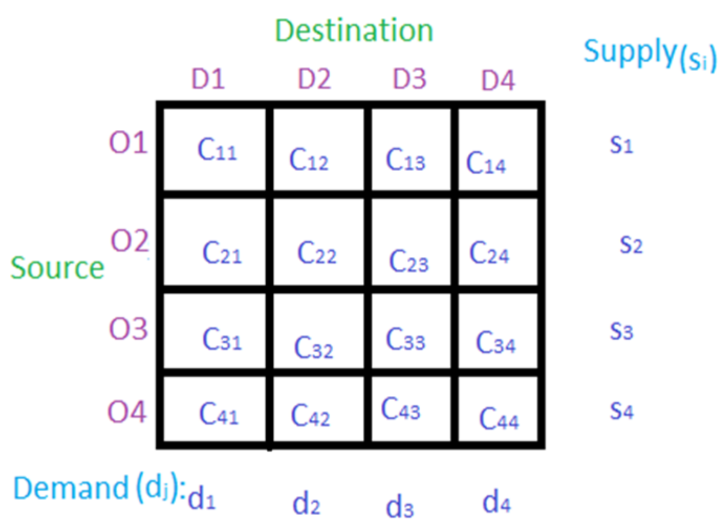
UNIT III

Transportation problem

Introduction

Transportation problem is a particular class of linear programming, which is associated with day-to-day activities in our real life and mainly deals with logistics. It helps in solving problems on distribution and transportation of resources from one place to another. The goods are transported from a set of sources (e.g., factory) to a set of destinations (e.g., warehouse) to meet the specific requirements. In other words, transportation problems deal with the transportation of a product manufactured at different plants (supply origins) to a number of different warehouses (demand destinations). The objective is to satisfy the demand at destinations from the supply constraints at the minimum transportation cost possible. To achieve this objective, we must know the quantity of available supplies and the quantities demanded. In addition, we must also know the location, to find the cost of transporting one unit of commodity from the place of origin to the destination. The model is useful for making strategic decisions involved in selecting optimum transportation routes so as to allocate the production of various plants to several warehouses or distribution centers.

Basic structure of transportation problem:



Mathematical formulation of Transportation problem

$$\text{Minimize } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

Subject to constraints,

$$\sum_{j=1}^{n} x_{ij} = a_{i,} \qquad i = 1,2,\ldots..m \text{ (supply constraints)}$$

$$\sum_{i=1}^{n} x_{ij} = b_{j,} \qquad j = 1,2,\ldots..m \text{ (demand constraints)}$$

and $x_{ij} \geq 0$ for all $i = 1,2,\ldots..m$ and,

$j = 1,2,\ldots..m$

cij = cost per unit distributed from source i to destination j
xij = the number of units to be distributed from source i to destination j

a_i= supply from source i;

b_j = demand at destination j;

Types of Transportation problems:

Balanced: When both supplies and demands are equal then the problem is said to be a balanced transportation problem**.**

i.e.,

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

. Such problems are called balanced transportation problems.

Unbalanced: When the supply and demand are not equal then it is said to be an unbalanced transportation problem. In this type of problem, either a dummy row or a dummy column is added according to the requirement to make it a balanced problem. Then it can be solved similar to the balanced problemIn many real life situations, however, the total availability may not be equal to the total demand. i.e.,

$$\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j$$

; such problems are called unbalanced transportation problems.

In these problems either some available resources will remain unused or some requirements will remain unfilled.

Since a feasible solution exists only for a balanced problem, it is necessary that the total availability be made equal to the total demand. If total capacity or availability is more than the demand and if there are no costs associated with the failure to use the excess capacity, we add a dummy (fictitious) destination to take up the excess capacity and the costs of shipping to this destination are set equal to zero. The zero cost cells are treated the same way as real cost cells and the problem is solved as a balanced problem. If there is, however, a cost associated with unused capacity (e.g., maintenance cost) and it is linear, it too can be easily treated.

In case the total demand is more than the availability, we add a dummy origin (source) to “fill” the balance requirement and the shipping costs are again set to equal to zero. However, in real life, the cost of unfilled demand is seldom zero since it may involve lost sales, lesser profits, and possibility of losing the customer or even business or the use of a more costly substitute. Solution of the problem under such situations may be more involved.

Feasible solution: A feasible solution to a transportation problem is a set of non-negative allocations, x_{ij} that satisfies the rim (row and column) restrictions.

Basic feasible solution: A feasible solution to a transportation problem is said to be a basic feasible solution if it contains no more than m + n – 1 non – negative allocations, where m is the number of rows and n is the number of columns of the transportation problem.

Optimal solution: A feasible solution (not necessarily basic) that minimizes (maximizes) the transportation cost (profit) is called an optimal solution.

Non -degenerate basic feasible solution: A basic feasible solution to a (m x n) transportation problem is said to be non – degenerate if,

1. the total number of non-negative allocations is exactly m + n – 1 (i.e., number of independent constraint equations), and
2. these m + n – 1 allocations are in independent positions.

Degenerate basic feasible solution: A basic feasible solution in which the total number of non-negative allocations is less than m + n – 1 is called degenerate basic feasible solution.

Methods to Solve Transportation problem:

To find the initial basic feasible solution there are three methods:

1. North West Corner Cell Method.
2. Least Cost Cell Method.
3. Vogel’s Approximation Method (VAM).

Problem 1:Find the initial basic feasible solution using the North West Corner Cell

Destination

Source	D1	D2	D3	D4	Supply
O1	3	1	7	4	300
O2	2	6	5	9	400
O3	8	3	3	2	500
Demand:	250	350	400	200	1200

Method

Solution:

Step 1: The problem is Balanced Transportation problem. i.e.,

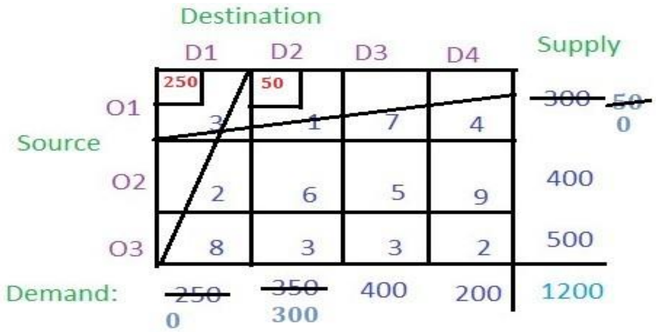
$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j = 1200, \quad \text{Balanced transportation problem.}$$

According to North West Corner method, **(O1, D1)** has to be the starting point i.e. the north-west corner of the table. Each and every value in the cell is considered as the cost per transportation. Compare the demand for column **D1** and supply from the source **O1** and allocate the minimum of two to the cell **(O1, D1)** as shown in the figure. The demand for Column **D1** is completed so the entire column **D1** will be canceled. The supply from the source **O1** remains **300 – 250 = 50**.

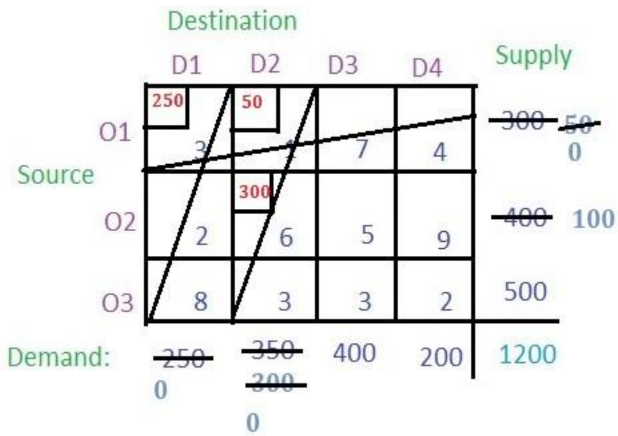
Destination

Source	D1	D2	D3	D4	Supply
O1	[250] 3	1	7	4	~~300~~ 50
O2	2	6	5	9	400
O3	8	3	3	2	500
Demand:	~~250~~ 0	350	400	200	1200

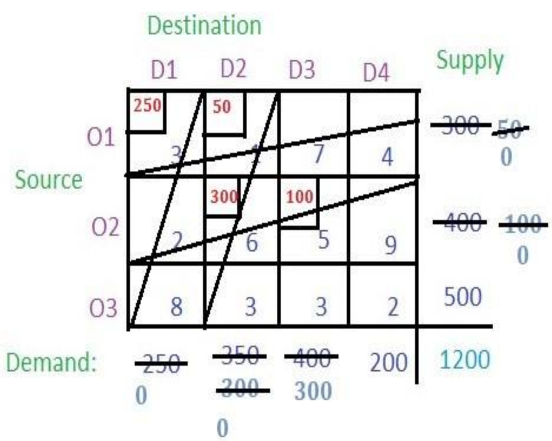
Step 2: Now from the remaining table i.e. excluding column **D1**, check the north-west corner i.e. **(O1, D2)** and allocate the minimum among the supply for the respective column and the rows. The supply from **O1** is **50** which is less than the demand for **D2** (i.e. 350), so allocate **50** to the cell **(O1, D2)**. Since the supply from row **O1** is completed cancel the row **O1**. The demand for column **D2** remain **350 – 50 = 300**.



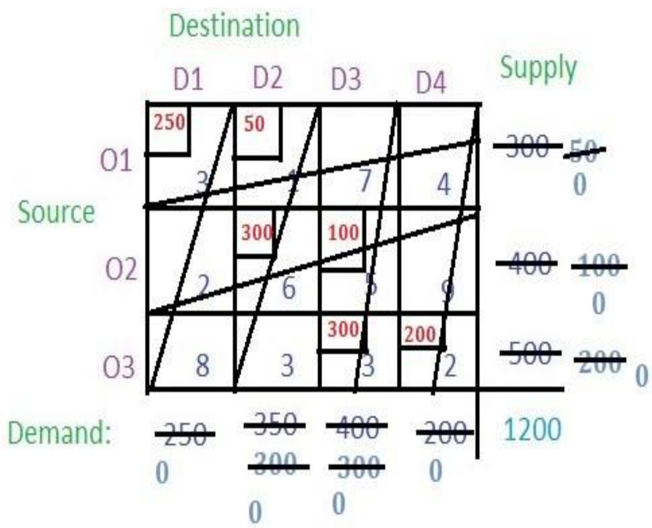
Step 3:From the remaining table the north-west corner cell is **(O2, D2)**. The minimum among the supply from source **O2** (i.e 400) and demand for column **D2** (i.e 300) is **300**, so allocate **300** to the cell **(O2, D2)**. The demand for the column **D2** is completed so cancel the column and the remaining supply from source **O2** is **400 – 300 = 100**.



Step 4: Now from remainig table find the north-west corner i.e. **(O2, D3)** and compare the **O2** supply (i.e. 100) and the demand for **D2** (i.e. 400) and allocate the smaller (i.e. 100) to the cell **(O2, D2)**. The supply from **O2** is completed so cancel the row **O2**. The remaining demand for column **D3** remains **400 – 100 = 300**.



Step5: Proceeding in the same way, the final values of the cells will be:



Note: In the last remaining cell the demand for the respective columns and rows are equal which was cell **(O3, D4)**. In this case, the supply from **O3** and the demand for **D4** was **200** which was allocated to this cell. At last, nothing remained for any row or column.

Source	Destination					
		D1	D2	D3	D4	SUPPLY
	O1	$^{250}3$	$^{50}1$	7	4	300
	O2	2	$^{300}6$	$^{100}5$	9	400
	O3	8	3	$^{300}3$	$^{200}2$	400
	DEMAND	250	350	400	200	1200

Step 6: The solution is Non degenerate basic feasible solution i.e;

$$m+n-1 = x_{ij}$$
$$3+4-1 = 6$$
$$6 = 6$$

Step 7:Now just multiply the allocated value with the respective cell value (i.e. the cost) and add all of them to get the basic solution

i.e. The Optimal Transportation cost = **(250 * 3) + (50 * 1) + (300 * 6) + (100 * 5) + (300 * 3) + (200 * 2)**

= 4400

Problem 2: Determine basic feasible solution to the following transportation problem using North west corner rule:

ORIGIN	A	B	C	D	E	SUPPLY
P	2	11	10	3	7	4
Q	1	4	7	2	1	8
R	3	9	4	8	12	9
DEMAND	3	3	4	5	6	

Solution:

Step 1: The problem is Balanced Transportation problem. i.e.,

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j = 21$$, Balanced transportation problems.

Step 2: The solution is Non degenerate basic feasible solution i.e;

$m+n-1 = x_{ij}$

$3 + 5 - 1 = 7$

ORIGIN	A	B	C	D	E	SUPPLY
P	3 2	111	10	3	7	4
Q	1	2 4	47	22	1	8
R	3	9	4	38	612	9
DEMAND	3	3	4	5	6	21

Step 3: The Optimal Transportation cost =3*2+1*11+2*4+4*7+2*2+3*8+6*12

= Rs 153/-

Transportation Problem - Least Cost Cell Method

Problem 3: Solve the Transportation problem using LCM

Destination

Source	D1	D2	D3	D4	Supply
O1	3	1	7	4	300
O2	2	6	5	9	400
O3	8	3	3	2	500
Demand:	250	350	400	200	1200

Solution:

Step 1: The problem is Balanced Transportation problem. i.e.,

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j = 1200,$$ Balanced transportation problem.

According to the Least Cost Cell method, the least cost among all the cells in the table has to be found which is **1** (i.e. cell **(O1, D2)**).

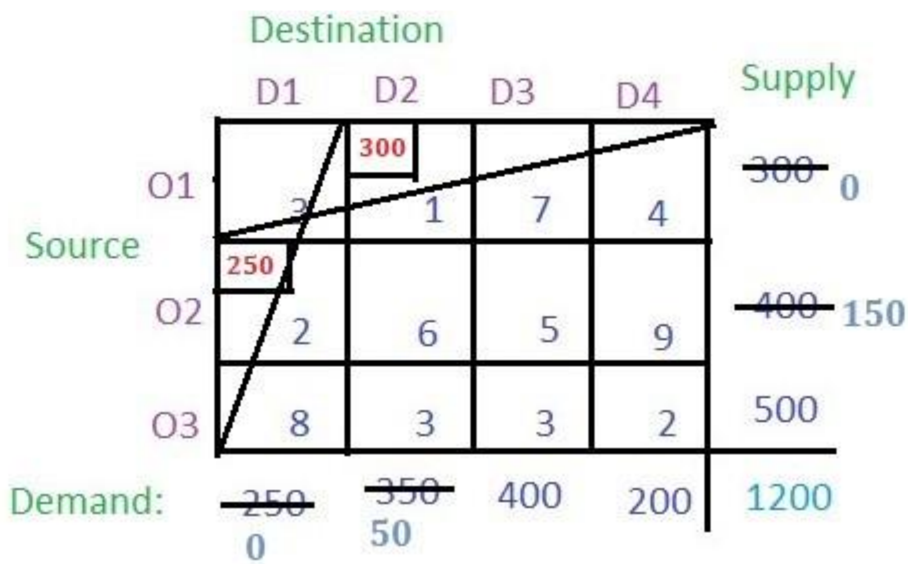
Now check the supply from the row **O1** and demand for column **D2** and allocate the smaller value to the cell. The smaller value is **300** so allocate this to the cell. The supply from **O1** is completed so cancel this row and the remaining demand for the column **D2** is **350 – 300 = 50**.

Destination

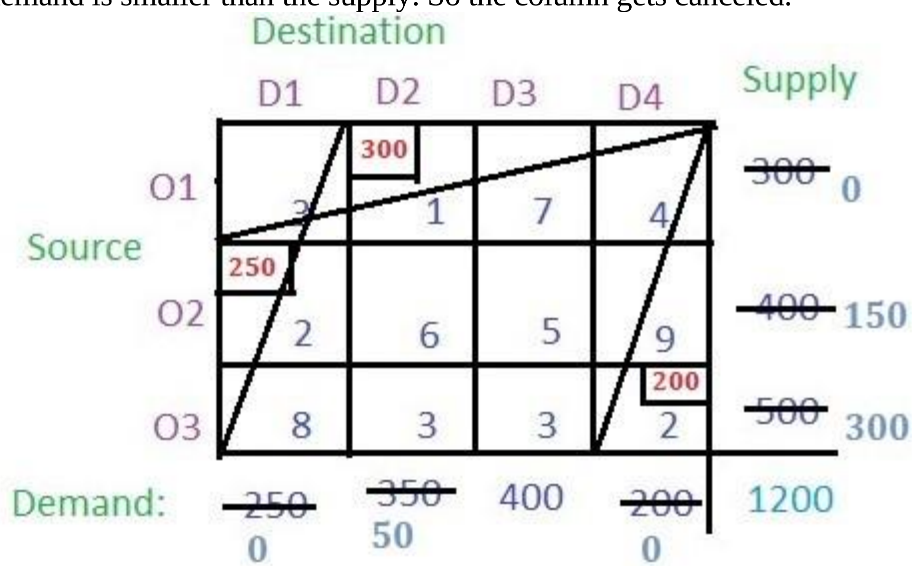
Source	D1	D2	D3	D4	Supply
O1	3	300 1	7	4	~~300~~ 0
O2	2	6	5	9	400
O3	8	3	3	2	500
Demand:	250	~~350~~ 50	400	200	1200

Step 2: Now find the cell with the least cost among the remaining cells. There are two cells with the least cost i.e. **(O2, D1)** and **(O3, D4)** with cost **2**. Let's select **(O2, D1)**. Now find the demand and supply for the respective cell and allocate the minimum among them to the cell and cancel the row or column whose supply or demand becomes **0** after allocation.

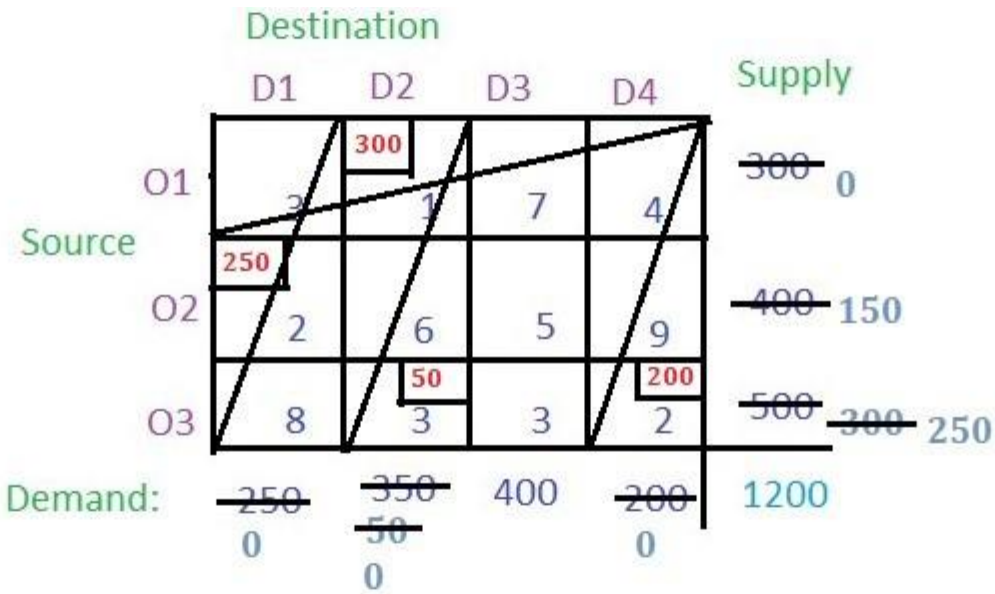
ADVERTISING



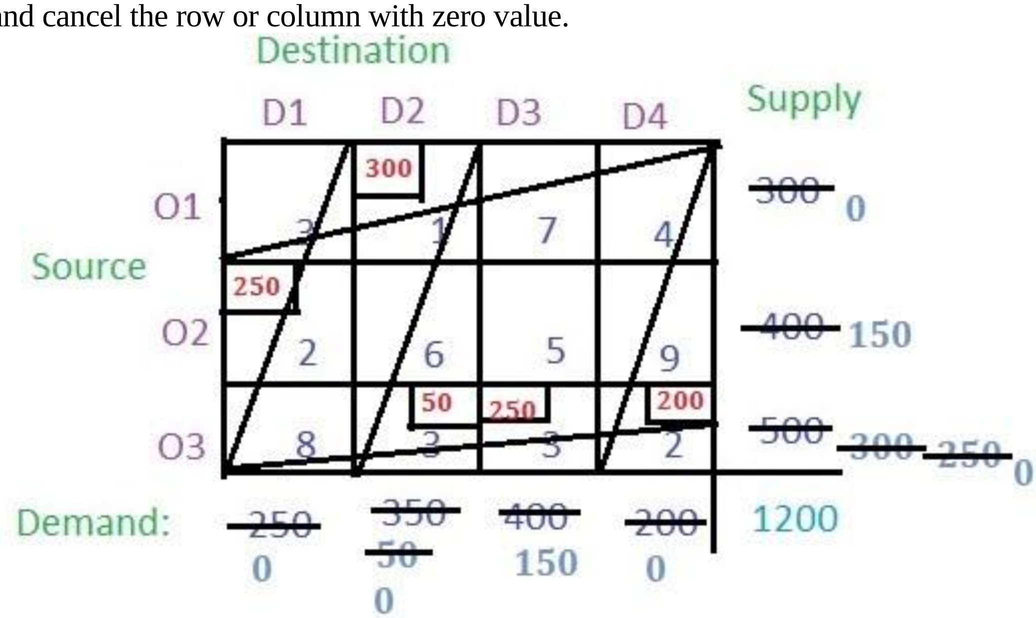
Step3: Now the cell with the least cost is **(O3, D4)** with cost **2**. Allocate this cell with **200** as the demand is smaller than the supply. So the column gets canceled.



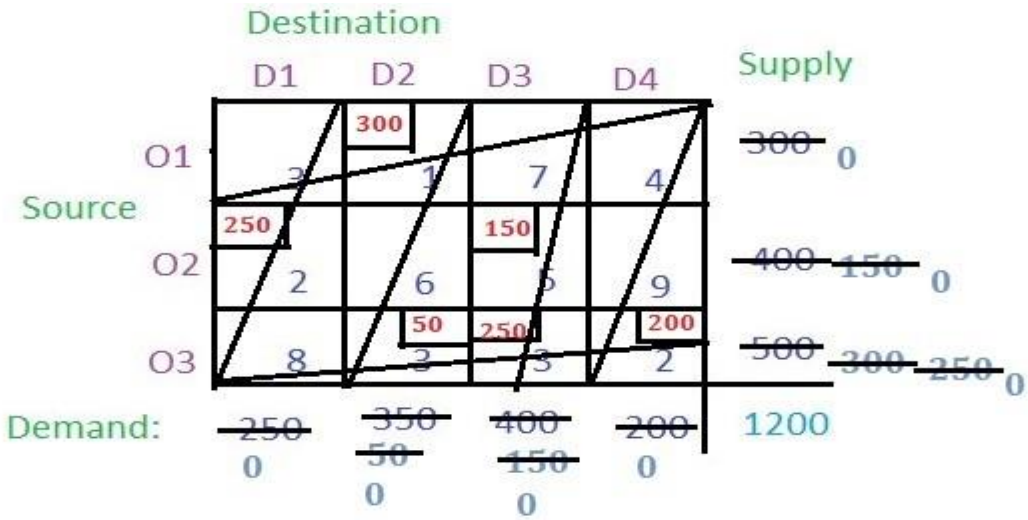
Step4: There are two cells among the unallocated cells that have the least cost. Choose any at random say **(O3, D2)**. Allocate this cell with a minimum among the supply from the respective row and the demand of the respective column. Cancel the row or column with zero value.



Step 5: Now the cell with the least cost is **(O3, D3)**. Allocate the minimum of supply and demand and cancel the row or column with zero value.



Step 6: The only remaining cell is **(O2, D3)** with cost **5** and its supply is **150** and demand is **150** i.e. demand and supply both are equal. Allocate it to this cell.



Step 7: The solution is Non degenerate basic feasible solution i.e;

$m+n-1 = x_{ij}$

$3+4-1 = 6$

Step 8:Now just multiply the allocated value with the respective cell value (i.e. the cost) and add all of them to get the basic solution i.e.

The Optimal Transportation cost = **(300 * 1) + (250 * 2) + (150 * 5) +**

(50 * 3) + (250 * 3) + (200 * 2)
= 2850

Transportation Problem - Vogel's Approximation Method

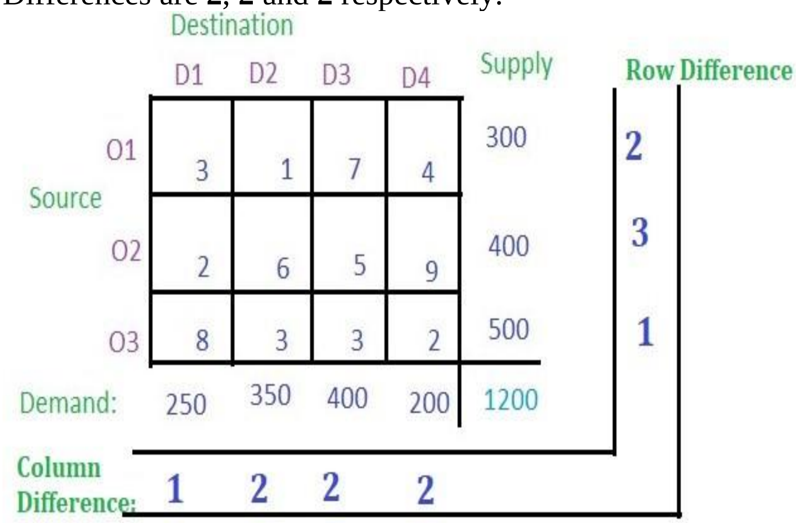
Problem 4: Solve the Transportation problem using **Vogel's Approximation** method.

Source \ Destination	D1	D2	D3	D4	Supply
O1	3	1	7	4	300
O2	2	6	5	9	400
O3	8	3	3	2	500
Demand:	250	350	400	200	1200

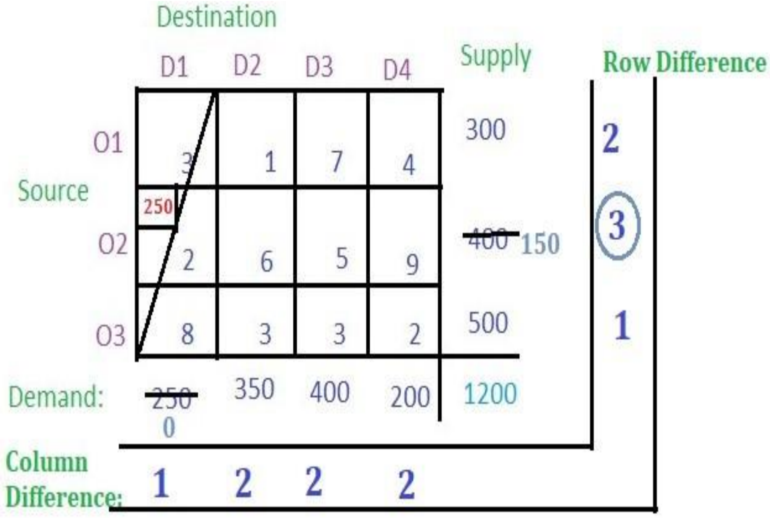
Solution: Step 1: The problem is Balanced Transportation problem. i.e.,

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j = 1200, \quad \text{Balanced transportation problem.}$$

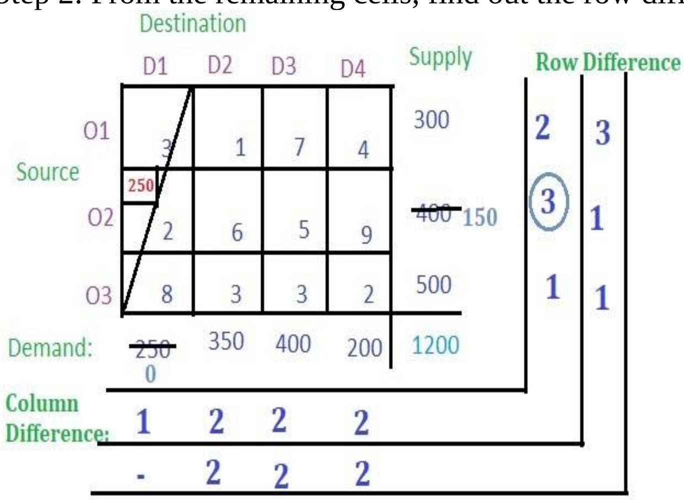
- For each row find the least value and then the second least value and take the absolute difference of these two least values and write it in the corresponding row difference as shown in the image below. In row **O1**, **1** is the least value and **3** is the second least value and their absolute difference is **2**. Similarly, for row **O2** and **O3**, the absolute differences are **3** and **1** respectively.
- For each column find the least value and then the second least value and take the absolute difference of these two least values then write it in the corresponding column difference as shown in the figure. In column **D1**, **2** is the least value and **3** is the second least value and their absolute difference is **1**. Similarly, for column **D2**, **D3** and **D3**, the absolute
- Differences are **2**, **2** and **2** respectively.



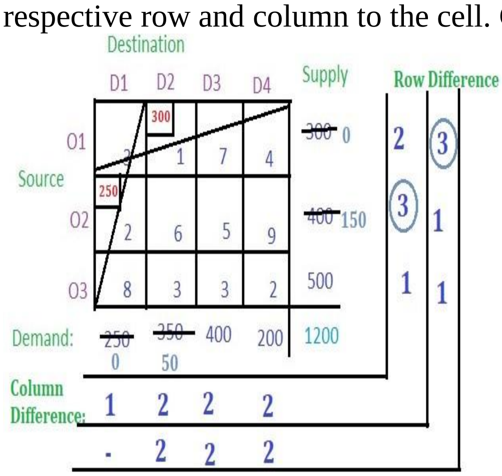
The value of row difference and column difference are also called as penalty. Now select the maximum penalty. The maximum penalty is **3** i.e. row **O2**. Now find the cell with the least cost in row **O2** and allocate the minimum among the supply of the respective row and the demand of the respective column. Demand is smaller than the supply so allocate the column’s demand i.e. **250** to the cell. Then cancel the column **D1**.



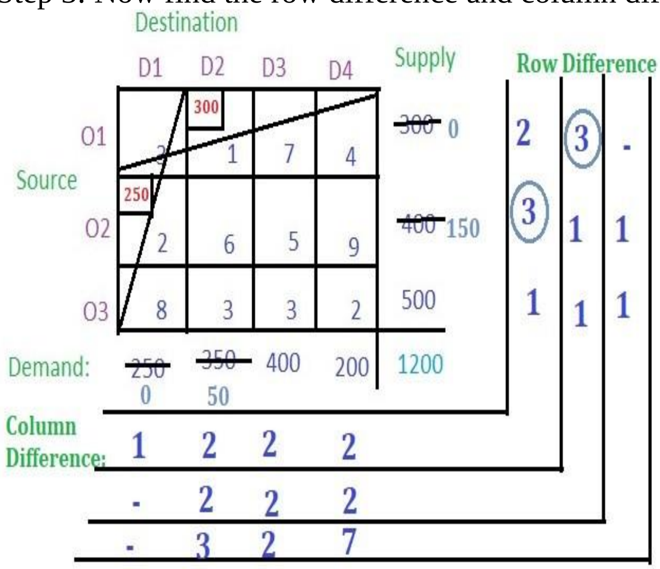
Step 2: From the remaining cells, find out the row difference and column difference.



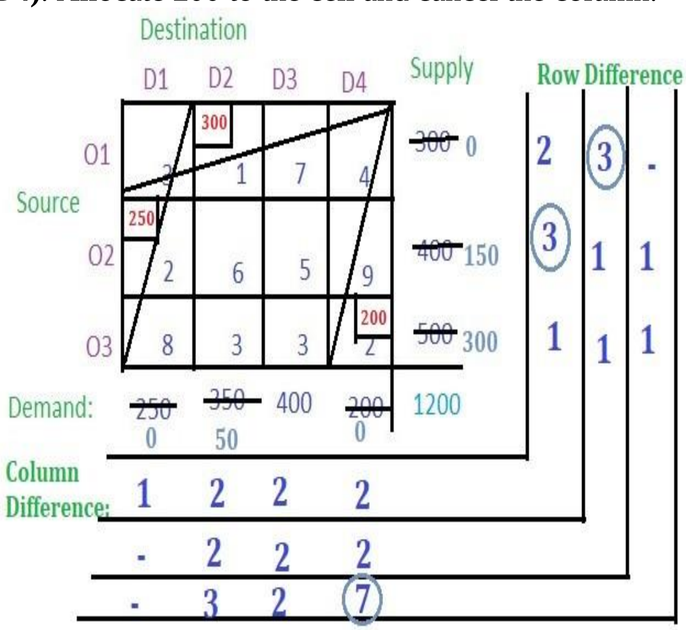
- Again select the maximum penalty which is **3** corresponding to row **O1**. The least-cost cell in row **O1** is **(O1, D2)** with cost **1**. Allocate the minimum among supply and demand from the respective row and column to the cell. Cancel the row or column with zero value.



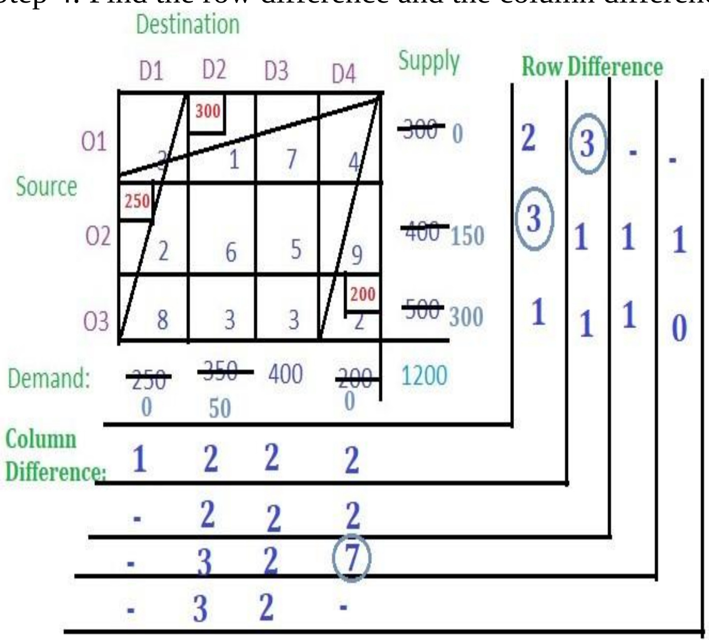
Step 3: Now find the row difference and column difference from the remaining cells.



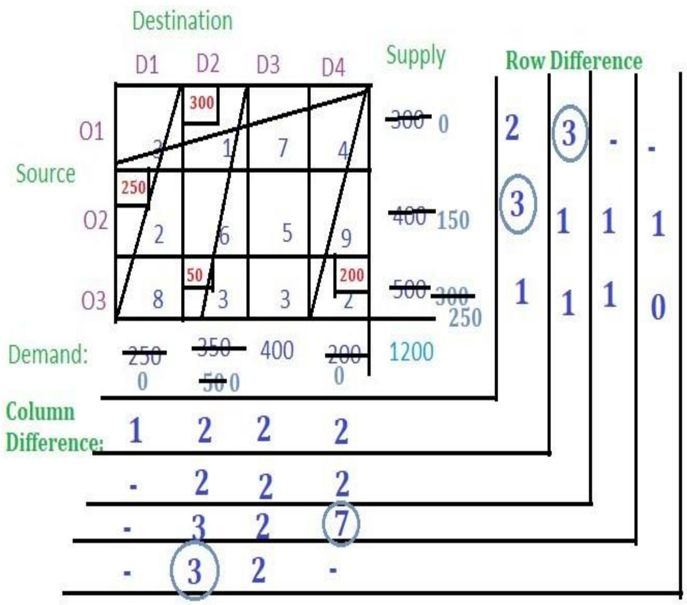
- Now select the maximum penalty which is **7** corresponding to column **D4**. The least cost cell in column **D4** is **(O3, D4)** with cost **2**. The demand is smaller than the supply for cell **(O3, D4)**. Allocate **200** to the cell and cancel the column.



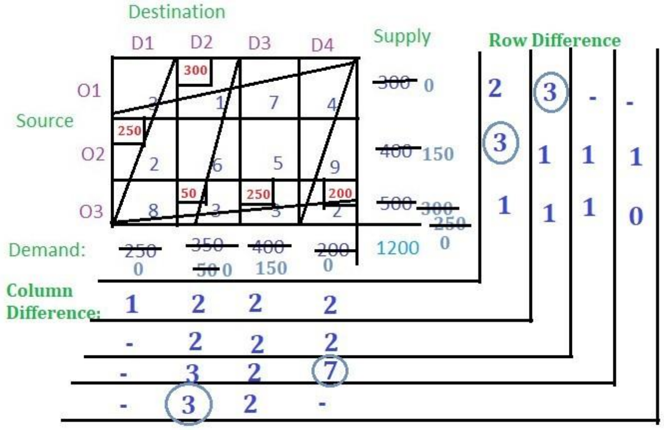
Step 4: Find the row difference and the column difference from the remaining cells.



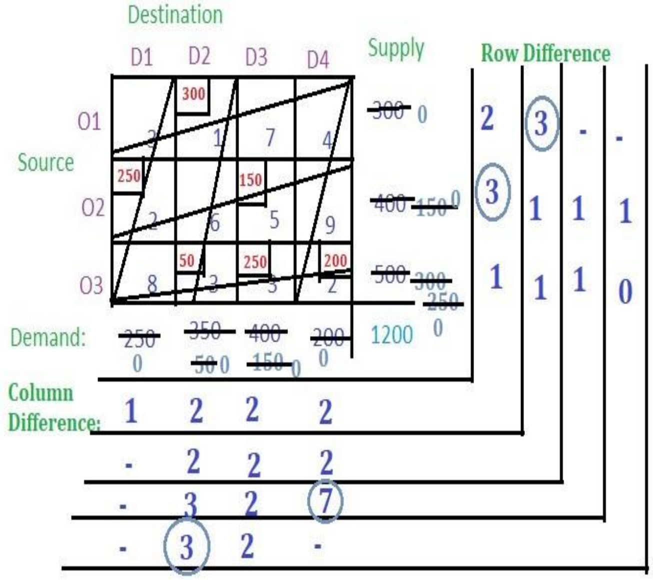
- Now the maximum penalty is **3** corresponding to the column **D2**. The cell with the least value in **D2** is **(O3, D2)**. Allocate the minimum of supply and demand and cancel the column.



Step 5: Now there is only one column so select the cell with the least cost and allocate the value.



- Now there is only one cell so allocate the remaining demand or supply to the cell



Step 6: The solution is Non degenerate basic feasible solution i.e;

$$m+n-1 = x_{ij}$$
$$3+4-1 = 6$$

Step 7:Now just multiply the allocated value with the respective cell value (i.e. the cost) and add all of them to get the basic solution i.e.

- The Optimal Transportation cost = **(300 * 1) + (250 * 2) + (50 * 3) + (250 * 3) + (200 * 2) + (150 * 5)=2850**

UNBALANCED TRANSPORTATION PROBLEM

Problem 5:

Solve the Transportation problem using North west corner method.

	A	B	C	D	Supply
1	11	20	7	8	50
2	21	16	20	12	40
3	8	12	18	9	70
Demand	30	25	35	40	

Solution:

Step 1: Since the total supply ($\Sigma\ a_i$ = 160) is greater than the total demand ($\Sigma\ b_j$ =130), the given problem is an Unbalanced transportation problem. To convert this into a balanced one we introduce a dummy destination E with zero unit transportation costs and having demand equal to 160 – 130 = 30 units then,

The problem is Balanced Transportation problem. i.e.,

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j = 160,$$ Balanced transportation problem.

By applying NWCM

	A	B	C	D	E	Supply
1	1130	2020	7	8	0 €	50
2	21	16 5	2035	12	0	40
3	8	12	18	940	030	70
Demand	30	25	35	40	30	160

Step 2: Since the number of non-negative allocations is 6 which is less than 7 then the basic solution is a degenerate one. To resolve this degeneracy, we allocate a small quantity € to the unoccupied cell so that the degeneracy become non degeneracy one.

Hence the solution is Non degenerate basic feasible solution i.e;

$m+n-1 = x_{ij}$

3+5-1 = 7

7 = 7

Step 3:Now just multiply the allocated value with the respective cell value (i.e. the cost) and add all of them to get the basic solution i.e.

The Optimal Transportation cost = 11*30+20*20+16*5+20*35+9*40+0*€+0*30

= Rs.1870/-

DEGENERACY IN TRANSPORTATION PROBLEM

Problem 6: Solve the Transportation problem using VAM

	TO				SUPPLY
FROM	10	20	5	7	10
	13	9	12	8	20
	4	5	7	9	30
	14	7	1	0	40
	3	12	5	19	50
DEMAND	60	60	20	10	

Solution:

Step 1: The problem is Balanced Transportation problem. i.e.,

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j = 150,$$ Balanced transportation problem.

	TO				SUPPLY
FROM	1010	20	5	7	10
	13	9 20	12	8	20
	4 €	5 30	7	9	30
	14	7 10	1 20	0 10	40
	3 50	12	5	19	50
DEMAND	60	60	20	10	150

Step 2: Since the number of non-negative allocations is 7 which is less than 8 then the basic solution is a degenerate one. To resolve this degeneracy, we allocate a small quantity € to the unoccupied cell so that the degeneracy become non degeneracy one.

Hence the solution is Non degenerate basic feasible solution i.e;

$m+n-1 = x_{ij}$

$5+4-1 = 8$

Step 3:Now just multiply the allocated value with the respective cell value (i.e. the cost) and add all of them to get the basic solution i.e.

The Optimal Transportation cost = 10*10+4*€+3*50+9*20+5*30+7*10+1*20+0*10 = Rs 670/-

MODIFIED DISTRIBUTION METHOD:

Problem 7:Find Solution using Voggel's Approximation method, also find optimal solution using MODI method,

	D1	D2	D3	D4	Supply
S1	19	30	50	10	7
S2	70	30	40	60	9
S3	40	8	70	20	18
Demand	5	8	7	14	

Solution:

Step 1: The problem is Balanced Transportation problem. i.e.,

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j = 34,$$ Balanced transportation problem.

By Applying VAM :

Initial feasible solution is

	$D1$	$D2$	$D3$	$D4$	Supply	Row Penalty
$S1$	19**(5)**	30	50	10**(2)**	7	9 | 9 | 40 | 40 | -- | -- |
$S2$	70	30	40**(7)**	60**(2)**	9	10| 20 | 20 | 20 | 20 | 40 |
$S3$	40	8**(8)**	70	20**(10)**	18	12 | 20 | 50 | -- | -- | -- |
Demand	5	8	7	14	34	
Column Penalty	21 21 -- -- -- --	22 -- -- -- -- --	10 10 10 10 40 40	10 10 10 50 60 --		

Step2: The solution is Non degenerate basic feasible solution i.e;

$m+n-1 = x_{ij}$

$3+4-1 = 6$

Step 3:The Optimal Transportation cost = 19×5+10×2+40×7+60×2+8×8+20×10

= 779

Optimality test using MODI method...
Allocation Table is

	D_1	D_2	D_3	D_4	Supply
S_1	19 **(5)**	30	50	10 **(2)**	7
S_2	70	30	40 **(7)**	60 **(2)**	9
S_3	40	8 **(8)**	70	20 **(10)**	18
Demand	5	8	7	14	34

Iteration-1 of optimality test

1. Find u_i and v_j for all occupied cells (i,j), where $c_{ij} = u_i+v_j$

	$D1$	$D2$	$D3$	$D4$	Supply	u_i
$S1$	19 **(5)**	30	50	10 **(2)**	7	$u1=10$
$S2$	70	30	40 **(7)**	60 **(2)**	9	$u2=60$
$S3$	40	8 **(8)**	70	20 **(10)**	18	$u3=20$
Demand	5	8	7	14	34	
v_j	$v1=9$	$v2= -12$	$v3=-20$	$v4=0$		

1. Substituting, $v4=0$, (because it has more allocated cells) we get,
2. $c14= u1+v4 \Rightarrow u1=c14-v4 \Rightarrow u1=10-0 \Rightarrow u1=10$
3. $c11=u1+v1 \Rightarrow v1=c11-u1 \Rightarrow v1=19-10 \Rightarrow v1=9$
4. $c24=u2+v4 \Rightarrow u2=c24-v4 \Rightarrow u2=60-0 \Rightarrow u2=60$
5. $c23=u2+v3 \Rightarrow v3=c23-u2 \Rightarrow v3=40-60 \Rightarrow v3=-20$
6. $c34=u3+v4 \Rightarrow u3=c34-v4 \Rightarrow u3=20-0 \Rightarrow u3=20$
7. $c32=u3+v2 \Rightarrow v2=c32-u3 \Rightarrow v2=8-20 \Rightarrow v2=-12$

2. Find d_{ij} for all unoccupied cells(i,j), where $d_{ij}=c_{ij}-\left(u_i+v_j\right)$

	$D1$	$D2$	$D3$	$D4$	Supply	ui
$S1$	19 **(5)**	30 [32]	50 [60]	10 **(2)**	7	$u1$=10
$S2$	70 [1]	30 [-18]	40 **(7)**	60 **(2)**	9	$u2$=60
$S3$	40 [11]	8 **(8)**	70 [70]	20 **(10)**	18	$u3$=20
Demand	5	8	7	14	34	
vj	$v1$=9	$v2$=-12	$v3$=-20	$v4$=0		

1.$d12=c12-(u1+v2)=30-(10-12)=32$

2.$d13=c13-(u1+v3)=50-(10-20)=60$

3.$d21=c21-(u2+v1)=70-(60+9)=1$

4.$d22=c22-(u2+v2)=30-(60-12)=-18$

5.$d31=c31-(u3+v1)=40-(20-9)=11$

6.$d33=c33-(u3+v3)=70-(20-20)=70$

3. Now choose the minimum negative value from all dij (opportunity cost) = $d22$ = [-18] and draw a closed path from $S2D2$.

Closed path and plus/minus sign allocation...

	$D1$	$D2$	$D3$	$D4$	Supply	ui

*S*1	19 **(5)**	30 [32]	50 [60]	10 **(2)**	7	*u*1=10
*S*2	70 [1]	30 [-18] **(+)**	40 **(7)**	60 **(2)** **(-)**	9	*u*2=60
*S*3	40 [11]	8 **(8)** **(-)**	70 [70]	20 **(10)** **(+)**	18	*u*3=20
Demand	5	8	7	14		
vj	*v*1=9	*v*2=-12	*v*3=-20	*v*4=0		

Closed path is *S*2*D*2→*S*2*D*4→*S*3*D*4→*S*3*D*2

4. Minimum allocated value among all negative position **(-)** on closed path = 2, Substract 2 from all **(-)** and Add it to all **(+)**

	*D*1	*D*2	*D*3	*D*4	Supply
*S*1	19 **(5)**	30	50	10 **(2)**	7
*S*2	70	30 **(2)**	40 **(7)**	60	9
*S*3	40	8 **(6)**	70	20 **(12)**	18
Demand	5	8	7	14	

5. Repeat the step 1 to 4, until an optimal solution is obtained.

Iteration-2 of optimality test

1. Find u_i and v_j for all occupied cells(i,j), where $c_{ij}=u_i+v_j$

	*D*1	*D*2	*D*3	*D*4	Supply	*ui*

$S1$	19 **(5)**	30	50	10 **(2)**	7	$u1=0$
$S2$	70	30 **(2)**	40 **(7)**	60	9	$u2=32$
$S3$	40	8 **(6)**	70	20 **(12)**	18	$u3=10$
Demand	5	8	7	14		
vj	$v1=19$	$v2=-2$	$v3=8$	$v4=10$		

1. Substituting, $u1=0$, we get

   2.$c11=u1+v1 \Rightarrow v1=c11-u1 \Rightarrow v1=19-0 \Rightarrow v1=19$

   3.$c14=u1+v4 \Rightarrow v4=c14-u1 \Rightarrow v4=10-0 \Rightarrow v4=10$

   4.$c34=u3+v4 \Rightarrow u3=c34-v4 \Rightarrow u3=20-10 \Rightarrow u3=10$

   5.$c32=u3+v2 \Rightarrow v2=c32-u3 \Rightarrow v2=8-10 \Rightarrow v2=-2$

   6.$c22=u2+v2 \Rightarrow u2=c22-v2 \Rightarrow u2=30+2 \Rightarrow u2=32$

   7.$c23=u2+v3 \Rightarrow v3=c23-u2 \Rightarrow v3=40-32 \Rightarrow v3=8$

   2. Find d_{ij} for all unoccupied cells(i,j), where $dij=cij-(ui+vj)$

	D_1	D_2	D_3	D_4	Supply	u_i
S_1	19 **(5)**	30 [32]	50 [42]	10 **(2)**	7	$u_1=0$
S_2	70 [19]	30 **(2)**	40 **(7)**	60 [18]	9	$u_2=32$
S_3	40 [11]	8 **(6)**	70 [52]	20 **(12)**	18	$u_3=10$
Demand	5	8	7	14		
v_j	$v_1=19$	$v_2=-2$	$v_3=8$	$v_4=10$		

1.$d12=c12-(u1+v2)=30-(0-2)=32$

2.$d13=c13-(u1+v3)=50-(0+8)=42$

3.$d21=c21-(u2+v1)=70-(32+19)=19$

4.$d24=c24-(u2+v4)=60-(32+10)=18$

5.$d31=c31-(u3+v1)=40-(10+19)=11$

6. $d_{33}=c_{33}-(u_3+v_3)=70-(10+8)=52$

Since all $d_{ij} \geq 0$, the final optimal solution is arrived.

	D_1	D_2	D_3	D_4	Supply
S_1	19 **(5)**	30	50	10 **(2)**	7
S_2	70	30 **(2)**	40 **(7)**	60	9
S_3	40	8 **(6)**	70	20 **(12)**	18
Demand	5	8	7	14	

Therefore our final minimum total transportation / optimal cost $=19\times5+10\times2+30\times2+40\times7+8\times6+20\times12=743$

MODI Method Steps (Rule)	
Step-1:	Find an initial basic feasible solution using any one of the three methods NWCM, LCM or VAM.
Step-2:	Find u_i and v_j for rows and columns. To start a. assign 0 to u_i or v_j where maximum number of allocation in a row or column respectively. b. Calculate other u_i's and v_j's using $c_{ij}=u_i+v_j$, for all occupied cells.
Step-3:	For all unoccupied cells, calculate $d_{ij}=c_{ij}-(u_i+v_j)$, .
Step-4:	Check the sign of d_{ij} a. If $d_{ij}>0$, then current basic feasible solution is optimal and stop this procedure. b. If $d_{ij}=0$ then alternative soluion exists, with different set allocation and same transportation cost. Now stop this procedure. b. If $d_{ij}<0$, then the given solution is not an optimal solution and further improvement in the solution is possible.
Step-5:	Select the unoccupied cell with the largest negative value of d_{ij}, and included in the next solution.
Step-6:	Draw a closed path (or loop) from the unoccupied cell (selected in the previous step). The right angle turn in this path is allowed only at occupied cells and at the original unoccupied cell. Mark (+) and (-) sign alternatively at each corner, starting from the original unoccupied cell.
Step-7:	1. Select the minimum value from cells marked with (-) sign of the closed path. 2. Assign this value to selected unoccupied cell (So unoccupied cell becomes occupied cell). 3. Add this value to the other occupied cells marked with (+) sign. 4. Subtract this value to the other occupied cells marked with (-) sign.
Step-8:	Repeat Step-2 to step-7 until optimal solution is obtained. This procedure stops when all $d_{ij}\geq 0$ for unoccupied cells.