UNIT V

Inventory Theory

Introduction

The word ‘inventory’ means simply a stock of idle resources of any kind having an economic value. In other words, inventory means a physical stock of goods, which is kept in hand for smooth and efficient running of future affairs of an organization. It may consist of raw materials, work-in-progress, spare parts/consumables, finished goods, human resources such as unutilized labor, financial resources such as working capital, etc. It is not necessary that an organization has all these inventory classes but whatever may be the inventory items, they need efficient management as generally a substantial amount of money is invested in them. The basic inventory decisions include: 1) *How much to order?* 2) *When to order?* 3) *How much safety stock should be kept?* The problems faced by different organizations have necessitated the use of scientific techniques in the management of inventories known as inventory control. Inventory control is concerned with the acquisition, storage, and handling of inventories so that the inventory is available whenever needed and the associated total cost is minimized.

Reasons for Carrying Inventory

Inventories are carried by organisations because of the following major reasons :

1. Improve customer service- An inventory policy is designed to respond to indi- vidual customer’s or organization’s request for products and services.

2. Reduce costs- Inventory holding or carrying costs are the expenses that are incurred for storage of items. However, holding inventory items in the warehouse can indirectly reduce operating costs such as loss of goodwill and/or loss of po- tential sale due to shortage of items. It may also encourage economies of pro- duction by allowing larger, longer and more production runs.

3. Maintenance of operational capability- Inventories of raw materials and work- in-progress items act as buffer between successive production stages so that downtime in one stage does not affect the entire production process.

4. Irregular supply and demand- Inventories provide protection against irregular supply and demand; an unexpected change in production and delivery sched- ule of a product or a service can adversely affect operating costs and customer service level.

5. Quantity discount- Large size orders help to take advantage of price-quantity discount. However, such an advantage must keep a balance between the storage cost and costs due to obsolescence, damage, theft, insurance, etc.

6. Avoiding stockouts (shortages)- Under situations like, labor strikes, natural disasters, variations in demand and delays in supplies, etc., inventories act as buffer against stock out as well as loss of goodwill.

Costs Associated with Inventories

Various costs associated with inventory control are often classified as follows:

1. *Purchase (or production) cost*: It is the cost at which an item is purchased, or if an item is produced.

2. *Carrying (or holding) cost*: The cost associated with maintaining inventory is known as holding cost. It is directly proportional to the quantity kept in stock and the time for which an item is held in stock. It includes handling cost, maintenance cost, depreciation, insurance, warehouse rent, taxes, etc.

3. *Shortage (or stock out) cost*: It is the cost which arises due to running out of stock. It includes the cost of production stoppage, loss of goodwill, loss of profitability, special orders at higher price, overtime/idle time payments, loss of opportunity to sell, etc.

4. *Ordering (or set up) cost*: The cost incurred in replenishing the inventory is known as ordering cost. It includes all the costs relating to administration (such as salaries of the persons working for purchasing, telephone calls, computer costs, postage, etc.), transportation, receiving and inspection of goods, processing payments, etc. If a firm produces its own goods instead of purchasing the same from an outside source, then it is the cost of resetting the equipment for production.

Basic Terminologies

The followings are some basic terminologies which are used in inventory theory:

1. Demand

It is an effective desire which is related to particular time, price, and quantity. The demand pattern of a commodity may be either deterministic or probabilistic. In case of deterministic demand, the quantities needed in future are known with certainty. This can be fixed (static) or can vary (dynamic) from time to time. On the contrary, probabilistic demand is uncertain over a certain period of time but its pattern can be described by a known probability distribution.

2. Ordering cycle

An ordering cycle is defined as the time period between two successive replenishments. The order may be placed on the basis of the following two types of inventory review systems:

- *Continuous review*: In this case, the inventory level is monitored continuously until a specified point (known as reorder point) is reached. At this point, a new order is placed.
- *Periodic review*: In this case, the orders are placed at equally spaced intervals of time. The quantity ordered each time depends on the available inventory level at the time of review.

3. Planning period

This is also known as time horizon over which the inventory level is to be controlled. This can be finite or infinite depending on the nature of demand.

4. Lead time or delivery lag

The time gap between the moment of placing an order and actually receiving it is referred to as lead time. Lead time can be deterministic (constant or variable) or probabilistic.

5. Buffer (or safety) stock

Normally, demand and lead time are uncertain and cannot be predetermined completely. So, to absorb the variation in demand and supply, some extra stock is kept. This extra stock is known as buffer stock.

6. Re-order level

Min.
The level between maximum and minimum stocks at which purchasing activity must start for replenishment is known as re-order level.

Economic Order Quantity (EOQ)

The concept of economic ordering quantity (EOQ) was first developed by F. Harris in 1916. Management of inventory is confronted with a set of opposing costs. As the lot size increases, the carrying cost increases while the ordering cost decreases. On the other hand, as the lot size decreases, the carrying cost decreases but the ordering cost increases. The two opposite costs can be shown graphically by plotting them against the order size as shown in Fig. 1.1 below :

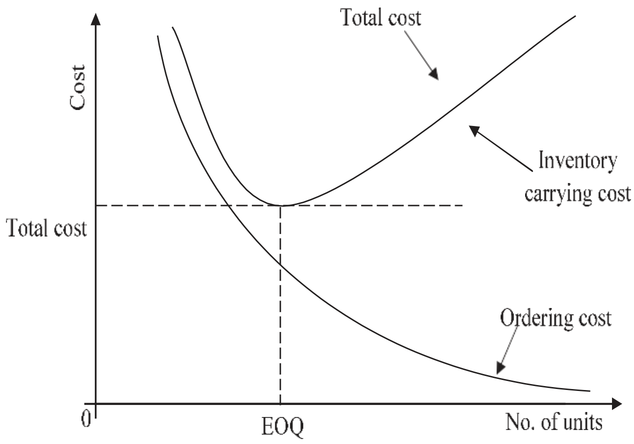

Fig. 1.1: Graph of EOQ

Economic ordering quantity(EOQ) is that size of order which minimizes the average total cost of carrying inventory and ordering under the assumed conditions of certainty and the total demand during a given period of time is known.

List of Symbols

The following symbols are used in connection with the inventory models presented in this chapter :

c = purchase (or manufacturing) cost of an item
c_1 = holding cost per quantity unit per unit time
c_2 = shortage cost per quantity unit per unit item
c_3 = ordering (set up) cost per order (set up)
R = demand rate
P = production rate
t = scheduling period which is variable
t_p = prescribe scheduling period
D = total demand or annual demand
q = lot (order) size
L = lead time

Deterministic Inventory Models

Model I(a): EOQ model without shortage or purchasing model with no shortages

The basic assumptions of the model are as follows:

- Demand rate R is known and uniform.
- Lead time is zero or a known constant.
- Replenishment rate is infinite, i.e., replenishments are instantaneous.

- Shortages are not permitted.
- Inventory holding cost is c_1 per unit per unit time.
- Ordering cost is c_3 per order.

Our objective is to determine the economic order quantity q^* which minimizes the average total cost of the inventory system. An inventory-time diagram with inventory level on the vertical axis and time on the horizontal axis is shown in Fig. 1.2. Since the actual consumption of inventory varies constantly, the concept of average inventory is applicable here.

$$Average\ Inventory = 1/2[maximum\ level + minimum\ level]$$
$$= (q + 0)/2 = q/2.$$

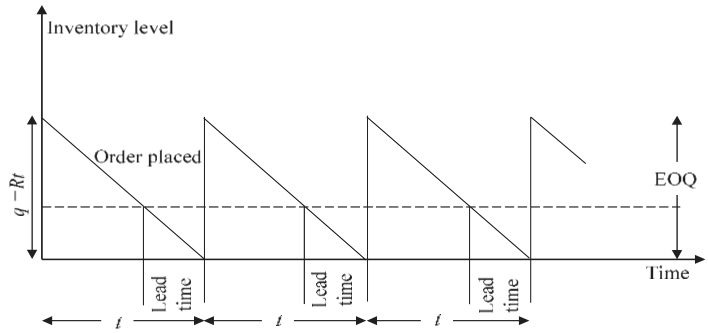

Fig. 1.2: Inventory-time diagram when lead time is a known constant

Thus, the average inventory carrying cost is = *average inventory* × *holding cost* $= \frac{1}{2}qc_1$.

The average ordering cost is $(R/q)c_3$. Therefore, the average total cost of the inventory system is given by

$$C(q) = \frac{1}{2}c_1q + \frac{c_3R}{q}. \tag{1.1}$$

Since the minimum average total cost occurs at a point when average ordering cost and average inventory carrying cost are equal, therefore, we have $\frac{1}{2}c_1q = \frac{c_3R}{q}$ which gives the optimal order quantity

$$q^* = \sqrt{\frac{2c_3R}{c_1}}. \tag{1.2}$$

This result was derived independently by F.W. Harris and R.H. Wilson in the year 1915. Thats why the model is called *Harris-Wilson model*.

Characteristics of Model I(a)

(i) Optimal ordering interval $t^* = q^*/R = \sqrt{\frac{2c_3}{c_1R}}$

(ii) Minimum average total cost $C_{\min} = C(q^*) = \sqrt{2c_1c_3R}$

If in Model I(a), the ordering cost is taken as $(c_3 + kq)$ where k is the ordering cost per unit item ordered then there will be no change in the optimal order quantity q^*.
In this case, the average total cost is

$$C(q) = \frac{1}{2}c_1q + \frac{c_3R}{q} + kR. \tag{1.3}$$

Model I(b): EOQ Model with Different Rates of Demand OR Manufacturing model with no shortages

This inventory system operates on the assumptions of Model I(a) except that the demand rates are different in different cycles but order quantity is fixed in each cycle. The objective is to determine the order size in each reorder cycle that will minimize the total inventory cost. Suppose that the total demand D is specified over the planning period T . If t_1, t_2, ..., t_n denote the lengths of successive n inventory cycles and D_1, D_2, ..., D_n are the demand rates in these cycles, respectively, then the total period T is given by $T = t_1+t_2+...+t_n$. Fig. 1.3 depicts the inventory system under consideration.

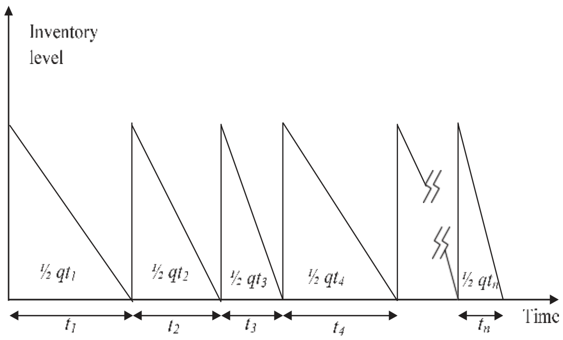

Fig. 1.3: Inventory-time diagram for different cycles

Suppose that each time a fixed quantity q is ordered. Then the number of orders in the time period T is $n = D/q$. Thus, the inventory carrying for the time period T is

$$\frac{1}{2}qt_1c_1 + \frac{1}{2}qt_2c_1 + ... + \frac{1}{2}qt_nc_1 = \frac{1}{2}qc_1(t_1 + t_2 + ... + t_n) = \frac{1}{2}qc_1T$$

Total ordering cost = (Number of orders) $\times\, c_3 = \frac{D}{q}c_3$

Hence, the total inventory cost is $C(q) = \frac{1}{2}c_1qT + \frac{c_3D}{q}$

The optimal ordering quantity (q^*) is then determined by the first order condition as

$$q^* = \sqrt{\frac{2c_3(D/T)}{c_1}}$$

The minimum total inventory cost is obtained by substituting the value of q^* in the cost equation, i.e.,

$$C_{\min} = \sqrt{2c_1c_3(D/T)}$$

Here we observe from the optimal results that the fixed demand rate R in Model I(a) is replaced by the average demand rate (D/T) in this model.

ABC Analysis

ABC Analysis also referred to as ABC Classification, is an integral part of material management. It is an inventory categorization method, which classifies the inventory primarily into three distinct categories based on the revenue generation. ABC inventory helps business entrepreneurs and stock owners identify the essential products in the stock and prioritize their management based on the value. The inventory analysis is based on the Pareto Principle.

Pareto Principle states that 80% of the sales volume gets generated from the top 20% of the items. It says that in any group, there are significant few and insignificant many. It is also known as the 80/20 rule.

If one implements the Pareto Principle to ABC Analysis, then A consists of 20% of the total products with almost 80% revenue generation. Hence, it demands a robust and consistent control. B regulates approximately 30% of the goods with 15% revenue, while C has the lion's share controlling almost 50% of the stock but only powering 5% of the total revenue. Hence the stock managers are quite lenient while calculating this category of inventory.

Procedure for ABC Analysis

A stock manager can perform ABC calculations on both individual product groups or a wide range of inventory. An ABC Calculation is usually carried out within five steps, which are as follows-

1. First, multiply the annual number of products with each item's cost and find the utility of that product.
2. Make a category of every product in the descending order based on its usage value.
3. Add the usage value of the products, including the total number of items.
4. Find out the cumulative percentages of items sold and annual consumption value.
5. Now, it's time to divide your data into three categories, finally, in an approximate ratio of 80:15:5.

VED ANALYSIS

VED analysis is an inventory management technique that classifies inventory based on its functional importance. It categorizes stock under three heads based on its importance and necessity for an organization for production or any of its other activities. VED analysis stands for Vital, Essential, and Desirable.

V-VITAL CATEGORY

As the name suggests, the category “Vital” includes inventory, which is necessary for production or any other process in an organization. The shortage of items under this category can severely hamper or disrupt the proper functioning of operations. Hence, continuous checking, evaluation, and replenishment happen for such stocks. If any of such inventories are unavailable, the entire production chain may stop. Also, a missing essential component may be of need at the time of a breakdown. Therefore, order for such inventory should be before-hand. Proper checks should be put in place by the management to ensure the continuous availability of items under the “vital” category.

E- ESSENTIAL CATEGORY

The essential category includes inventory, which is next to being vital. These, too, are very important for any organization because they may lead to a stoppage of production or hamper some other process. But the loss due to their unavailability may be temporary, or it might be possible to repair the stock item or part.

The management should ensure optimum availability and maintenance of inventory under the “Essential” category too. The unavailability of inventory under this category should not cause any stoppage or delays.

D- DESIRABLE CATEGORY

The desirable category of inventory is the least important among the three, and their unavailability may result in minor stoppages in production or other processes. Moreover, the easy replenishment of such shortages is possible in a short duration of time.

USAGE OF VED ANALYSIS

Small and big organizations both widely use VED analysis. The most important application of this analysis is in maintaining medical inventory in hospitals and their drug stores. Drugs and related supplies comprise a significant portion of a hospital’s budget. Moreover, maintaining the right quantity of the right drugs is an extremely challenging task for management. While a shortage of critical medicine can lead to crises and even loss of lives, an abundance of non-important medications can lead to blockage of money and space, both.

VED analysis helps in dividing medicines into the three categories as per their usage and importance. Therefore, medication in the vital group is to be kept in stock compulsorily, as they would be critical for patients. Medicines which are a bit less risky, or which can be obtained from other sources too at short notice, become part of an essential category. Those that are least critical and their shortage will not pose any danger to a patient’s health, and lives get its place in the desired class. As a result, the hospital’s management can wisely allocate resources on medical inventory as per their respective VED categories.

UNIT V : Model I + II FORMULAS

Symbols

1) k = Purchase (or Manufacting) cost of an it
2) C_1 = Holding cost per quantity unit per unit
3) C_2 = Shortage cost per quantity unit per unit
4) C_3 = ordering (set up) cost per order (set up)
5) R = Demand rate
6) P = Production rate
7) D = Total demand or annual demand
8) q = lot (order) size.

Cost in inventory	Model I: Purchasing model with no Shortage	Model II: Manufacturing mod with no shortages.
1. Economic order quantity or optimum lot size (EOQ)	EOQ: $q^* = \sqrt{\frac{2C_3R}{C_1}}$	$q_0 = \sqrt{\frac{K}{K-R}}\sqrt{\frac{2C_3R}{C_1}}$
2. Time order	$t_Q^* = \frac{q^*}{R}$	$t_0 = \frac{q_0}{R}$
3. Minimum Avg cost	$C_{min} = \sqrt{2C_1C_3R}$	$C_{min} = \sqrt{2C_1C_3R}\sqrt{\frac{K-R}{K}}$
4. Optimal Total cost	$= \frac{R}{q}C_3 + \left(\frac{q}{2} \times C_1\right) + RP$	$= \frac{1}{2}\frac{K-R}{K}C_1 q + \frac{C_3R}{q}$
5. No. of orders	$= 1/t_0$	—
6. Optimum no. of orders	$N^* = \frac{\text{demand}}{EOQ}$	—
7. Optimum period of supply	$t^* = \frac{1}{N^*}$	—
8. Maximum Inventory	—	$I_{m_0} = \frac{K-R}{K} q_0$
9. Manufacturing time	—	$t_1 = \frac{I_{m_0}}{K-R}$

Example 1 : **The annual demand for an item is 3200 units. The unit cost is Rs. 6/- and inventory carrying charges 25% per annum. If the cost of one procurement is Rs.150/- determine *(i)* Economic order quantity *(ii)* time between two consecutive orders *(iii)* number of orders per year *(iv)* the optimal total cost.**

Solution :

$$R = 3200 \text{ Units}, \quad C_1 = \frac{25}{100} \times 6 = \frac{3}{2}$$

$$C_3 = 150 \text{ Rs}, \qquad \therefore\ q^* = \sqrt{\frac{2C_3R}{C_1}}$$

$$= \sqrt{\frac{2 \times 150 \times 3200}{3/2}}$$

$$= 800 \text{ units.}$$

(ii)

$$t_0 = \{= t^*\} = \frac{q^*}{R}$$

$$= \frac{800}{3200} = \frac{1}{4} \text{th of a year}$$

$$= 3 \text{ months}$$

(iii) Number of orders

$$= \frac{1}{t_0} = \frac{1}{1/4} = 4$$

(iv)

$$\text{Total Optimal cost} = (R \times \text{price per unit}) + C_0$$

$$= (6 \times 3200) + \sqrt{2C_1C_3R}$$

$$= \text{Rs. } 20{,}400$$

[Ans]

Otherwise Optimal Total cost $= \dfrac{R}{q}\, c_3 + \left(\dfrac{q}{2} \times C_1\right) + RP$ 3200×6

600 + 600 + 19200

Example 2 : **A manufacturing company purchases 9000 parts of a machine for its annual requirements, ordering one month usage at a time. Each part costs Rs 20. The ordering cost per order is Rs.15 and the carrying charges are 15% of the average inventory per year. You have been asked to suggest a more economical purchasing policy for the company. What advice would you offer, and how much would it save the company per year ?**

[MU. BE (Mech). Apr 97, MU. BE. Apr 99]

Solution : Here R = 9000 parts per year

C_1 = 15% unit cost

(Here 15% of average Inventory per year means that the carrying cost per unit per year is 15% of the unit cost)

$$= 20 \times \frac{15}{100} = \text{Rs } 3 \text{ each part per year}$$

$$C_3 = \text{Rs. } 15 \text{ per order}$$

$$\therefore q^* = \sqrt{\frac{2C_3R}{C_1}}$$

$$= \sqrt{\frac{2 \times 15 \times 9000}{3}} = 300 \text{ units}$$

$$t^* = \frac{q^*}{R} = \frac{300}{9000}$$

0.033

$$= \frac{1}{30} \text{ year} = \frac{365}{30} = 12 \text{ days}$$

$$C_{min} = \sqrt{2\,C_1C_3R}$$

$$= \sqrt{2 \times 3 \times 15 \times 9000}$$

$$= \text{Rs. } 900.$$

If the company follows the policy of ordering every month, then the annual ordering cost becomes

$$= 12 \times 15 = \text{Rs. } 180.$$

and lot-size of Inventory each month $q = \frac{9000}{12} = 750$ parts.

Average Inventory at any time $= \frac{1}{2}q = 375$ parts

Storage cost at any time $= 375\,C_1$

$= 375 \times 3 = \text{Rs.}1125.$

Total annual cost $= 1125 + 180 = \text{Rs. } 1305.$

$\therefore$ The company purchases 300 parts at time intervals of 12 days instead of ordering 750 parts each month. So there will be a net saving of Rs 1305 – Rs. 900 = Rs. 405 per year

[Ans]

Example 3 : A certain item costs Rs. 235 per ton. The monthly requirements are 5 tons, and each time the stock is replenished, there is a setup cost of Rs. 1000. The cost of carrying inventory has been estimated at 10% of the average inventory per year. What is the optimum order quantity.

Solution :

$R = 5$ tons/month

$= 60$ tons/year

$C_3 = \text{Rs. } 1000$

$C_1 = 10\%$ of unit cost per year

$$= \text{Rs. } 235 \times \frac{10}{100}$$

$$= \text{Rs. } 23.5 \text{ per item per year}$$

$$\therefore q^* = \sqrt{\frac{2C_3R}{C_1}}$$

$$= \sqrt{\frac{2 \times 1000 \times 60}{23.5}}$$

$$= 71.458 \text{ tons}$$

[Ans]

Example 4 : For an item, the production is instantaneous. The storage cost of one item is Re one per month and the set up cost is Rs. 25 per run. If the demand is 200 units per month, Find the optimum quantity to be produced per set-up and hence determine the total cost of storage and set-up per month.

Solution :

$$R = 200 \text{ units/month}$$

$$C_1 = \text{Rs } 1 \text{ per unit per month}$$

$$C_3 = \text{Rs. } 25 \text{ per run}$$

$$\therefore q^* = \sqrt{\frac{2C_3R}{C_1}}$$

$$= \sqrt{\frac{2 \times 25 \times 2000}{1}} = 100 \text{ units.}$$

$$C_{min} = C^* = \sqrt{2C_1C_3R}$$

$$= \sqrt{2 \times 1 \times 25 \times 200}$$

$$= \text{Rs. } 100.$$

Total costs of storage and set up

$$= 25 + (1 \times 100)$$

$$= \text{Rs. } 125$$

Example 9 : [illegible]

[Ans]

Example 5 : A manufacturer has to supply his customer with 600 u[illegible]s of his products per year. Shortage are not allowed and storage c[illegible] amounts to 60 paise per unit per year. The set up cost is Rs. 80.00 fi[illegible]

(i) the economic order quantity

(ii) the minimum average yearly cost

(iii) the optimum number of orders per year

(iv) the optimum period of supply per optimum order

[MKU. B.Sc 72]

Solution :

$$R = 600 \text{ units/year}$$
$$C_3 = \text{Rs.}80$$
$$C_1 = 0.60 \text{ per unit/year}$$

(i)
$$\therefore q^* = \sqrt{\frac{2C_3R}{C_1}} = \sqrt{\frac{2 \times 600 \times 80}{0.60}}$$
$$= 400 \text{ units/year}$$

(ii)
$$C^* = \sqrt{2C_1C_3R} = \sqrt{2 \times 0.60 \times 80 \times 600}$$
$$= \text{Rs. } 240.$$

(iii)
$$N^* = \frac{\text{demand}}{\text{EOQ}} = \frac{600}{400} = \frac{3}{2} = 1.5$$

(iv)
$$t^* = \frac{1}{N^*} = \frac{2}{3} \text{ of a year} \quad 0.66$$

Example 6 : A company uses rivets at a rate of 5000 kg per year, rivets costing Rs. 2.00/kg. It costs Rs. 20 to place an order and carrying cost of inventory is 10% per year. How frequently should the order for rivets be placed and how much?

Solution :
$$R = 5000 \text{ per year} \;;\; C_3 = \text{Rs.}20$$
$$C_1 = \frac{10}{100} \times 2 = 0.2$$
$$\therefore q^* = \sqrt{\frac{2C_3R}{C_1}} = \sqrt{\frac{2 \times 20 \times 5000}{0.2}}$$
$$= 1000 \text{ kgs}$$
[Ans]

and
$$t_0 = \frac{q^*}{R} = \frac{1000}{5000} = \frac{1}{5} \text{ year} = 0.2$$
$$C_{min} = \sqrt{2C_1C_3R} = \sqrt{2 \times 0.2 \times 20 \times 5000}$$
$$= \text{Rs. } 200.$$
[Ans]

Example 7 : A contractor ... has to supply 10,000 bearings ... month to an automobile manufacturer. He finds that when he start... production run he can produce 25,000 bearings per month. The c... of holding a bearing in stock for one year is Rs. 2 and the set up c... of a production run is Rs. 180. How frequently should the producti... run be made ?

Solution :

$$R = 10{,}000/\text{month} \times 12 = 1{,}20{,}000 \text{ per year} \;;\; C_3 = \text{Rs } 180$$
$$K = 25{,}000 \times 12 = 3{,}00{,}000 \text{ per year} \;;\; C_1 = \text{Rs. } 2 \text{ per year}$$

$$\therefore q^* = \sqrt{\frac{K}{K-R}} \sqrt{\frac{2C_3R}{C_1}}$$

$$= \sqrt{\frac{3,00,000}{30,0000 - 120000}} \times \sqrt{\frac{2 \times 180 \times 120000}{2}}$$

$$= 1.29 \times \sqrt{21600000} = 6000 \text{ units}$$

$$t^* = \frac{q^*}{R} = \frac{6000}{120000} = 0.05 \text{ year } (i.e.,) \text{ 18 Days.} \quad \textit{[Ans]}$$

Example 8 : Find the most economic batch quantity of a product on a machine if the production rate of that item on the machine is 200 pieces per day and the demand is uniform at the rate of 100 pieces per day. The set up cost is Rs. 200 per batch and the cost of holding one item in Inventory is Rs. 0.81 per day. How will the batch quantity vary if the machine production rate is Infinite ?

Solution :

$$R = \text{Demand rate/day} = 100$$
$$C_3 = \text{Rs. 200 per set up}$$
$$C_1 = \text{Rs. 0.81 per day}$$
$$K = \text{production rate} = 200 \text{ per day}$$

$$\therefore EOQ = \sqrt{\frac{K}{K-R}} \sqrt{\frac{2C_3R}{C_1}}$$

$$= \sqrt{\frac{200}{100}} \times \sqrt{\frac{2 \times 200 \times 100}{0.81}}$$

$$= 1.414 \times 222.22 = 314 \text{ pieces}$$

when $K \to \infty$ then,

$$EOQ = \sqrt{\frac{2C_3R}{C_1}} = 222.22 \qquad \textit{[Ans]}$$

Example 9 : The annual demand for a product is 1,00,000 units. The rate of production is 2,00,000 units per year. The set-up cost per production run is Rs. 5000, and the variable production cost of each item is Rs. 10. The annual holding cost per unit is 20% of the value of the unit. Find the optimum production lot-size, and the length of production run.

Solution :

$$R = 1,00,000 \text{ per year}$$
$$C_1 = \frac{20}{100} \times 10 \text{ Rs. per year}$$
$$C_3 = \text{Rs. 5000}$$

$$K = 2,00,000$$

$$\therefore \text{EOQ} = \sqrt{\frac{K}{K-R}}\sqrt{\frac{2C_3R}{C_1}}$$

q_0

$$= \sqrt{\frac{2,00,000}{1,00,000}} \times \sqrt{\frac{2 \times 1,00,000 \times 5000}{\frac{20}{100} \times 10}}$$

$$= 1.4142 \times 22360.6$$

$$= 31622 \text{ Units. } (= q^*)$$

$$t^0 = \frac{q^0}{R} = \frac{31622}{1,00,000} = 0.31622 \text{ years}$$

$$\approx 115 \text{ days}$$

[Ans]

Example 10 : An item is produced at the rate of 50 items per day. The demand occurs at the rate of 25 items per day. If the set up cost is Rs. 100 per set up and holding cost is Re 0.01 per unit of item per day, find the economic lot size for one run, assuming that shortages are not permitted. Also find the time of cycle and minimum total cost for one run.

Solution : R = 25 items per day

C_1 = Rs.0.01 per unit per day

C_3 = Rs.100 per set up

K = 50 items per day

$$\therefore q_0 = \sqrt{\frac{K}{K-R}}\sqrt{\frac{2C_3R}{C_1}}$$

$$= \sqrt{\frac{2 \times 100 \times 25}{0.01}} \times \sqrt{\frac{50}{25}}$$

$$= 1000 \text{ items}$$

$$t_0 = \frac{q_0}{R} = \frac{1000}{25} = 40 \text{ days}$$

$$\text{Minimum daily cost} = \sqrt{2C_1C_3R}\sqrt{\frac{K-R}{K}}$$

$$= \text{Rs. } \sqrt{2 \times 0.01 \times 100 \times 25 \times \frac{25}{50}}$$

$$= \text{Rs. } 5$$

$$\text{Minimum total cost per run} = 5 \times 40$$

$$= \text{Rs. } 200$$

[Ans]

Example 11 : **A company has a demand of 12,000 units/year for an item and it can produce 2000 such items per month. The cost of one setup is Rs. 400 and the holding cost/unit/month is Rs. 0.15. Find the optimum lot size, max inventory, manufacturing time, total time.**

Solution :

$$R = 12{,}000 \text{ units/year}$$

$$C_3 = \text{Rs. } 400/\text{set up}$$

$$C_1 = \text{Rs. } 0.15 \times 12$$

$$= \text{Rs. } 1.80/\text{unit/year.}$$

$$K = 2000 \times 12 = 24{,}000 \text{ units/year}$$

$$\therefore q_0 = \sqrt{\frac{K}{K-R}}\sqrt{\frac{2C_3R}{C_1}}$$

$$= \sqrt{\frac{2 \times 400 \times 12{,}000}{1.80}} \times \sqrt{\frac{24{,}000}{12{,}000}}$$

$$= 3266 \text{ units/set up.}$$

$$\text{Max Inventory } I_{m_0} = \frac{K-R}{K} q_0$$

$$= \frac{24{,}000 - 12{,}000}{24{,}000} \times 3266$$

$$= 1632 \text{ units.}$$

$$\text{Manufacturing time } t_1 = \frac{Im_0}{K-R} = \frac{1632}{12{,}000}$$

$$= 0.136 \text{ years.}$$

$$\text{Total time } t_0 = \frac{q_0}{R} = \frac{3264}{12{,}000}$$

$$= 0.272 \text{ years.}$$ *[Ans]*

Example 12 : **A certain item costs Rs. 250 per ton. The monthly requirements are 10 tons and each time the stock is replenished there is a setup cost of Rs. 1000. The cost of carrying inventory has been estimated as 12% of the value of the stock per year. What is the optimal order quantity and how frequently should orders be placed ?**

[MU. MBA Apr 96]

Solution :

$$C_1 = \frac{12}{100} \times 250$$

$$C_3 = 1000 \text{ Rs}$$

$$R = 10 \times 12 = 120 \text{ tons/year}$$

$$\therefore \text{ EOQ} = \sqrt{\frac{2C_3R}{C_1}}$$

$$= \sqrt{\frac{2 \times 1000 \times 120}{\frac{12}{100} \times 250}}$$

$$= \sqrt{\frac{24,000}{30}} = \sqrt{8000}$$

$$= 89.44 \text{ units}$$ *[Ans]*

$$t_0 = \frac{q_0}{R} = \frac{89.44}{120} = 0.745 \text{ year}$$

$$\approx 9 \text{ months}$$ *[Ans]*

EXERCISE

1. [1] What are the different forms of inventory ?

[2] Discuss briefly the reasons for maintaining inventory in Business Management and Industry ?

[3] Explain various types of inventory.

[4] Write short notes on the losts involved in inventory problems ?

[5] What are the variables in an inventory problems ?

[6] Explain briefly *(a)* Lead time and *(b)* Reorder level.

[7] Define economic order quantity.

[8] Discuss various types of deterministic inventory models.

[9] Derive the optimal lot size formula due to R.H. Wilson.

[10] Discuss the purchasing model with no shortages assuming the demand rate to be inform and the production rate infinite.

[11] Derive the formula for optimum lot size for the manufacturing model with no shortages (demand rate uniform, production rate infinite.)

[12] What is an inventory system ? Explain the terms :

(a) Shortage cost *(b)* Lead time *(c)* Reorder point

[M.U., M.B.A., Apr. '98]

2. A contractor has to supply 10,000 bearings per day to an automobile manufacturer. He finds that, when he starts a production run, he can produce 25,000 bearings per day. The cost of holding a bearing in stock for one year is 2 paise, and the set up cost of a production run is Rs. 18. What is the optimum lot size and how frequently should the order be placed ?

Ans : 1,05,000 ; 10.5 days.

3. The XYZ manufacturing company has determined from an analysis of its accounting and production data for part number 625, that its cost Rs. 36 to purchase per order and Rs. 2 per part. Its inventory carrying charge is 18% of the average inventory. The demand for this part is 10,000 units per annum. Find *(a)* What should be the economic order Quantity be *(b)* What is the optimal number of days supply per optimum order? *[MU. BE. Oct 96]*

Ans : 1,144 units, 0.1414 year

4. A shop keeper has a uniform demand of an item at the rate of R: **600. Items per year. The buys from a supplier at a cost of Rs. 8 per item and the cost of ordering is** c_3 **12 each time. If the stock holding costs are 20% per year of stock value, how frequently should be replenish his stocks and what is the optimal order quantity ?** 0.1581 $C_1 = \frac{20}{100} \times 8$

[MU. MBA. Nov. 96] [*Ans :* $q^* = 95$ units, $t^* = 57$ Days]

5. The demand for an item in a company is 18,000 units per year, and the company can produce the item at the rate of 3000 per month. The cost of one set up is Rs. 500, and the holding cost of 1 unit per month is 15 paise. Deter-mine the optimum manufacturing quantity and the total cost per year assuming the cost of 1 unit as Rs. 2.00

Ans : $q^* = 4470$ units ; cost = Rs. 40,026

6. A stockist has to supply 400 units of a product every Monday to his customers. He gets the product at Rs.50 per unit from the manufacturer. The cost of ordering and transportation from the manufacturer is Rs. 75 per order. The cost of carrying inventory is 7.5% per year of the cost of the product. Find *(i)* the economic lot size *(ii)* the total optimal cost (including the capital cost)

Ans : $q_0 = 912$ units/order, $c_0 =$ Rs. 20,065.80 per week.

7. In a paints manufacturing unit, each type of paint is to be ground to a specified degree of fitness. The manufacturer uses the same ball mill for a variety of paints and after completion of each

batch, the mill has to be cleaned and the ball charge properly made up. The change over from one type of paint to another is estimated to cost Rs. 80 per batch. The annual sales of a particular grade of paint is 30,000 litres and the inventory carrying cost is Re.1 per litre. Given that the rate of production is 3 times the sales rate, determine EOQ, number of batches per year.

Ans : 2683.28 ; 11.18

8. A product is to be manufactured on a machine. The cost, production and demand *etc.*, are as follows :

Fixed cost per lot = Rs. 30,

Variable cost per unit = Rs. 0.10.

Charges for Insurance, taxes *etc.* = 50%

Production rate = 1,00,000 units per year

Demand rate = 10,000 units per year.

Find EOQ.

Ans : 3651 = q^* units.