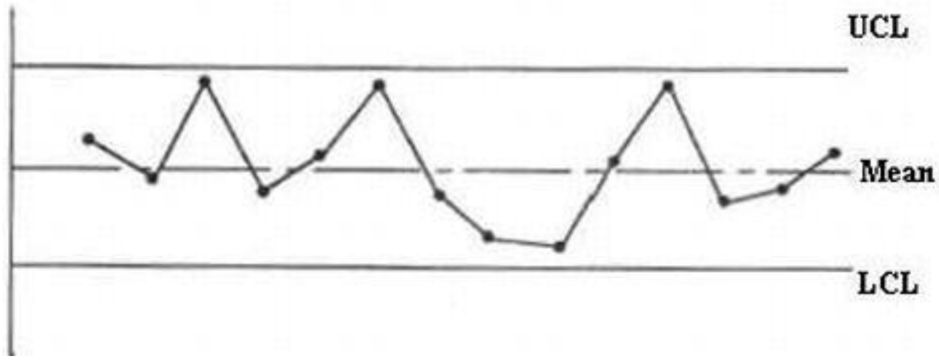
Some definitive guidelines are developed to interpret control charts. Keeping in mind that the main principle is none of the points should cross UCL or LCL, the developed standards can be grouped as follows showing that process is out-of-control:

Point/Points crossing the control limits: Examples of control charts showing point/points crossing the control limits are given in Figure. If there is an assignable cause in the $\bar{X}$ control chart, this is related with either raw material, erratic method, or human error. The latter may be attributable to either changes in raw material lot, changes in microstructure, changes in measuring and control methods, changes in machine adjustments, or wrong reading by the operator. If there is an assignable cause in the R-control chart, this is related with either machine or measuring instruments. Some noteworthy cases are, say, not calibrated measuring instruments, showing as such a low sensitivity value, the systematic causes of deterioration of production machines, as well as low machine maintenance. If there are points lying out of the control limits in both the $\bar{X}$ and R chart, this means that the calculation of the UCL and LCL would have been either wrong, or that the points were placed erratically. In addition to this, the process would have been out-of-control, the measuring system might have changed, or the measuring instrument may not be working properly. If one or more points fall sharp beyond or get close to the UCL or LCL, this is evidence that the process is out-of-control. A detailed investigation of the current circumstance has to be done, and corrective action has to be taken.



Examples of control charts showing point/points crossing the control limits.

Many points very near to the control limits: An example of a control chart showing many points that are very near to the control limits is given in Figure This pattern may be toward UCL or LCL.

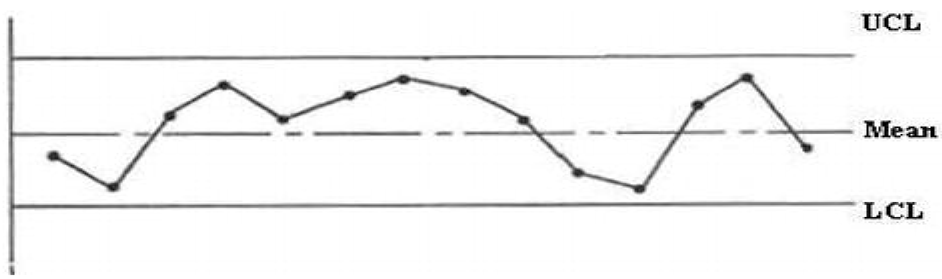


Example of a control chart showing many points that are very near to the control limits.

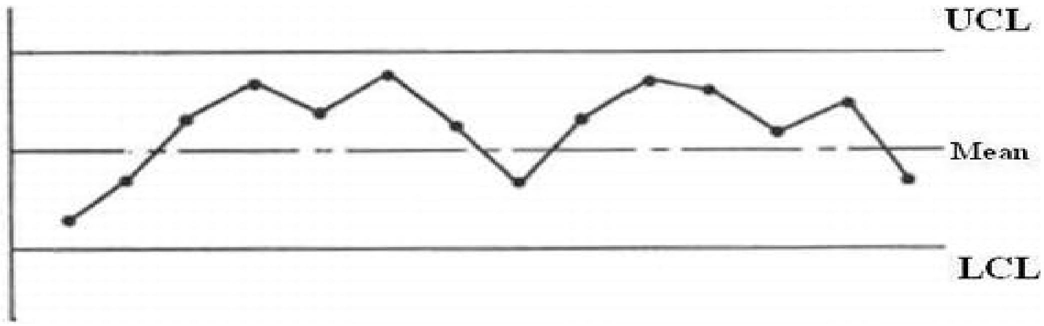
Points gather around a value: An example of a control chart showing points gathering around a value is given in Figure

Example of a control chart showing points gathering around a value.

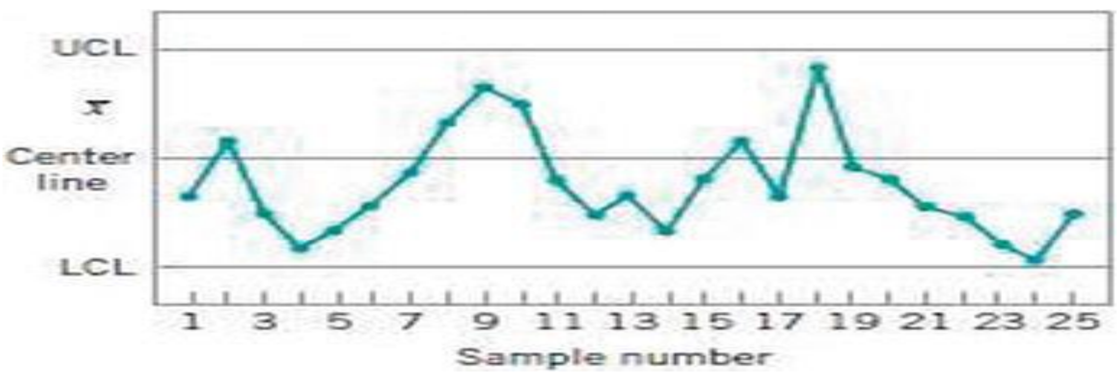
Consecutive points: All the consecutive seven points which are placed on one side of the center line is given in Figure . About 10 out of 11 consecutive points that are placed on one side of the center line is shown in Figure .



All of the consecutive 7 points are placed on one side of the center line.



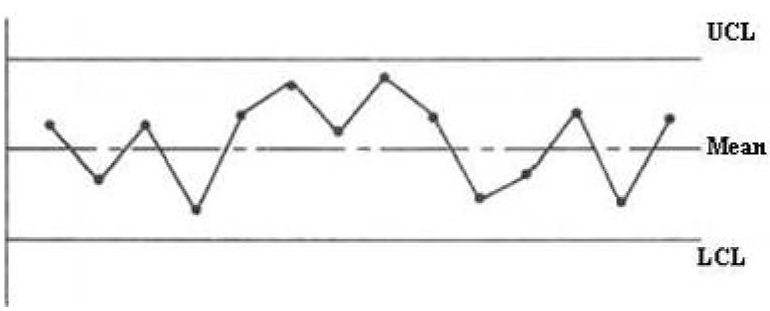
10 out of 11 consecutive points that are placed on one side of the center line. This expression can be widened as 12 out of 14 consecutive points, 14 out of 17 consecutive points, 16 out of 20 consecutive points, and 19 out of 25 consecutive points (Figure) are placed on one side of the center line. They all indicate very nonrandom appearance and an out-of-control production.



19 out of 25 consecutive points are placed on one side of the center line.

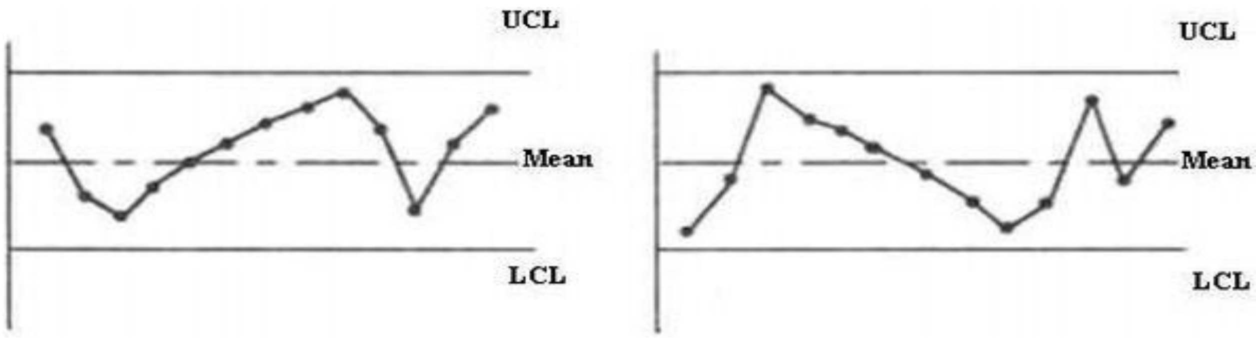
Runs: Average run length is the average number of points that must be plotted assignable before it can be said that it is an out-of-control condition. They describe the performance of the control charts. Some examples are:

A run of 2 points out of 3 near the control limits is given in Figure .



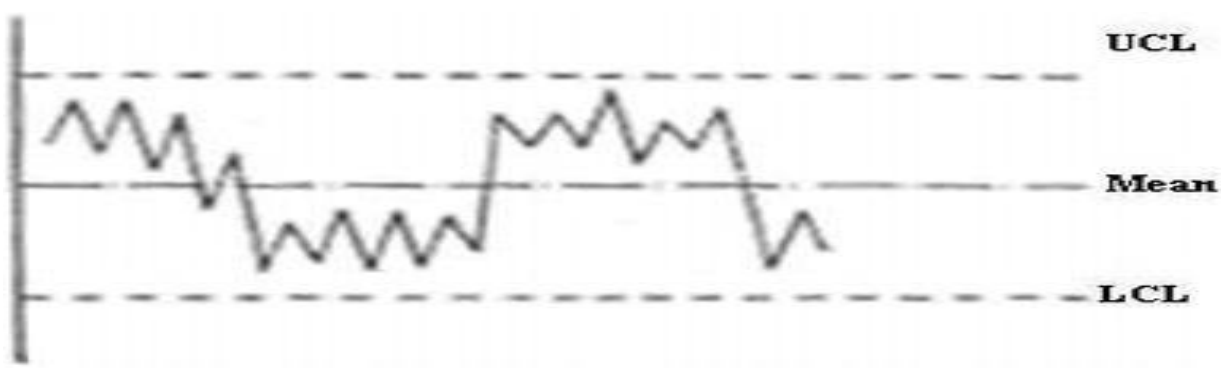
A run of 2 points out of 3 is near the control limits.

Others may be a run of 4 points out of 5 at a 1σ distance from the center line, a run of 8 points lie at one side of the center line, and a run of 7 points rises or falls (Figure).



Less than 2/3 of points lie in the middle 1/3 of the control limits.

An example of clear shifts for different periods is given in Figure. The reason for these shifts would be that the process is changing periodically, and so, different limits have to be calculated for different periods. Another reason would be that the lot had been changed, but the person in charge is not aware of it and continues to plot two different lots on the same chart rather than preparing a new one for the new lot.



Example of clear shifts for different periods.

CONTROL CHARTS FOR ATTRIBUTES

The term attributes in quality control refers to those quality characteristics, which classify the items/units into one of the two classes: conforming or non-conforming, defective or non-defective, good or bad.

There are two types of attributes:

1.Where numerical measurements of the quality characteristics are not possible, for example, colour, scratches, damages, missing parts, etc.

2. Where numerical measurements of the quality characteristics are possible, and items are classified as defective or non-defective on the basis of the inspection. For example, the diameter of a cricket ball can be measured by the micrometer but sometimes it may be more convenient to classify the balls as defective and non-defective using a Go-No-Go gauge (read the margin remark).

In inspection of attributes, actual measurements are not done, but the number of defective items (defectives) or number of defects in the item is counted. The size of defect and its location is not important. Items are inspected and either accepted or rejected.

There are different types of control charts for attributes for different situations.

These control charts for attributes into two groups as follows:

- Control charts for defectives, and
- Control charts for defects.

Control charts for defectives are mainly of two types as given below:

1. Control chart for fraction defective (p-chart), and

2. Control chart for number of defectives (np-chart).

Both control charts for defectives are based on the binomial distribution.

Control charts for defects are also of two types as given below:

1. Control chart for number of defects (c-chart), and

2. Control chart for number of defects per unit (u-chart).

CONTROL CHARTS FOR FRACTION DEFECTIVE (p-CHART)

The most widely used control chart for attributes is the fraction (proportion) defective chart, that is, the p-chart.

The p-chart may be applied to quality characteristics, which cannot be measured or impracticable and uneconomical to measure it.

These items/units are classified as defective or non-defective based on certain criteria (defects).

While dealing with attributes, a process will be said to be in statistical control if all the samples or subgroups are ascertained to have the same population proportion P.

If 'd' is the number of defectives in a sample of size n, then the sample proportion defective is p = d/n.

Hence, d is a binomial variate with parameters n and P.

Therefore, E(d) = n P and Var(d) = n PQ, Q = 1 – P

Thus E(p) = E (d/n) = (1/n) E(d) = P and

Var (p) = Var(d/n) = (1 / n^2) Var (d) = PQ /n

Thus, the 3 – σ limits for p – chart are given by:

$$E(p) \pm 3\ S.E.\ (p) = P \pm 3\sqrt{PQ/n} = P \pm A\sqrt{PQ}$$

Where A = 3/ √n has been tabulated for different values of n.

Case i) standards specified.

If P′ is the given or known value of P, then

$$UCL_p = P' + A\sqrt{P'(1-P')} \quad ; \quad LCL_p = P' - A\sqrt{P'(1-P')} \quad ; \quad CL_p = P'$$

Case ii) standards not specified.

Let d_i be the number of defectives and pi is the fraction defective for the i^{th} sample (i= 1,2,3…k) of size n_i. Then the population proportion P is estimated by the statistic $\bar{p}$ given by:

$$\bar{p} = \frac{\sum d_i}{\sum n_i} = \frac{\sum_i (n_i p_i)}{\sum_i n_i}$$

It may be remarked that $\bar{p}$ is an unbiased estimate of P, since

$$E(\bar{p}) = \frac{\sum E(d_i)}{\sum n_i} = \frac{\sum_i (n_i P)}{\sum_i n_i} = P$$

In this case

$$UCL_p = \bar{p} + A\sqrt{\bar{p}(1-\bar{p})} \quad ; \quad LCL_p = = \bar{p} - A\sqrt{\bar{p}(1-\bar{p})} \quad ; \quad CL_p = \bar{p}$$

Where A = 3/ √n

CONTROL CHARTS FOR NUMBER OF DEFECTIVES (d-CHART)

If instead of p, the sample proportion defective, we use d, the number of defectives in the sample, then the 3-σ control limits for d-chart are given by:

$$E(d) \pm 3\ S.E.\ (d) = n\ P \pm 3\sqrt{nP(1 = P)}$$

Case i) standards specified.

If P′ is the given or known value of P, then

$UCL_d = n\ P' + 3\sqrt{nP'(1-P')}$; $LCL_d = n\ P' - 3\sqrt{nP'(1-P')}$; $CL_d = n\ P'$

Case ii) standards not specified. Using $\bar{p}$ as an estimate of P, we get

$UCL_d = n\bar{p} + 3\sqrt{n\ \bar{p}\ (1-\bar{p})}$; $LCL_d = n\ \bar{p} - 3\sqrt{n\ \bar{p}\ (1-\bar{p})}$; $CL_d = n\ \bar{p}$

Since p cannot be negative, if LCL as given by the formula comes out to be negative, then it is taken as zero.

Interpretation of p-chart and d-chart:

1. If all the sample points fall within the control limits without exhibiting any specific pattern, the process is said to be in control. In such a case, the observed variations in fraction defective are attributed to the stable pattern of chance causes and the average fraction defective $\bar{p}$ is taken as the standard fraction defective P.

2. Points outside the UCL are termed as high spots. These suggest deterioration in the quality and should be regularly reported to the production engineers. The reason for such deterioration could possibly be known and removed if the details of conditions under which data are collected, were known. Of particular interest and importance is, if there was any change of inspection or inspection standards.

3. Points below LCL are called low spots. Such points represent a situation showing improvement in the product quality. However, before taking this improvement for guaranteed, it should be investigated if there was any slackness in inspection or not.

4. When the number of points fall outside the control limits, a revised estimate of P should be obtained by eliminating all the points that fall above UCL (it is assumed that the points that fall below LCL are due to faulty inspection). The standard fraction defective P should be revised periodically in this way.

The following are the figures of defectives in 22 lots each containing 2,000 rubber belts:

425, 430, 216, 341, 225, 322, 280, 306, 337, 305, 356, 402, 216, 264, 126, 409, 193, 326, 280, 389, 451, 420

Draw control chart for fraction defective and comment on the state of control of the process

s. no.	d	p=(d/2000)	s. no.	d	p=(d/2000)
1	425	0.2125	12	402	0.2010
2	430	0.2150	13	216	0.1080
3	216	0.1080	14	264	0.1320
4	341	0.1705	15	126	0.0630
5	225	0.1125	16	409	0.2045
6	322	0.1610	17	193	0.0965
7	280	0.1400	18	326	0.1630
8	306	0.1530	19	280	0.1400
9	337	0.1685	20	389	0.1945
10	305	0.1525	21	451	0.2255
11	356	0.1780	22	420	0.2100
	3543	1.7715		3476	1.7380

In the usual notations, we have

$$\bar{p} = \frac{\sum p_i}{k} = \frac{1.7715+1.7380}{22} = \frac{3.5095}{22} = 0.1595, \quad \bar{q} = 1- \bar{p} = 0.8405$$

$$\text{Or} \quad \bar{p} = \frac{\sum d_i}{nk} = \frac{3543+3476}{22 \; X \; 2000} = \frac{7019}{44000} = 0.1595,$$

3-σ control limits for p-chart are given by:

$$\bar{p} \pm 3\sqrt{\bar{p}\bar{q} \;\; /n} = 0.1595 \pm 3\sqrt{(0.1595 \; X \; 0{,}8405)/2000}$$

$$= 0.1595 \pm 3\sqrt{(0.000067} = 0.1595 \pm 0.0246$$

Therefore, $UCL_p = 0.1595 + 0.0246 = 0.1841$

$LCL_p = 0.1595 - 0.0246 = 0.1349$

$CL_p = \bar{p} = 0.1595$

CONTROL CHARTS FOR VARIABLES

The Variable Control Chart tracks characteristics that can be measured on a continuous scale. Many quality characteristics can be expressed in terms of a numerical measurement. A single measurable quality characteristic, such as a dimension, weight, or volume, is called a variable. The variable Control Charts usually leads to more efficient control feedback and provides more information about process performance than Attribute Control Charts. When working with variables data, both the mean and variability are usually tracked.

$\bar{X}$ and R charts

No production process is perfect enough to produce all the items exactly alike. Some amount of variation, in the produced items, is inherent in any production scheme. This variation is the totality of numerous characteristics of the production process viz., raw material, machine setting and handling, operators etc., As pointed earlier, this variation is the result of i) chance causes, and ii) assignable causes. The control limits in the $\bar{X}$ and R charts are so placed that they reveal the presence or absence of assignable causes of variation in the

a) Average- mostly related to machine setting, and
b) Range- mostly related to negligence on the part of the operator.

Steps for $\bar{X}$ and R charts

Measurement. Actually the work of a control chart starts first with measurements. Any method of measurement has its own inherent variability. Errors in measurement can enter into the data by:

i) The use of faulty instruments
ii) Lack of clear-cut definitions of quality characteristics and the method of taking measurements, and
iii) Lack of experience in the handling or use of the instrument, etc.

Since the conclusions drawn from control chart are broadly based on the variability in the measurements as well as the variability in
the quality being measured, it is important that the mistakes in reading measurement instruments or errors in recording data should be minimized so as to draw valid conclusions from control charts.

Selection of Samples or sub-groups. In order to make the control chart analysis effective, it is essential to pay due regard to the rational selection of the samples or sub-groups. The choice of the sample size n and the frequency of sampling, i.e., the time between the selection of two groups, depend upon the process and no hard and fast rules can be laid down for this purpose. Usually, n is taken to be 4 or 5 while the frequency of the sampling depends on the state of control exercised. Initially more frequent samples will be required and once a state of control is maintained, the frequency may be relaxed. Normally 25 samples of size 4 each or 20 samples of size 5 each under control will give good estimate o the process average and dispersion.

Calculation of $\bar{X}$ and R for each sub-group. Let X_{ij}, j=1,2,….,n be the measurements on the i^{th} sample (i = 1,2,…,k). the mean $\bar{X}_i$, the range R_i and the standard deviation s_i for the i^{th} sample are given by:

$$\bar{X}_i = \frac{1}{n} \sum_j X_{ij}\,, \qquad R_i = max_j X_{ij} - min_j X_{ij},$$

$$s_i^2 = \frac{1}{n} \sum_j \left(X_{ij} - \bar{X}_i\right)^2 \text{ (i = 1,2,…,k)}$$

Next we find $\bar{\bar{X}}$, $\bar{R}$ and $\bar{s}$, the averages of sample means, sample ranges and sample standard deviations, resp., as follows

$$\bar{\bar{X}} = \frac{1}{k} \sum_i \bar{X}_i, \qquad \bar{R} = \frac{1}{k} \sum_i R_i, \qquad \bar{s} = \frac{1}{k} \sum_i s_i$$

4. Setting of control limits: it is known that if σ is ht process standard deviation, then the standard error of the sample mean is σ / √n, where n is the sample size, i.e., S.E($\bar{X}$) = σ / √n, (i = 1,2...k).

From sampling distribution of range, we know that

E(R) = d_2 σ, where d_2 is a constant depending on the sample size. Thus an estimate of σ can be obtained from $\bar{R}$ by the relation: $\bar{R}$ = d_2 σ $\quad \rightarrow \hat{\sigma} = \bar{R}/d_2$

Also $\bar{\bar{X}}$ gives an unbiased estimate of the population mean μ, since

$$E\,(\bar{\bar{X}}) = \frac{1}{k} \sum_{i=1}^{k} E(\bar{\bar{X}}_i) = \frac{1}{k} \sum_{i=1}^{k} \mu = \mu$$

CONTROL LIMITS FOR $\bar{X}$ - CHART

case i. When standards are given i.e., both μ and σ are known. The 3-σ control limits for $\bar{X}$ - chart are given by E ($\bar{X}$) ± 3 S.E. ($\bar{X}$) = μ ± 3σ/√n = μ ± A σ where A = 3/√n.

If μ' and σ' are known values of μ and σ resp., then

$UCL_{\bar{X}} = \mu' + A\,\sigma'$, $LCL_{\bar{X}} = \mu' - A\,\sigma'$, $CL_{\bar{X}} = \mu'$

where $A = 3/\sqrt{n}$. is a constant depending on n and its values are tabulated for different values of n from 2 to 25 in the tables.

case ii. When standards are not given i.e., both μ and σ are unknown, then using their estimates $\bar{\bar{X}}$ and $\hat{\sigma}$, we get the 3-σ control limits for $\bar{X}$ - chart as:

$$\bar{\bar{X}} \pm 3\frac{\bar{R}}{d_2}\frac{1}{\sqrt{n}} = \bar{\bar{X}} \pm (3/d_2\sqrt{n})\,\bar{R} = \bar{\bar{X}} \pm A_2\bar{R} \text{ where } A_2 = 3/d_2\sqrt{n}.$$

$UCL_{\bar{X}} = \bar{\bar{X}} + A_2\bar{R}$, $LCL_{\bar{X}} = \bar{\bar{X}} - A_2\bar{R}$, $CL_{\bar{X}} = \bar{\bar{X}}$

Since d_2 is a constant depending on n, $A_2 = 3/d_2\sqrt{n}$. also depends only on n and its values are tabulated for different values of n from 2 to 25 in the tables.

CONTROL LILMITS FOR R-CHART

R- chart is constructed for controlling the variation in the dispersion (variability) of the product. The procedure of constructing R-chart is similar to that of $\bar{X} -$ chart and involves the following steps:

1. Compute the range $R_i = max_j X_{ij} - min_j X_{ij}$, (i= 1,2,…n) for each sample.
2. Compute the mean of the sample ranges: $\bar{R} = \frac{1}{k}\sum_i R_i$,
3. Computation of control limits
   The 3-σ control limits for R - chart are:
   $E(R) \pm 3\sigma_R$, $E(R)$ is estimated by $\bar{R}$ and σ_R is estimated from the relation:
   $\sigma_R = d_3\hat{\sigma} = d_3\frac{\bar{R}}{d_2}$, where d_2 and d_3 are constants depending on n.
   Therefore, $UCL_R = E(R) + 3\sigma_R = \bar{R} + \frac{3d_3}{d_2}\bar{R}$
   $$\rightarrow UCL_R = \left(1 + \frac{3d_3}{d_2}\right)\bar{R} = D_4\bar{R}$$
   Similarly, $LCL_R = \left(1 - \frac{3d_3}{d_2}\right)\bar{R} = D_3\bar{R}$
   The values of D_3 and D_4 depend only on n and have been computed for different values of n from 2 to 25 and tabulated.

However, if σ is known, then

$UCL_R = E(R) + 3\sigma_R = d_2\sigma + 3d_3\sigma = (d_2 + 3d_3)\sigma = D_2\sigma$

$LCL_R = E(R) - 3\sigma_R = d_2\sigma - 3d_3\sigma = (d_2 - 3d_3)\sigma = D_1\sigma$

In each case, (σ known or unknown), the central line is given by:

$$CL_R = \bar{R}$$

Since range cannot be negative, LCL_R must be greater than or equal to 0. In case it comes out to be negative, it is taken as zero.

Construction of Control Chart for $\bar{X}$ and R. Control charts are plotted on a rectangular co-ordinate axis – vertical scale representing the statistical measures $\bar{X}$ and R, and the horizontal scale representing the sample number. Sample points (mean or range) are indicated on the charts by points.

For $\boldsymbol{\bar{X} - chart},$ the central line is drawn as a solid horizontal line at $\bar{R}$ and $UCL_{\bar{X}}$ and $LCL_{\bar{X}}$ are drawn at computed values as dotted horizontal lines.

For $\boldsymbol{R - chart},$ the central line is drawn as a solid horizontal line at $\bar{\bar{X}}$ and UCL_R are drawn at computed values as dotted horizontal lines. If the sample size is seven or more LCL_R is drawn as dotted horizontal line at the computed value, otherwise i.e., if $n < 7$ LCL_R is taken as zero.