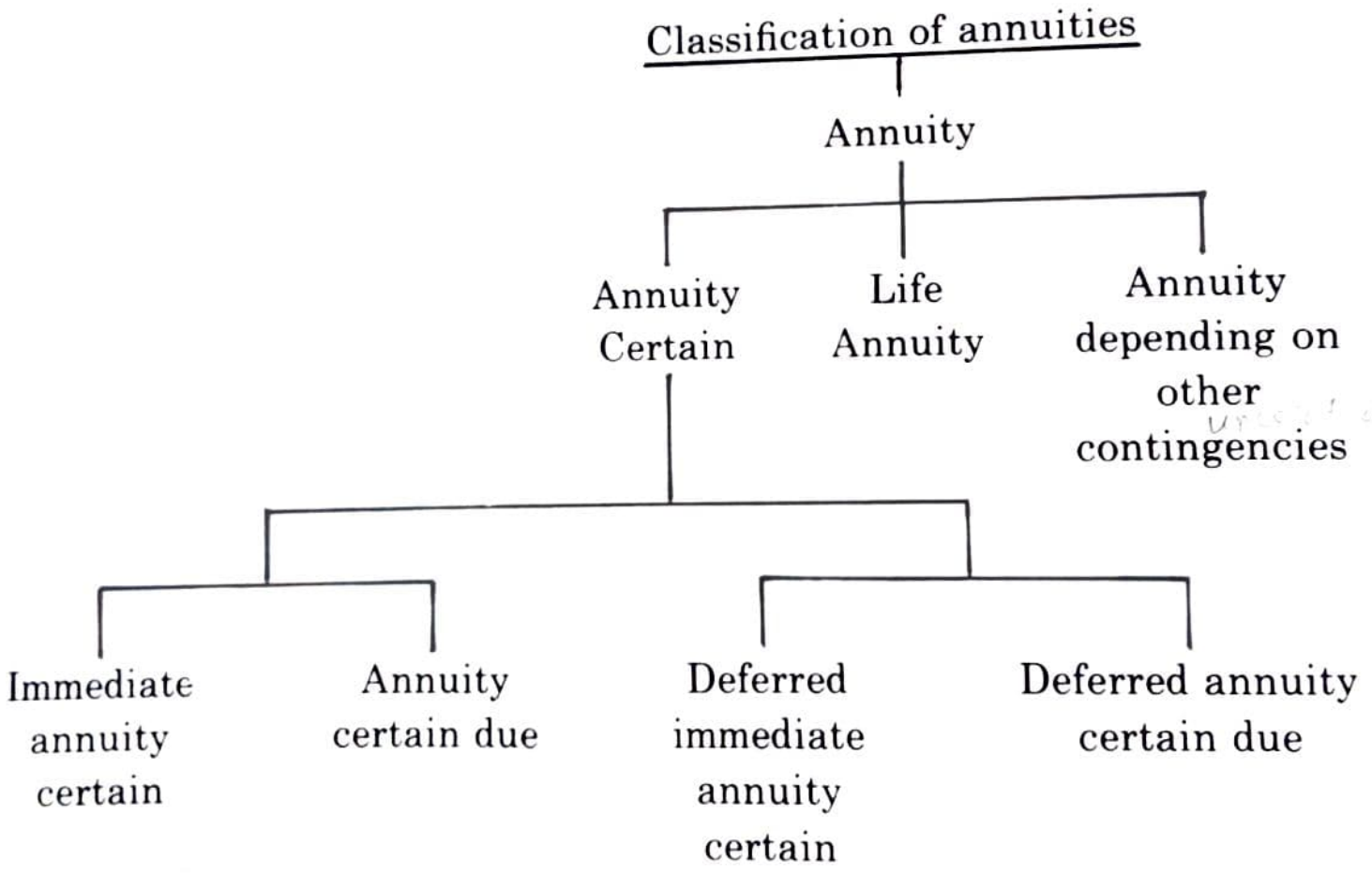


for 10 years with a deferment period of 6 years.

Unit-III
Annuities

The following chart shows various types of annuities :



series of payments during the given period...

(2)

1. [A series of payments made at successive periods (intervals) of time is called an annuity. If all the payments are equal, then the annuity may be called a level or uniform annuity, otherwise it may be called a variable annuity. If the total interval for which the payments of the annuity are to be made is a definite number of years, not depending upon any contingency, then the annuity is called annuity certain. If the payments are to be made during the life of a person it is called a life annuity.]

any sum
at rate
of interest

2. An annuity is usually referred to as so much per annum even when the frequency of payment is different from yearly. Thus if a payment of Rs. 100 falls due every year the annuity is of Rs. 100 p.a., while if a payment of Rs. 25 falls due every quarter, we call it an annuity of Rs. 100 p.a. payable quarterly or an annuity with annual rent of Rs. 100 payable quarterly. Where the frequency is not specified it is understood to be yearly.

3. [If the successive payments of the annuity are made at the end of the successive periods, then the annuity is called an immediate annuity. If these successive payments are made at the beginning of each successive period, then the annuity is called an annuity-due. In the case of an immediate annuity certain of 1 p.a. for 10 years there are ten yearly payments in all, the first payment of 1 being made at the end of the first year and the last at the end of the 10th year. In the case of an annuity certain due of 1 p.a. for 10 years, there are again 10 payments but the first

... are considered.

Present Value of an annuity is the sum of the present values of all the instalment payments and is denoted by V /

Amount of an annuity is sum of the amounts (end values) of all the instalment payments and is denoted by M . /

Let i be the rate of interest per Re.1 per annum. i.e., $i = \frac{r}{100}$, A be the instalment amount and n be the number of payments.

The four formulae can be derived as follows:

Present Value of An Immediate Annuity:

From compound interest, $A = P(1+i)$ when $n=1$. That is, $P = \frac{A}{1+i}$. That is, if A is the amount at the end of one year its present value is $\frac{A}{1+i}$.



In Immediate Annuity, the first payment, A, is at the end of the first year and its present value $= \frac{A}{1+i}$

Similarly, the present value of A due at the end of the second year $= \frac{A}{(1+i)^2}$

the present value of A due at the end of the third year $= \frac{A}{(1+i)^3}$

⋮

and the present value of A due at the end of nth (last) year $= \frac{A}{(1+i)^n}$

∴ the present value of an Immediate Annuity,

$$V = \frac{A}{1+i} + \frac{A}{(1+i)^2} + \frac{A}{(1+i)^3} + \dots + \frac{A}{(1+i)^n}$$

$$V = \frac{A}{1+i} \left[\frac{1 - \frac{1}{(1+i)^n}}{1 - \frac{1}{1+i}} \right] \text{ from G.P. with } a = \frac{A}{1+i}, n=n \text{ and } r = \frac{1}{1+i}$$

$$= \frac{A}{i} [1 - (1+i)^{-n}] \quad (1+i) \left(1 - \frac{1}{1+i}\right) = 1+i-1 = i.$$

Hence, the present value of an Immediate Annuity,

$$V = \frac{A}{i} [1 - (1+i)^{-n}]$$

Present Value of an Annuity Due:

In Annuity Due, each payment is due at the beginning of the year. Hence, except the first payment others can be

$v = \frac{1}{1+i}$ under immediate annuity and $v = \frac{1}{1+i}$ under annuity due

5

Example 35: A sum of Rs.1,000 is to be paid at the end of every year for a period of 5 years at the rate of 10% per annum compound interest. If the first instalment is paid at the end of the first year, how much amount will be accrued to the credit of the depositor? What is its present worth?

(B.Com. Bharathiar, D84)

Solution: Given: $A=1,000$; $n=5$; $r=10$; $\therefore i = \frac{r}{100} = \frac{10}{100} = 0.1$.

Required to find M and V of an Immediate Annuity. Hence, the given values are substituted in the formulae and the following answers are found out.

$$M = \frac{A}{i} [(1+i)^n - 1]$$

$$= \frac{1000}{0.1} [(1.1)^5 - 1] = \text{Rs. } 6,105.10$$

$$V = \frac{A}{i} [1 - (1+i)^{-n}]$$

$$= \frac{1000}{0.1} [1 - (1.1)^{-5}] = \text{Rs. } 3,790.79$$

Example 36: A person deposits Rs.5,000 every year with a company which pays him interest at 12% per annum. He allows his deposits to accumulate with the company at compound interest. What would be the amount standing to his credit one year after he has made his deposit for 15th time?

(I.C.W.A. Inter., J94)

Solution: Given: $A = 5,000$; $r = 12$; $n = 15$;

$$\therefore i = \frac{r}{100} = \frac{12}{100} = 0.12.$$

M under Annuity Due is to be found out. Consider,

$$M = \frac{A}{i} (1+i) [(1+i)^n - 1]$$

$$= \frac{5000}{0.12} (1.12) [(1.12)^{15} - 1] = \text{Rs. } 2,08,766.40$$

Example 37: The annual membership fee of an association is Rs.3,000. Dr.Pandian is eager to become a life member by paying a lump sum. Assuming the expectation of life of a man of his age is 20 years and rate of compound interest is 15%, how much should the association charge now?

Solution: Given $A=3000$; $n = 20$; $r=15 \therefore i = 0.15$.

Under Annuity Due, V is to be found out.

$$\text{Consider } V = A + \frac{A}{i} [1 - (1+i)^{-(n-1)}]$$

$$= 3000 + \frac{3000}{0.15} [1 - (1.15)^{-19}] = \text{Rs. } 21,594.69$$

Example 38: Mr. X borrows Rs.20,000 at 4% compound interest and agrees to pay both the principal and the interest in 10 equal instalments at the end of each year. Find the amount of these instalments. (I.C.W.A. Inter., D 88)

Solution: Given : $V = 20,000$; $r=4$; $n=10$. $\therefore i = 0.04$

Required to find A under Immediate Annuity.

Consider
$$V = \frac{A}{i} [1 - (1+i)^{-n}]$$

$\therefore 20,000 = \frac{A}{0.04} [1 - (1.04)^{-10}]$

\therefore Instalment amount , $A = \frac{20000 \times 0.04}{0.3244358} = \text{Rs.}2,465.82$

Example 39: A person wishes to collect Rs.1,20,000 for a house at the time of retirement due after 18 years. If the rate of compound interest is 6% per annum, how much should he deposit annually to receive this amount? (B.Com. Madras. N81)

Solution: Given: $M = 1,20,000$; $n=18$; $r=6$. $\therefore i = 0.06$.

Required to find A under Annuity Due.

Consider
$$M = \frac{A}{i} (1+i) [(1+i)^n - 1]$$

i.e., $1,20,000 = \frac{A}{0.06} (1.06) [(1.06)^{18} - 1]$

\therefore His annual deposit. $A = \frac{1,20,000 \times 0.06}{1.06 \times 1.8543392} = \text{Rs.}3,663$

Note: Earlier the formula for finding the effective annual rate of interest of s percent p.a. corresponding to a nominal rate of r percent per annum (i.e. $\frac{r}{m}$ per unit of time when there are m units of time in one year) was seen. If the rate required is $\frac{r}{m}$ (per unit of time) and the rate known is s, the same formula can be rewritten as

6. Present Value of an Immediate Annuity Certain :

We frequently come across problems of finding the present value or accumulated amount of an annuity. The present value of an annuity is simply the sum of the present values of each of its periodical payments as at the commencement of the annuity. Similarly the amount or accumulated value of the annuity is simply the sum of the accumulated amounts of the periodical payments as at the end of the period of the annuity.

Let us find the present value of an immediate annuity of 1 p.a. payable for n years certain at rate of interest of i per unit per annum. This present value is denoted by the symbol $a_{\overline{n}|}$. Under this annuity, n payments are to be made, the first one at the end of the 1st year, the second one at the end of the 2nd year and so on, the last payment being made at the end of n years. To get the present value of the annuity, we have to find the present value of each of these n payments as at the commencement, and add them up.

- As at the commencement of the annuity,
- value of the 1st payment of 1 payable at the end of 1 year = v
- value of the 2nd payment of 1 payable at the end of 2 years = v^2
- value of the nth payment of 1 payable at the end of n yrs. = v^n

Adding, we get the present value of the annuity

$$a_{\overline{n}|} = v + v^2 + v^3 + \dots + v^n.$$

The expression above is a geometric progression whose first term is v, common ratio is v and number of terms is n. Therefore applying the formula for the sum of a G.P. we see that the present value of the annuity

$$a_{\overline{n}|} = \frac{v(1-v^n)}{(1-v)}$$

This expression can be further simplified. We know that $v = \frac{1}{1+i}$ and

so writing $\frac{1}{1+i}$ for v in the numerator, we have

$$\begin{aligned}
 a_{\overline{n}|} &= \frac{(1-v^n)}{(1+i)(1-v)} \\
 &= \frac{1-v^n}{1(1+i)-v(1+i)}
 \end{aligned}$$

But $v(1+i) = 1$ and so
 we have

$$a_{\overline{n}|} = \frac{1-v^n}{1+i-1} = \frac{1-v^n}{i}$$

$\frac{i}{\overline{}}$
For example (9)

..... (4.1)

Thus instead of adding n terms which are successive powers of v , the value of $a_{\overline{n}|}$ can be obtained by finding the value of v^n , subtracting it from 1 and then dividing by i , the rate of interest. For example the present value @ 5% p.a. of an immediate annuity of 1 p.a. for 10 years is given by

$$a_{\overline{10}|} = \frac{1-v^{10}}{.05} = \frac{1-.61391}{.05} = \frac{.38609}{.05} = 7.7218$$

It may be verified that $a_{\overline{10}|}$ is just the sum of the first 10 terms of the column of values of v^n @ 5%.

The values of $a_{\overline{n}|}$ are printed alongside the values of $(1+i)^n$ and v^n for different values of i and n .

7. In section 6 we have obtained the present value of an annuity of 1. p.a. payable yearly, the rate of interest being i p.a. The value of a similar annuity of S per annum is clearly $Sa_{\overline{n}|} = S \left[\frac{1-v^n}{i} \right]$ and can be readily obtained by multiplying the annual payment by the value of $a_{\overline{n}|}$ taken from the tables.

9. Accumulated Value of Annuity ✓

10

We shall now find an expression for the accumulated value or amount of an immediate annuity of 1 p.a. for n years. The accumulated value of an immediate annuity of 1 p.a. for n years at rate of interest of i per unit per annum is denoted by the symbol $s_{\overline{n}|}$. The procedure is similar to that for finding the present value of an annuity i.e., we accumulate each individual payment at the end of n years and add up the accumulated values.

Thus at the end of the period of n years the accumulated value of the last payment due = 1.

- The accumulated value of the last payment but one = $(1 + i)$
- The accumulated value of the last payment but two = $(1 + i)^2$
- Lastly, the accumulated value of the first payment = $(1 + i)^{n-1}$

Adding, we see that the accumulated value
= $1 + (1 + i) + (1 + i)^2 + \dots + (1 + i)^{n-1}$

The above expression is a geometric progression whose 1st term is 1 with common ratio $(1 + i)$ and no. of terms n .

$$\therefore s_{\overline{n}|} = \frac{1[(1 + i)^n - 1]}{(1 + i) - 1} = \frac{(1 + i)^n - 1}{i} \quad \dots\dots (2.2)$$

As a corollary, we see that if the annuity is not 1 p.a. but C p.a., the accumulated value of the annuity is $C \cdot s_{\overline{n}|i}$

10 The argument of Section 7 holds good in this case also. If the annuity is payable more frequently than once a year the formula can still be applied to the number of periods involved instead of to the number of years, provided the rate of interest is also reckoned for the same period. Thus an immediate annuity of 20 p.a. payable quarterly for 12 years can be regarded as an annuity of 5 per quarter for 48 quarters; and if the rate of interest is 6% p.a. payable quarterly i.e. $1\frac{1}{2}\%$ per quarter the amount of the annuity = $5s_{\overline{48}|1.5\%}$ @ $1\frac{1}{2}\%$.

11. As in the case of Section 8, we can obtain the value of $s_{\overline{n}|i}$, also by general reasoning. Suppose a unit of money is invested at rate i p.a. Then in lieu of the unit, the investor receives (i) i p.a. for n years, and (ii) in addition, at the end of n years, he receives back the capital invested i.e., 1. The first item constitutes an immediate annuity of i p.a. and its value at the end of n years is $i \cdot s_{\overline{n}|i}$. The second item has value 1. The value of these two items at the end of n years is thus $1 + i s_{\overline{n}|i}$. This must be clearly equal to the accumulated value of the original investment of 1 at the end of n years i.e., $(1 + i)^n$. Thus we have

$$1 + i s_{\overline{n}|i} = (1 + i)^n$$

$$\therefore i s_{\overline{n}|i} = (1 + i)^n - 1$$

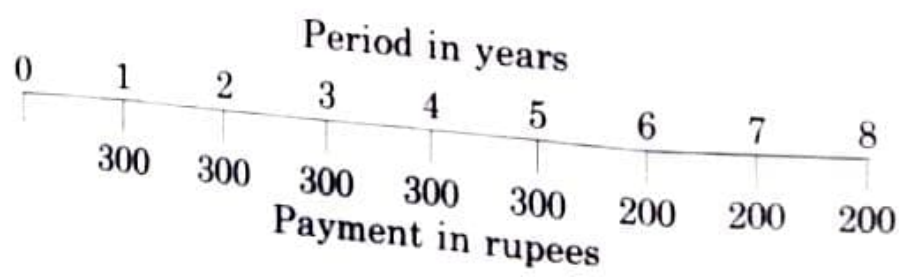
$$\text{or } s_{\overline{n}|i} = \frac{(1 + i)^n - 1}{i}$$

The values of $s_{\overline{n}|i}$ are also available for various values of i and n .

Example 1 : A series of 8 annual sums of money is payable, the first payment taking place at the end of one year from now. The first five payments are Rs. 300 each and the last three payments are Rs. 200 each. Find the present value of the eight payments @ 8% p.a.

Solution :

First Method : The present value of 8 annual sums of money can be found by adding the present values of each payment as at the commencement of the period.



- Present value of 1st payment of Rs. 300/-
= $300 \times v @ 8\% = 300 \times .92593 = 277.779$
 - Present value of 2nd payment of Rs. 300/- = $300 \times v^2$
= $300 \times .85734 = 257.202$
 - Present value of 3rd payment of Rs. 300/- = $300 \times v^3$
= $300 \times .79383 = 238.149$
 - Present value of 4th payment of Rs. 300/- = $300 \times v^4$
= $300 \times .73503 = 220.509$
 - Present value of 5th payment of Rs. 300/- = $300 \times v^5$
= $300 \times .68058 = 204.174$
 - Present value of 6th payment of Rs. 200/- = $200 \times v^6$
= $200 \times .63017 = 126.034$
 - Present value of 7th payment of Rs. 200/- = $200 \times v^7$
= $200 \times .58349 = 116.698$
 - Present value of 8th payment of Rs. 200/- = $200 \times v^8$
= $200 \times .54027 = 108.054$
- The present value of eight payments is 1548.599, i.e. Rs. 1548.60.

Second Method : If we express the payments as a series of uniform payments, we can use the table of a_n and find the present value of the given eight payments. For this purpose, consider the first five payments of Rs. 300/- each as made of two parts viz. (Rs. 200 + Rs. 100).

Taking out of Rs. 300/-, Rs. 200/- of first five payments alongwith last 3 payments of Rs. 200/- each, we get a series of uniform payments of Rs. 200/- payable for 8 years. The present value of these 8 payments of Rs. 200/- each is given by $200 a_{\overline{8}|8\%} @ 8\% = 200 \times 5.7466 = 1149.32 \dots\dots (I)$

Period in years								
0	1	2	3	4	5	6	7	8
	200	200	200	200	200	200	200	200
	+ 100	+ 100	+ 100	+ 100	+ 100			
Payments in rupees								

Now we have to consider the present value of Rs. 100/-, left out of Rs. 300/- payable for first five years. The present value is given by
 = $100 \times a_{\overline{5}|8\%} @ 8\%$
 = 100×3.9927
 = 399.27 (II)

The present value of the 8 payments is obtained by adding (I) and (II)
 = $1149.32 + 399.27 = \text{Rs. } 1548.59$ (The difference of 1 paise is due to using

$a_{\overline{n}|}$ values nearest to 4 decimal places whereas v^n values are used nearest to 5 decimal places).

Example 2 : Show that $\frac{a_{\overline{6n}|}}{a_{\overline{3n}|}} = 1 + v^{3n}$

Solution : $a_{\overline{6n}|}$ represents an immediate annuity of 1 p.a. payable for 6n years.

$$\therefore a_{\overline{6n}|} = v + v^2 + v^3 + \dots + v^{3n} + v^{3n+1} + v^{3n+2} \dots + v^{6n}$$

$a_{\overline{3n}|}$ represents an immediate annuity of 1 p.a. payable for 3n years.

$$\begin{aligned} \therefore a_{\overline{6n}|} &= v + v^2 + v^3 \dots + v^{3n} + v^{3n} (v + v^2 + \dots + v^{3n}) \\ &= a_{\overline{3n}|} + v^{3n} (a_{\overline{3n}|}) \\ &= a_{\overline{3n}|} (1 + v^{3n}) \end{aligned}$$

$$\therefore \frac{a_{\overline{6n}|}}{a_{\overline{3n}|}} = \frac{a_{\overline{3n}|} (1 + v^{3n})}{a_{\overline{3n}|}} = 1 + v^{3n}$$

Example 3 : Find the accumulated value of 8 annual sums given in Example 1 as at the end of eight years.

Solution :

First Method : We can find the desired value by adding the accumulated value of each of eight payments.

The first payment will earn interest for 7 years.

\therefore The accumulated value of 1st payment of 300/- at the end of 8 years = $300 (1+i)^7$ @ 8% = $300 \times 1.71382 = 514.146$.

The accumulated value of 2nd payment of 300/-

$$\begin{aligned} &= 300 \times (1+i)^6 = 300 \times 1.58687 = 476.061 \\ \text{- do -} \quad 3\text{rd " of 300/-} &= 300 \times (1+i)^5 = 300 \times 1.46933 = 440.799 \\ \text{- do -} \quad 4\text{th " of 300/-} &= 300 \times (1+i)^4 = 300 \times 1.36049 = 408.147 \\ \text{- do -} \quad 5\text{th " of 300/-} &= 300 \times (1+i)^3 = 300 \times 1.25971 = 377.913 \\ \text{- do -} \quad 6\text{th " of 200/-} &= 200 \times (1+i)^2 = 200 \times 1.16640 = 233.280 \\ \text{- do -} \quad 7\text{th " of 200/-} &= 200 \times (1+i) = 200 \times 1.08000 = 216.000 \\ \text{- do -} \quad 8\text{th " of 200/-} &= 200 \\ &= 200.000 \end{aligned}$$

Adding all the accumulated values, we get 2866.346.

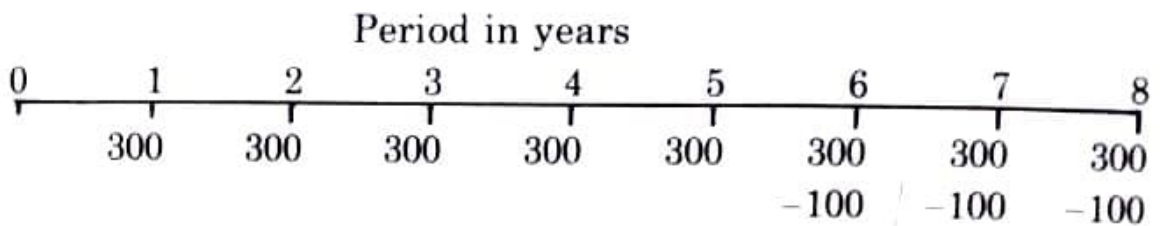
\therefore The accumulated value of given eight payments is Rs. 2866.35.

Second Method : If we can express the payments as a series of uniform payments, we can use the table of $s_{\overline{n}|}$ and find the accumulated

value of the eight payments.

However, since the accumulated values at the end of eight years is required the series should be uniform with reference to the end of the period.

For this purpose consider the last 3 payments of Rs. 200/- as made up of two parts viz. Rs. 300/- less Rs. 100/- :



The accumulated value of Rs. 300/- p.a. as at the end of eight years

$$\begin{aligned}
 &= 300 \times s_{\overline{8}|} @ 8\% = 300 \times 10.6366 \\
 &= 3190.98 \quad \dots\dots (I)
 \end{aligned}$$

Since we have taken Rs. 100/- more in last 3 payments; we should find the value of these 3 additional amounts as at the end of eight years and deduct the same from value obtained above.

Accumulated value of Rs. 100/- for last 3 years

$$\begin{aligned}
 &= 100 \times s_{\overline{3}|} @ 8\% \\
 &= 100 \times 3.2464 \\
 &= 324.64 \quad \dots\dots (II)
 \end{aligned}$$

$$(I) - (II) = 3190.98 - 324.64 = \text{Rs. } 2866.34$$

For Exercise 2.1 see page 81.

12. Relation between $s_{\overline{n}|}$ and $a_{\overline{n}|}$.

$a_{\overline{n}|}$ is the present value of a series of n payments of 1 per period under an immediate annuity, as at the commencement of the interval of n periods. $s_{\overline{n}|}$ is the accumulated value of the same series of payments, as at the end of the interval of n periods. They represent the values of the same set of payments, at different points of time, the first of them $a_{\overline{n}|}$ as at commencement of n periods and the other $s_{\overline{n}|}$ as at the end of n periods.

$$\therefore a_{\overline{n}|} = v^n s_{\overline{n}|} \quad \dots\dots (2.3)$$

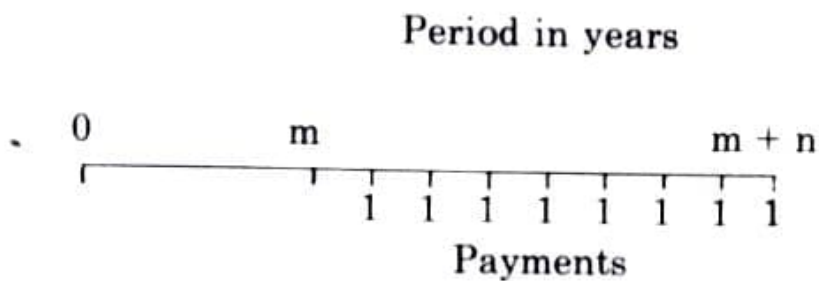
Conversely $s_{\overline{n}|}$ is the accumulated value of $a_{\overline{n}|}$ as at the end of n period

$$\therefore s_{\overline{n}|} = (1+i)^n a_{\overline{n}|} \quad \dots\dots (2.4)$$

The relations (2.3) and (2.4) above can easily be verified algebraically.

$$v^n s_{\overline{n}|} = v^n \left[\frac{(1+i)^n - 1}{i} \right] = \frac{v^n (1+i)^n - v^n}{i} = \frac{1 - v^n}{i} = a_{\overline{n}|}$$

The value $v^m a_n$ of the deferred annuity can be calculated by noting the values of v^m and a_n from the tables and multiplying the two together. This involves multiplication. However we can express the value in a slightly different form which enables us to get the value from tables without the necessity of multiplication.



Under the deferred annuity, there are n payments with no payments for the first m years. Suppose we include m additional payments at the end of 1st year, 2nd year, 3rd year etc. and at the end of the m th year. With m additional payments, we now have a total number of $(m+n)$ annual payments, which constitute an immediate annuity for $(m+n)$ years, whose present value is a_{m+n} . To get the present value of the deferred annuity, we have to subtract from a_{m+n} the value of the additional payments introduced. These additional payments constitute an immediate annuity for m years and so their present value = a_m . Therefore the value of the deferred annuity is given by

$$m | a_n = a_{m+n} - a_m \quad \dots\dots (2.7)$$

We have observed that $m | a_n$ can be expressed in two different ways, namely $v^m a_n$ or $a_{m+n} - a_m$. This enables us to simplify the evaluation of expressions of the form $v^m a_n$.

$$\begin{aligned} \text{Thus } v^7 a_6 @ 10\% &= a_{13} - a_7 \\ &= 7.6061 - 5.7590 \\ &= 1.8471 \end{aligned}$$

Example 8 : Find the present value of a series of 6 payments of Rs. 200 p.a., the first one being made at the end of 8 years, assuming a rate of interest of 6% p.a.

m=7, n=6, i=6%, P=200

Solution : The payments constitute a deferred annuity of Rs. 200 p.a. for 6 years, the deferment period being 7 years (and not 8 years). Hence the value is $7 | a_6 = v^7 a_6 = a_{13} - a_7 = 8.8527 - 5.5824 = 3.2703$.

Example 9 : Find the present value at the rate of 6% of 4 annual payments of Rs. 200/- p.a. followed by 7 annual payments of Rs. 350/- p.a. the first payment being made at the end of one year.

Solution : The 4 payments of Rs. 200 p.a. constitute an immediate annuity for 4 years and their present value is Rs. $200 a_4$.

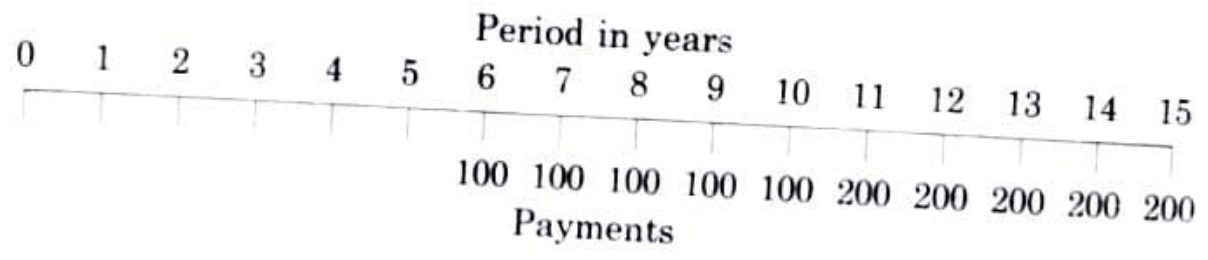
The value of the next 7 payments as at the end of 4 years is $350 a_7$. Therefore the value at present is $350 v^4 a_7$ which is equal to $350 (a_{11} - a_4)$.

$$\begin{aligned} \text{The required value} &= 200a_4 + 350(a_{11} - a_4) @ 6\% \\ &= 200 \times 3.4651 + 350(7.8869 - 3.4651) \\ &= \text{Rs. } 2240.65. \end{aligned}$$

Example 10 : Calculate the present value of a deferred annuity payable for 10 years certain, the first payment falling due at the end of 6 years from the present time. The annuity is payable at the rate of Rs. 100 p.a. for the first 5 years and Rs. 200 p.a. thereafter. *@ 5% p.a.*

Given : $a_{\overline{5}|} = 4.3295$ $a_{\overline{10}|} = 7.7217$ $a_{\overline{15}|} = 10.3797$.

Solution :



First consider the five payments of Rs. 100/- p.a. for the first 5 years, the first payment being due after 6 years. These five payments constitute a deferred annuity certain for five years, the deferment period being 5 years.

Its present value is given by

$$\begin{aligned} 100(5|a_{\overline{5}|}) &= 100(a_{\overline{10}|} - a_{\overline{5}|}) \\ &= 100(7.7217 - 4.3295) \\ &= 100(3.3922) \\ &= \text{Rs. } 339.22 \dots\dots\dots (I) \end{aligned}$$

v = 0.7524 *n = 5*

$$v^n a_{\overline{n}|}$$

x = 0.7835 (1 - v^n)

Now consider last five payments of Rs. 200/- p.a., the first payment of Rs. 200/- being due at the end of 11 years. These five payments also constitute a deferred annuity certain for five years, the deferment period being 10 years.

Its present value is given by

$$\begin{aligned} 200(10|a_{\overline{5}|}) &= 200 (a_{\overline{15}|} - a_{\overline{10}|}) \\ &= 200(10.3797 - 7.7217). \end{aligned}$$

m = 16 *n = 5*

$$v^n a_{\overline{n}|}$$

0.6140 x x 4.3279

$$= 200 (2.6580)$$

$$= \text{Rs. } 531.60 \dots\dots\dots(\text{II})$$

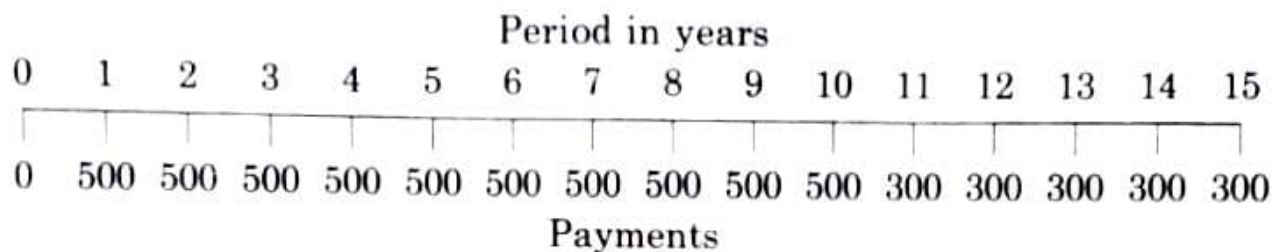
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The present value of 10 payments is given by (I) + (II)
 = Rs. 339.22 + Rs. 531.60
 = Rs. 870.82 .

Example 11 : An annuity is payable for 15 years certain, the first payment falling due at the end of first year. The annuity is payable at the rate of Rs. 500/- per annum during the first 10 years and at Rs. 300/- per annum during the remaining 5 years. Calculate the present value of the annuity on the basis of interest at 4% per annum.

Given : at 4 per cent $a_{\overline{10}|} = 8.1109$, $a_{\overline{15}|} = 11.1184$.

Solution :



Consider the first 10 payments of Rs. 500/- p.a. It constitutes an annuity certain for a period of 10 years. Its present value is given by

$$500 a_{\overline{10}|} @ 4\%$$

$$= 500 (8.1109)$$

$$= \text{Rs. } 4055.45 \dots\dots\dots (\text{I})$$

Now consider the last 5 payments of Rs. 300/- p.a., the first payment of Rs. 300/- being due at the end of 11 years. It constitutes a deferred annuity certain for 5 years, the deferment period being 10 years.

Its present value is given by :

$$(300) (10 | a_{\overline{5}|}) @ 4\%$$

$$= 300 (a_{\overline{15}|} - a_{\overline{10}|})$$

$$= 300 (11.1184 - 8.1109)$$

$$= 300 (3.0075)$$

$$= \text{Rs. } 902.25 \dots\dots\dots (\text{II})$$

Handwritten notes:
 $v^m a_{\overline{n}|}$
 $v^{10} a_{\overline{5}|}$
 $0.1756 \cdot 1 - v^n$
 902.25
 $v = 0.9615$

The present value of desired 15 payments is given by

$$(I) + (II) = \text{Rs. } 4055.45 + \text{Rs. } 902.25$$

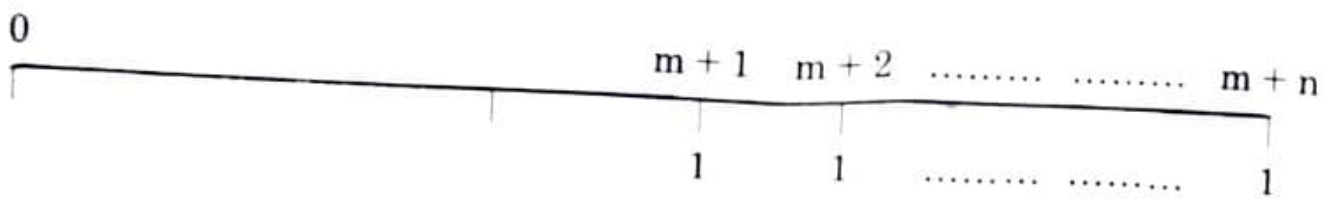
$$= \text{Rs. } 4957.70 .$$

16. Accumulated Value of a Deferred Annuity Certain :

Consider the accumulated value of a deferred annuity of 1 p.a. for n years certain, having deferment period m years, as at the end of m + n

years i.e. immediately after n th payment is made.

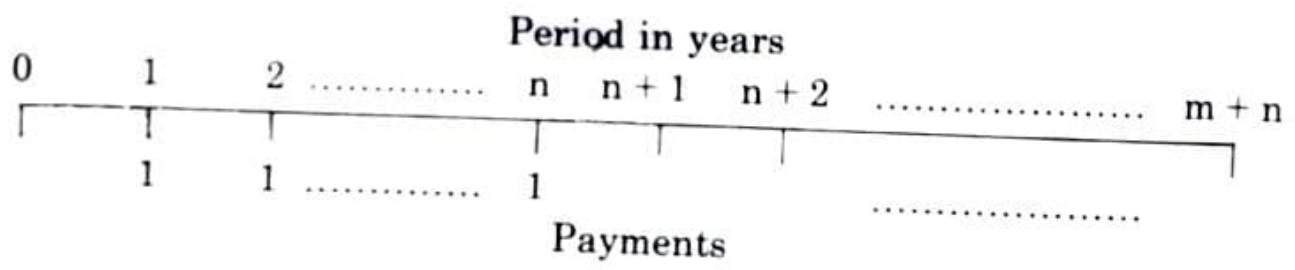
Under this annuity there will be no payments during the first m years.



Consider 1st payment under the above annuity. It is to be accumulated for a period of $(m+n)-(m+1) = n-1$ years, which is the same as the period for which 1st payment is to be accumulated under an immediate annuity certain for n years. This argument holds good for all the n payments.

\therefore The accumulated value of a deferred annuity of 1 p.a. for n years certain, having deferment period m years as at the end of $(m+n)$ years will be the same as accumulated value of an immediate annuity of 1 p.a. for n years as at the end of n years. i.e. $s_{\overline{n}|}$.

17. Accumulated Value of an Immediate Annuity of 1 p.a. for n years Certain, as at m years after the annuity payments have ceased.



The accumulated value of above annuity as at the end of n years is $s_{\overline{n}|}$.

\therefore The accumulated value of annuity after further m years is given by $(1+i)^m s_{\overline{n}|}$ (I)

Alternative Method :

If, instead of accumulating the payments of 1 p.a. after n years, they are continued till the end of $(m+n)$ years, the accumulated value would be $s_{\overline{m+n}|}$ as at end of $(m+n)$ years.

To find the accumulated value of above annuity, we have to subtract from $s_{\overline{m+n}|}$ the accumulated value of last additional m payments of 1 p.a. which is $s_{\overline{m}|}$.