ELEMENTS OF OPERATIONS RESEARCH

UNIT – I

Introduction

The British/Europeans refer to "operational research", the Americans to "operations research" - but both are often shortened to just "OR" - which is the term we will use. Another term which is used for this field is "management science" ("MS"). The Americans sometimes combine the terms OR and MS together and say "OR/MS" or "ORMS". Yet other terms sometimes used are "industrial engineering" ("IE") and "decision science" ("DS"). In recent years there has been a move towards a standardization upon a single term for the field, namely the term "OR". Operation Research is a relatively new discipline. The contents and the boundaries of the OR are not yet fixed. Therefore, to give a formal definition of the term Operations Research is a difficult task. The OR starts when mathematical and quantitative techniques are used to substantiate the decision being taken. The main activity of a manager is the decision making. In our daily life we make the decisions even without noticing them. The decisions are taken simply by common sense, judgment and expertise without using any mathematical or any other model in simple situations. But the decision we are concerned here with are complex and heavily responsible. Examples are public transportation network planning in a city having its own layout of factories, residential blocks or finding the appropriate product mix when there exists a large number of products with different profit contributions and production requirement etc.

Operations Research tools are not from any one discipline. Operations Research takes tools from different discipline such as mathematics, statistics, economics, psychology, engineering etc. and combines these tools to make a new set of knowledge for decision making. Today, O.R. became a professional discipline which deals with the application of scientific methods for making decision, and especially to the allocation of scarce resources. The main purpose of O.R. is to provide a rational basis for decisions making in the absence of complete information, because the systems composed of human, machine, and procedures may do not have complete information

Definition of Operations Research

Miller and Starr state, "O.R. is applied decision theory, which uses any scientific, mathematical or logical means to attempt to cope with the problems that confront the executive, when he tries to achieve a thorough-going rationality in dealing with his decision problem".

Pocock stresses that O.R. is an applied Science. He states "O.R. is scientific methodology (analytical, mathematical, and quantitative) which by assessing the overall implication of various alternative courses of action in a management system provides an improved basis for management decisions"

Scope ,Uses and Applications of Operations Research

Today, almost all fields of business and government utilizing the benefits of Operations Research. There are voluminous of applications of Operations Research. Although it is not feasible to cover all applications of O.R. in brief. The following are the abbreviated set of typical operations research applications to show how widely these techniques are used today:

- Accounting:
  Assigning audit teams effectively
  Credit policy analysis
  Cash flow planning

Developing standard costs
Establishing costs for byproducts

- <u>Construction:</u>
  Project scheduling, monitoring and control
  Determination of proper work force
  Deployment of work force
  Allocation of resources to projects

- <u>Facilities Planning:</u>
  Factory location and size decision
  Estimation of number of facilities required
  Hospital planning
  International logistic system design
  Transportation loading and unloading
  Warehouse location decision

- <u>Finance:</u>
  Building cash management models
  Allocating capital among various alternatives
  Building financial planning models
  Investment analysis
  Portfolio analysis
  Dividend policy making

- <u>Manufacturing:</u>
  Inventory control
  Marketing balance projection
  Production scheduling
  Production smoothing

- <u>Marketing:</u>
  Advertising budget allocation
  Product introduction timing
  Selection of Product mix
  Deciding most effective packaging alternative
  Organizational Behavior / Human Resources:
  Personnel planning
  Recruitment of employees
  Skill balancing
  Training program scheduling
  Designing organizational structure more effectively

- <u>Purchasing:</u>
  Optimal buying
  Optimal reordering
  Materials transfer

- <u>Research and Development:</u>
  R & D Projects control
  R & D Budget allocation
  Planning of Product introduction

Limitations of Operations Research

Operations Research has number of applications; similarly it also has certain limitations. These limitations are mostly related to the model building and money and time factors problems involved in its application. Some of them are as given below:

- i)Distance between O.R. specialist and Manager

Operations Researchers job needs a mathematician or statistician, who might not be aware of the business problems. Similarly, a manager is unable to understand the complex nature of Operations Research. Thus there is a big gap between the two personnel.

- ii)Magnitude of Calculations

The aim of the O.R. is to find out optimal solution taking into consideration all the factors. In this modern world these factors are enormous and expressing them in quantitative model and establishing relationships among these require voluminous calculations, which can be handled only by machines.

- iii)Money and Time Costs

The basic data are subjected to frequent changes, incorporating these changes into the operations research models is very expensive. However, a fairly good solution at present may be more desirable than a perfect operations research solution available in future or after some time.

- iv)Non-quantifiable Factors

When all the factors related to a problem can be quantifiable only then operations research provides solution otherwise not. The non-quantifiable factors are not incorporated in O.R.models. Importantly O.R. models do not take into account emotional factors or qualitative factors.

- v)Implementation

Once the decision has been taken it should be implemented. The implementation of decisions is a delicate task. This task must take into account the complexities of human relations and behavior and in some times only the psychological factors.

Models in Operation Research

The OR models are

1. Allocation models
2. Replacement models
3. Waiting line models
4. Network models
5. Game theory
6. Inventory models
7. Markovian models
8. Job sequencing models
9. Simulation models

Allocation models (Distribution models) These models are related with the allocation of available resources so as to make the most of profit or minimize loss subject to existing and predicted limitations. Methods which are used for solving allocation models are

- Linear programming problems
- Assignment problems

- Transportation problems

Waiting line models (Queueing) This model is an attempt made to forecast

- How much average time will be used up by the customer waiting in a queue?
- What will be a standard length of the queue?
- What can be the utilization factor of a queue system?

This model provides to reduce the sum of costs of service providing and cost of getting service, linked with the value of time used up by the customer in a queue.

Game theory (Competitive strategy models) These models are generally used to decide the behavior of decision-making under conflict or competition. Methods for solving such models are not found suitable for industrial practices mainly because they are meant for an idealistic world neglecting many necessary characteristic of reality.

Inventory (Production) models These models are related with the finding of the best order quantity and ordering production intervals considering the factors like cost of placing orders, demand per unit time, costs related with goods held up in the inventory and the cost due to scarcity of goods, etc.

Replacement models These models deals with finding of best time to compensate an equipment in situations which arise when some items or machinery require replacement by a scientific advance or new one or deterioration due to wear and tear, accidents etc. Individual and group replacement principles can be used in case of such equipments that completely fail instantaneously.

Job sequencing models These models include the selection of a sequence of performing a series of jobs to be done on machines that maximizes the efficiency measure of working of the system.

Network models These models are pertinent in big projects involving interdependencies and intricacies of activities. CPM (Critical Path Method) and PERT (Project Evaluation and Review Technique) are used for planning, arranging or scheduling and controlling activities of intricate project which can be described through network diagram.

Simulation models This model is used mostly for solving problems when there are large number of variables and constrained relationships.

Markovian models These models are applicable in the situations where the state of the system can be stated by some explanatory measure of numerical value and where the system changes from one state to another on a probability basis.

Phases of O.R or Stages of Operations Research

The stages of development of O.R. are also known as phases and process of O.R, which has six important steps. These six steps are arranged in the following order:

Step I: Observe the problem environment
Step II: Analyze and define the problem
Step III: Develop a model
Step IV: Select appropriate data input
Step V: Provide a solution and test its reasonableness
Step VI: Implement the solution

Step I: Observe the problem environment

The first step in the process of O.R. development is the problem environment observation. This step includes different activities; they are conferences, site visit, research, observations etc. These activities provide sufficient information to the O.R. specialists to formulate the problem.

Step II: Analyze and define the problem

This step is analyzing and defining the problem. In this step in addition to the problem definition the objectives, uses and limitations of O.R. study of the problem also defined. The outputs of this step are clear grasp of need for a solution and its nature understanding.

Step III: Develop a model

This step develops a model; a model is a representation of some abstract or real situation. The models are basically mathematical models, which describes systems, processes in the form of equations, formula/relationships. The different activities in this step are variables definition, formulating equations etc. The model is tested in the field under different environmental constraints and modified in order to work. Sometimes the model is modified to satisfy the management with the results.

Step IV: Select appropriate data input

A model works appropriately when there is appropriate data input. Hence, selecting appropriate input data is important step in the O.R. development stage or process. The activities in this step include internal/external data analysis, fact analysis, and collection of opinions and use of computer data banks. The objective of this step is to provide sufficient data input to operate and test the model developed in

Step V: Provide a solution and test its reasonableness

This step is to get a solution with the help of model and input data. This solution is not implemented immediately, instead the solution is used to test the model and to find there is any limitations. Suppose if the solution is not reasonable or the behavior of the model is not proper, the model is updated and modified at this stage. The output of this stage is the solution(s) that supports the current organizational objectives.

Step VI: Implement the solution Inventory (Production) models

At this step the solution obtained from the previous step is implemented. The implementation of the solution involves many behavioral issues. Therefore, before implementation the implementation authority has to resolve the issues. A properly implemented solution results in quality of work and gains the support from the management.

Linear programming problem

Linear programming (LP) or **Linear Optimisation** may be defined as the problem of maximizing or minimizing a linear function which is subjected to linear constraints. The constraints may be equalities or inequalities. The optimisation problems involve the calculation of profit and loss. Linear programming problems are an important class of optimisation problems, that helps to find the feasible region and optimise the solution in order to have the highest or lowest value of the function.

Linear programming is the method of considering different inequalities relevant to a situation and calculating the best value that is required to be obtained in those conditions.

Some of the assumption taken while working with linear programming are:

- The number of constraints should be expressed in the quantitative terms
- The relationship between the constraints and the objective function should be linear
- The linear function (i.e., objective function) is to be optimised

Components of Linear Programming

The basic components of the LP are as follows:

- Decision Variables
- Constraints
- Data
- Objective Functions

Characteristics of Linear Programming

The following are the five characteristics of the linear programming problem:

Constraints – The limitations should be expressed in the mathematical form, regarding the resource.

Objective Function – In a problem, the objective function should be specified in a quantitative way.

Linearity – The relationship between two or more variables in the function must be linear. It means that the degree of the variable is one.

Finiteness – There should be finite and infinite input and output numbers. In case, if the function has infinite factors, the optimal solution is not feasible.

Non-negativity – The variable value should be positive or zero. It should not be a negative value.

Linear Programming Applications

A real-time example would be considering the limitations of labours and materials and finding the best production levels for maximum profit in particular circumstances. It is part of a vital area of mathematics known as optimisation techniques. The applications of LP in some other fields are

- Engineering – It solves design and manufacturing problems as it is helpful for doing shape optimisation
- Efficient Manufacturing – To maximise profit, companies use linear expressions
- Energy Industry – It provides methods to optimise the electric power system.
- Transportation Optimisation – For cost and time efficiency.

Advantages of Linear programming

The advantages of linear programming are:

- Linear programming provides insights to the business problems
- It helps to solve multi-dimensional problems
- According to the condition change, LP helps in makng the adjustments
- By calculating the cost and profit of various things, LP helps to take the best optimal solution

GENERAL LINEAR PROGRAMMING MODEL

A general representation of LP model is given as follows:

Maximize or Minimize, $Z = p_1 x_1 + p_2 x_2 \ldots\ldots\ldots\ldots\ldots\ldots p_n x_n$

Subject to constraints,

$w_{11} x_1 + w_{12} x_2 + \ldots\ldots\ldots\ldots\ldots\ldots w_{1n} x_n \leq \text{or} = \text{or} \geq w_1 \ldots\ldots\ldots\ldots\ldots (i)$

$w_{21} x_1 + w_{22} x_2 \ldots\ldots\ldots\ldots\ldots\ldots w_{2n} x_n \leq \text{or} = \text{or} \geq w_2 \ldots\ldots\ldots\ldots\ldots (ii)$

. . . .

. . . .

. . . .

$w_{m1} x_1 + w_{m2} x_2 + \ldots\ldots\ldots\ldots\ldots\ldots w_{mn} x_n \leq \text{or} = \geq w_m \ldots\ldots\ldots\ldots (iii)$

Non-negativity constraint,

$x_i \geq 0$ (where i = 1,2,3 .....n)

DEFINITIONS:

(i)Solution

Values of the decision variable x;(i = 1,2,3, in) satisfying the constraints of a general linear programming model is known as the solution to that linear programming model.

(ii) Feasible solution

Out of the total available solution a solution that also satisfies the non-negativity restrictions of the linear programming problem is called a feasible solution.

(iii) Basic solution

For a set of simultaneous equations in Q unknowns (p Q) a solution obtained by setting (P - Q) of the variables equal to zero & solving the remaining P equation in P unknowns is known as a basic solution. The variables which take zero values at any solution are detained as non-basic variables & remaining are known as-basic variables, often called basic.

(iv) Basic feasible solution

A feasible solution to a general linear programming problem which is also basic solution is called a basic feasible solution.

(v) Optimal feasible solution

Any basic feasible solution which optimizes (ie; maximise or minimises) the objective function of a linear programming modes known as the optimal feasible solution to that linear programming model.

(vi) Degenerate Solution

A basic solution to the system of equations is termed as degenerate if one or more of the basic variables become equal to zero.

GRAPHICAL SOLUTION

Solving a Linear Programming Problem Graphically

1. Define the variables to be optimized. The question asked is a good indicator as to what these will be.
2. Write the objective function in words, then convert to mathematical equation
3. Write the constraints in words, then convert to mathematical inequalities
4. Graph the constraints as equations
5. Shade feasible regions by taking into account the inequality sign and its direction. If,

a)A vertical line

≤≤, then shade to the left

≥≥, then shade to the right

b) A horizontal line

≤≤, then shade below

≥≥, then shade above

c) A line with a non-zero, defined slope

≤≤, shade below

≥≥, shade above

6. Identify the corner points by solving systems of linear equations whose intersection represents a corner point.

7. Test all corner points in the objective function. The “winning” point is the point that optimizes the objective function (biggest if maximizing, smallest if minimizing)

1. Unique Car Ltd. Manufacturers & sells three different types of Cars A, B, & C. These Cars are manufactured at two different plants of the company having different manufacturing capacities. The following details pertaining to the manufacturing process are provided:

Manufacturing Plants	A	B	C	Operating cost of Plants
1	50	100	100	2500
2	60	60	200	3500
DEMAND	2500	3000	7000	

Using the graphical method technique of linear programming , find the least number of days of operations per month so as to minimize the total cost of operations at the two plants.

SOLUTION

Let x1 = no-of days plant 1 operates; and

X2 = no-of days plant 2 operates.

The objective of uniques Car Ltd. Is to minimize the operating costs of both its plants.

The above problem can be formulated as follows: Ie;

Objective function

Minimize Z = 2,500x1+3,500x2

Subject to:

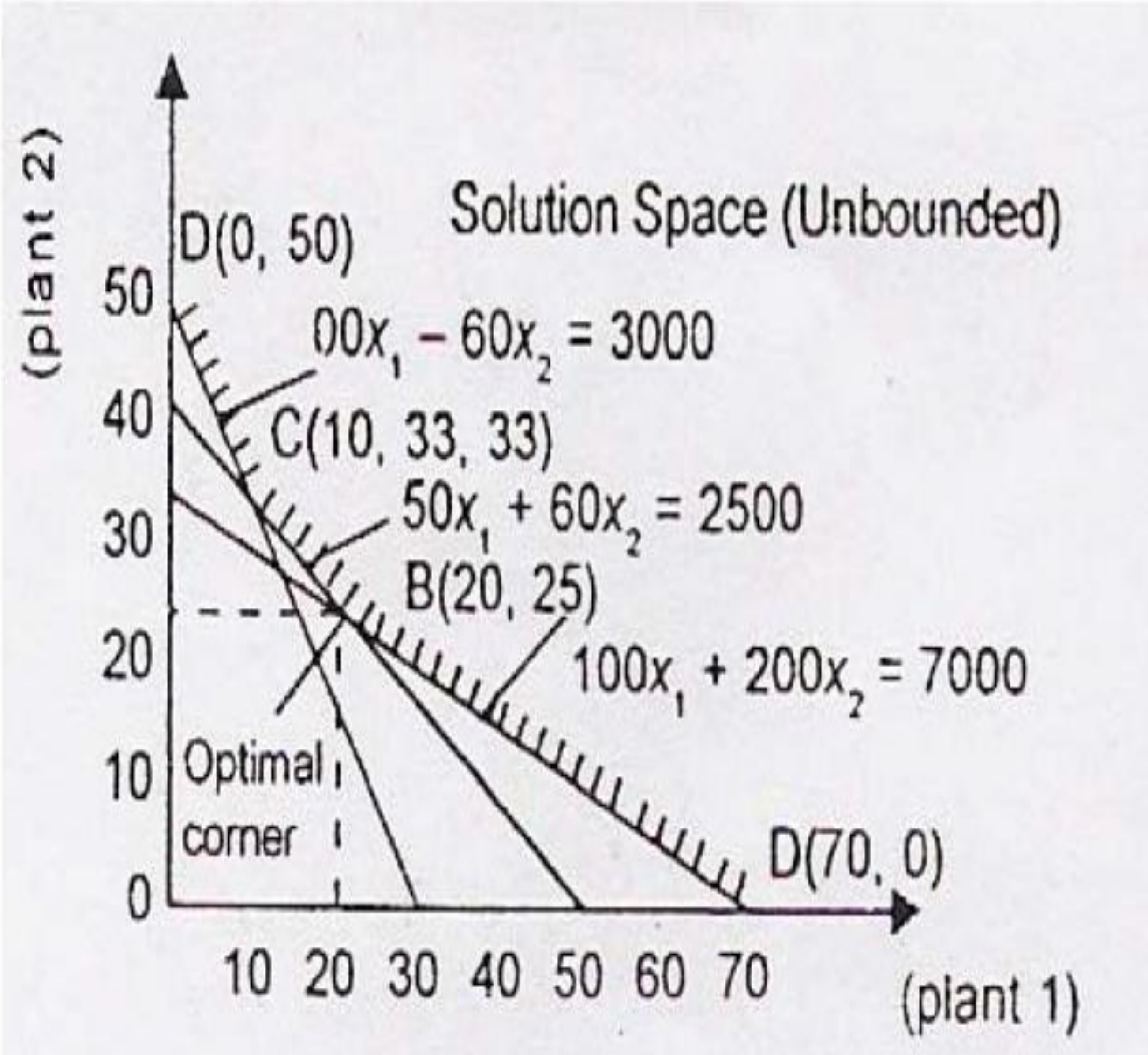
50 x1 + 60x2 $\geq$ 2,500

100 x1 + 60 x2 $\geq$ 3,000

100 x1 + 200x2 $\geq$ 7000

and , x1 x2 $\geq$ 0

Making the graphs of the above constraints:



(plant 2)
Solution Space (Unbounded)
D(0, 50)
00x1 – 60x2 = 3000
C(10, 33, 33)
50x1 + 60x2 = 2500
B(20, 25)
100x1 + 200x2 = 7000
Optimal corner
D(70, 0)
50
40
30
20
10
0
10 20 30 40 50 60 70
(plant 1)

The solution space lines at the points A,B,C & D Calculating the Optimal solution:

Points	***Co-ordinates***	***Objective – func***	***Value***
0	(0,0)	Z=0+0	0
A	(70,0)	Z=2500x70+0	1,75,000
B	(20,25)	Z=2500x20+3500x25	1,37,500
C	(10,33.33)	Z=2500x10+3500x33.33	1,41,655
D	(0,50)	Z=0+3500x50	1,75,000

Thus, the least monthly operately cost is at the point B. Where x1=20 days, x2=25 days & operating cost = Rs. 1,37,500.

2. Solve the following linear programming problem graphically:

Minimise Z = 200 x + 500 y

subject to the constraints:

x + 2y ≥ 10

3x + 4y ≤ 24

x ≥ 0, y ≥ 0

Solution:

Given,

Minimise Z = 200 x + 500 y … (1)

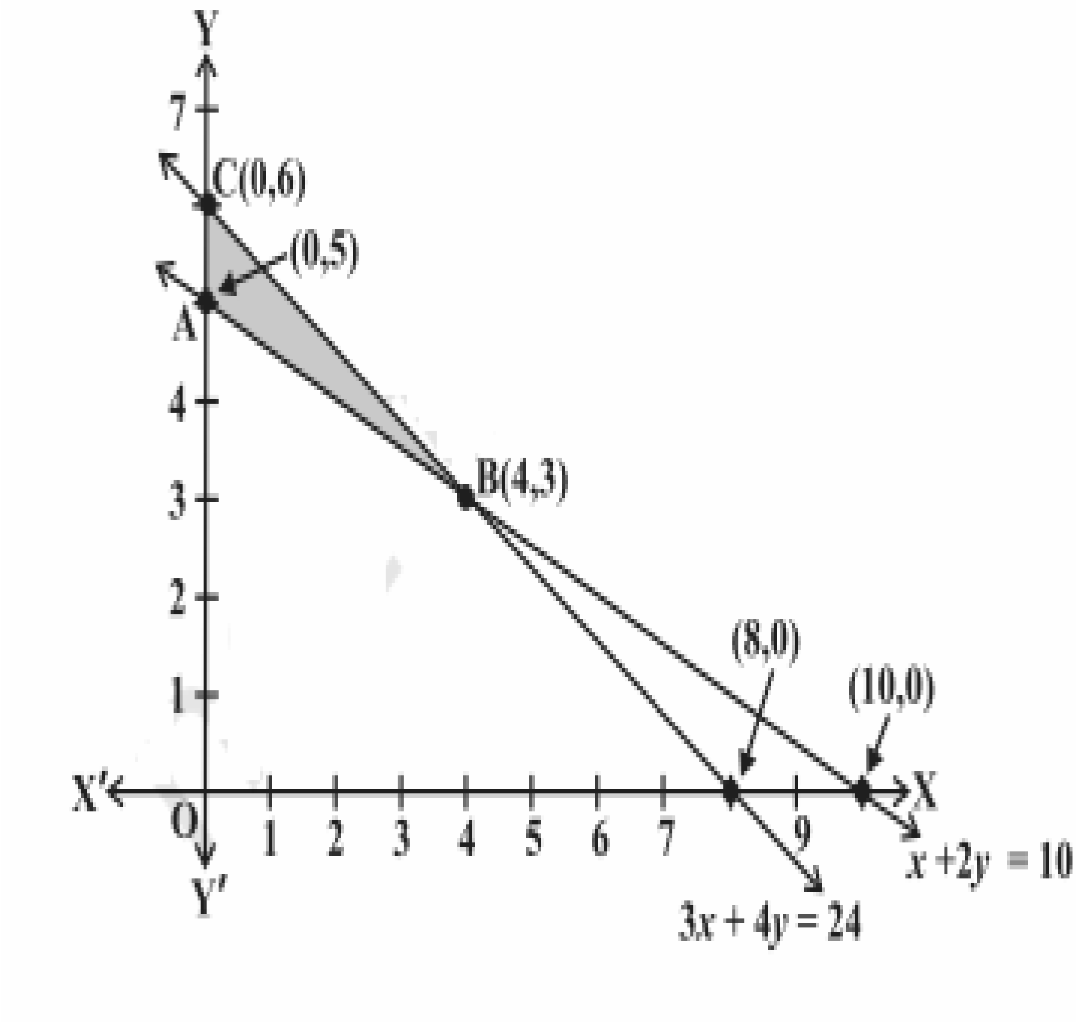
subject to the constraints:

x + 2y ≥ 10 … (2)

3x + 4y ≤ 24 … (3)

x ≥ 0, y ≥ 0 … (4)

Let us draw the graph of x + 2y = 10 and 3x + 4y = 24 as below.



The shaded region in the above figure is the feasible region ABC determined by the

system of constraints (2) to (4), which is bounded. The coordinates of corner point A, B and C are (0,5), (4,3) and (0,6) respectively.

Calculation of Z = 200x + 500y at these points.

Corner point	Value of Z
(0, 5)	2500
(4, 3)	2300← Minimum
(0, 6)	3000

Hence, the minimum value of Z is 2300 is at the point (4, 3).

3.A dietician wishes to mix two types of foods in such a way that vitamin contents of the mixture contain atleast 8 units of vitamin A and 10 units of vitamin C. Food ‘I’ contains 2 units/kg of vitamin A and 1 unit/kg of vitamin C. Food ‘II’ contains 1 unit/kg of vitamin A and 2 units/kg of vitamin C. It costs Rs 50 per kg to purchase Food ‘I’ and Rs 70 per kg to purchase Food ‘II’. Formulate this problem as a linear programming problem to minimise the cost of such a mixture.

Solution:

Let the mixture contain x kg of Food 'I' and y kg of Food 'II'.

Clearly, $x \geq 0$, $y \geq 0$.

Tabulate the given data as below.

Resources	Food		Requirement
	I (x)	II (y)	
Vitamin A (units/kg)	2	1	8
Vitamin C (units/kg)	1	2	10
Cost (Rs/kg)	50	70	

Given that, the mixture must contain at least 8 units of vitamin A and 10 units of vitamin C.

Thus, the constraints are:

$2x + y \geq 8$

$x + 2y \geq 10$

Total cost Z of purchasing x kg of food 'I' and y kg of Food 'II' is $Z = 50x + 70y$

Hence, the mathematical formulation of the problem is:

Minimise $Z = 50x + 70y$ … (1)

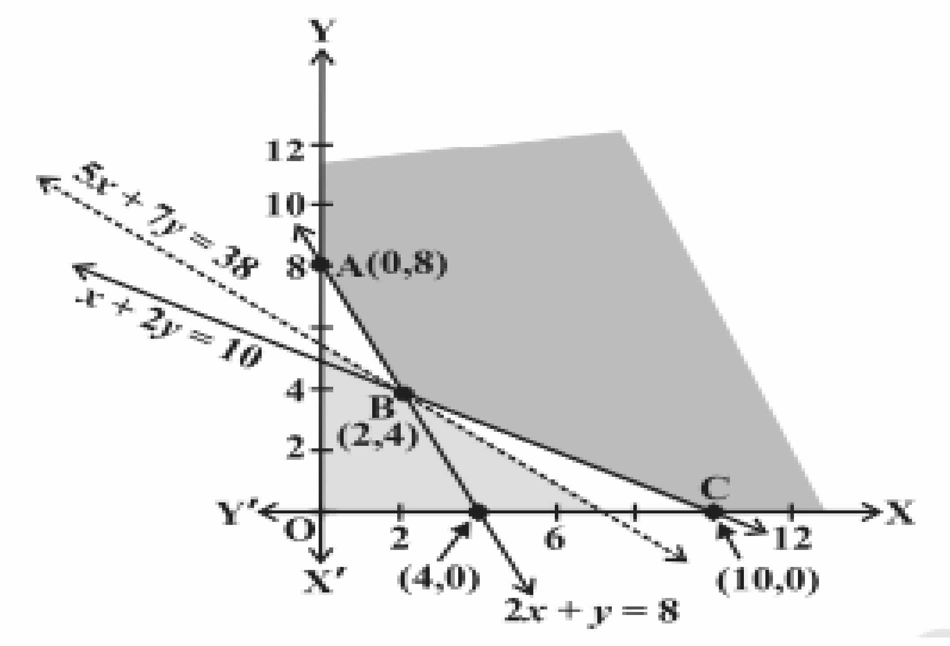
subject to the constraints:

$2x + y \geq 8$ … (2)

$x + 2y \geq 10$ … (3)

$x, y \geq 0$ … (4)

Let us draw the graph of $2x + y = 8$ and $x + 2y = 10$ as given below.



Here, observe that the feasible region is unbounded.

Let us evaluate the value of Z at the corner points A(0,8), B(2,4) and C(10,0).

Corner point	Value of Z
A(0, 8)	560
B(2, 4)	380 = Minimum
C(10, 0)	500

Therefore, the minimum value of Z is 380 obtained at the point (2, 4).

Hence, the optimal mixing strategy for the dietician would be to mix 2 kg of Food ‘I’ and 4 kg of Food ‘II’, and with this strategy, the minimum cost of the mixture will be Rs 380.

4. Solve the following LPP graphically:

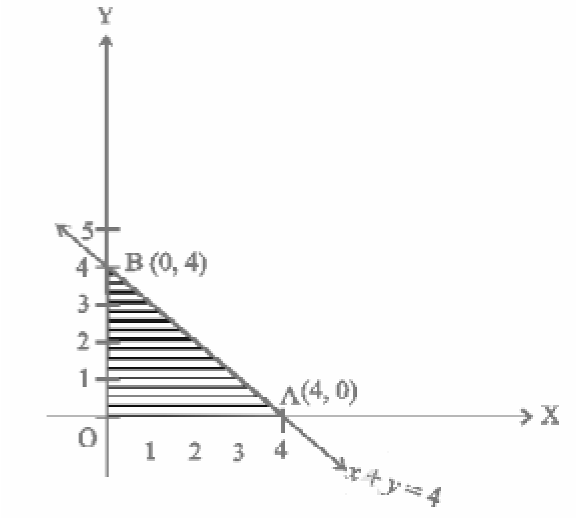
Maximise Z = 2x + 3y,

subject to x + y ≤ 4,

x ≥ 0, y ≥ 0

Solution:

Let us draw the graph pf x + y =4 as below.



The shaded region (OAB) in the above figure is the feasible region determined by the system of constraints x ≥ 0, y ≥ 0 and x + y ≤ 4.

The feasible region OAB is bounded and the maximum value will occur at a corner point of the feasible region.

Corner Points are O(0, 0), A (4, 0) and B (0, 4).

Evaluate Z at each of these corner points.

Corner Point	Value of Z
O(0, 0)	2 (0) + 3(0) = 0
A (4, 0)	2 (4) + 3(0) = 8
B (0, 4)	2 (0) + 3 (4)= 12 ← maximum

Hence, the maximum value of Z is 12 at the point (0, 4).

5.A manufacturing company makes two types of television sets; one is black and white and the other is colour. The company has resources to make at most 300 sets a week. It takes Rs 1800 to make a black and white set and Rs 2700 to make a coloured set. The company can spend not more than Rs 648000 a week to make television sets. If it makes a profit of Rs 510 per black and white set and Rs 675 per coloured set, how many sets of each type should be produced so that the company has a maximum profit? Formulate this problem as a LPP given that the objective is to maximise the profit.

Solution:

Let x and y denote, respectively, the number of black and white sets and coloured sets made each week.Thus x ≥ 0, y ≥ 0

The company can make at most 300 sets a week, therefore, x + y ≤ 300.

Weekly cost (in Rs) of manufacturing the set is 1800x + 2700y and the company can spend up to Rs. 648000.

Therefore, 1800x + 2700y ≤ 648000

or

2x + 3y ≤ 720

The total profit on x black and white sets and y coloured sets is Rs (510x + 675y).

Let the objective function be Z = 510x + 675y.

Therefore, the mathematical formulation of the problem is as follows.

Maximise Z = 510x + 675y

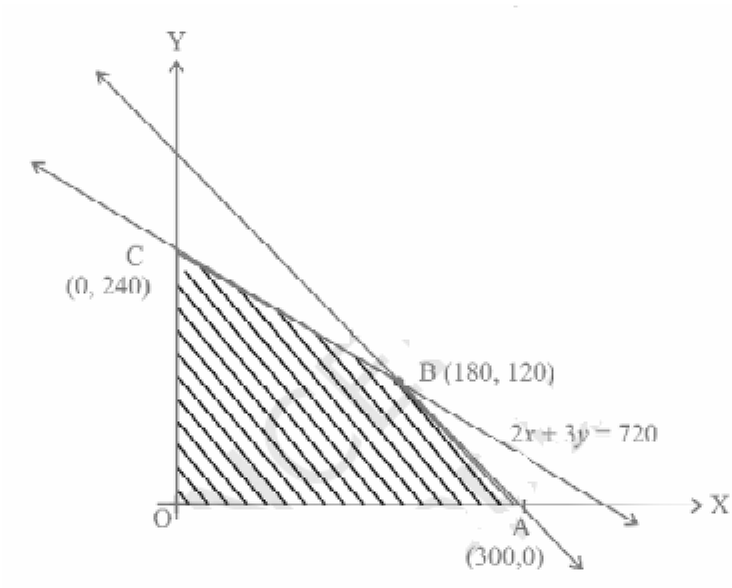
subject to the constraints :

x + y ≤ 300

2x + 3y ≤ 720

x ≥ 0, y ≥ 0

The graph of x + y = 30 and 2x + 3y = 720 is given below.



Corner point	Value of Z
A(300, 0)	153000
B(180, 120)	172800 = Maximum
C(0, 240)	162000

Hence, the maximum profit will occur when 180 black & white sets and 120 coloured sets are produced.

Exercise 6: A workshop has three (3) types of machines A, B and C; it can manufacture two (2) products 1 and 2, and all products have to go to each machine and each one goes in the same order; First to the machine A, then to B and then to C. The following table shows:

- The hours needed at each machine, per product unit
- The total available hours for each machine, per week
- The profit of each product per unit sold

Type of Machine	Product 1	Product 2	Available hours per week
A	2	2	16
B	1	2	12
C	4	2	28
Profit per unit	1	1.50	

Formulate and solve using the graphical method a Linear Programming model for the previous situation that allows the workshop to obtain maximum gains.

Decision Variables:

- X_1 : Product 1 Units to be produced weekly
- X_2 : Product 2 Units to be produced weekly

Objective Function:

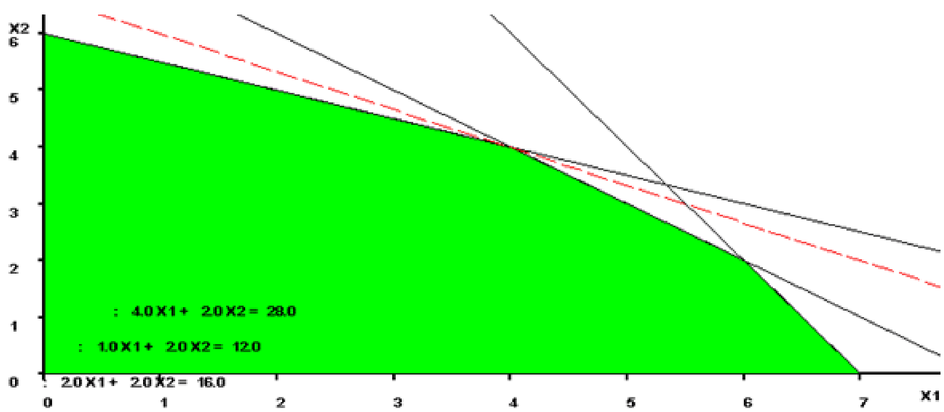
Maximize $X_1 + 1.5X_2$

Constraints:

- $2X_1 + 2X_2 \leq 16$ (8,0) (0,8)
- $X_1 + 2X_2 \leq 12$ (12,0) (0,6)
- $4X_1 + 2X_2 \leq 28$ (7,0) (0,14)
- $X_1, X_2 \geq 0$

The constraints represent the number of hours available weekly for machines A, B and C, respectively, and also incorporate the non-negativity conditions.

The ***green colored area*** corresponds to the set of feasible solutions and the level curve of the objective function that passes by the optimal vertex is shown with a ***red dotted line***.



4x1 + 2x2 =28 ---- 1
2x1 + 2x2 =16 ---- 2
Solving the above 2 equations
2x1 = 12 X1 = 6
Solve x1 = 6 in 2 then x2 = 2
2x1 + 2x2 =16
X1 + 2x2 = 12

X1 = 4
Sub x1 = 4 in x1 +2x2 =12 then x2 = 4

Vertex points Maximize z = $X_1 + 1.5X_2$

A (0,0) z= 0
B (7,0) z = 7
C (6,2) z = 9
D (4,4) z = 10
E (0,6) z = 9

The optimal solution is $X_1 = 4$ and $X_2 = 4$ with an optimal value $V(P) = 1(4) + 1,5(4) = 10$ that represents the workshop's profit.

Exercise #7: A winemaking company has recently acquired a 110 hectares piece of land. Due to the quality of the sun and the region's excellent climate, the entire production of Sauvignon Blanc and Chardonnay grapes can be sold. You want to know how to plant each variety in the 110 hectares, given the costs, net profits and labor requirements according to the data shown below:

Variety	Cost (US$/Hect)	Net Profit (US$/Hect)	Man-days/Hect
Sauvignon Blanc	100	50	10
Chardonnay	200	120	30

Suppose that you have a budget of US$10,000 and an availability of 1,200 man-days during the planning horizon. Formulate and solve graphically a Linear Programming model for this problem. Clearly outline the domain of feasible solutions and the process used to find the optimal solution and the optimal value.

Decision Variables:

- X_1 : Hectares intended for growing Sauvignon Blanc
- X_2 : Hectares intended for growing Chardonnay

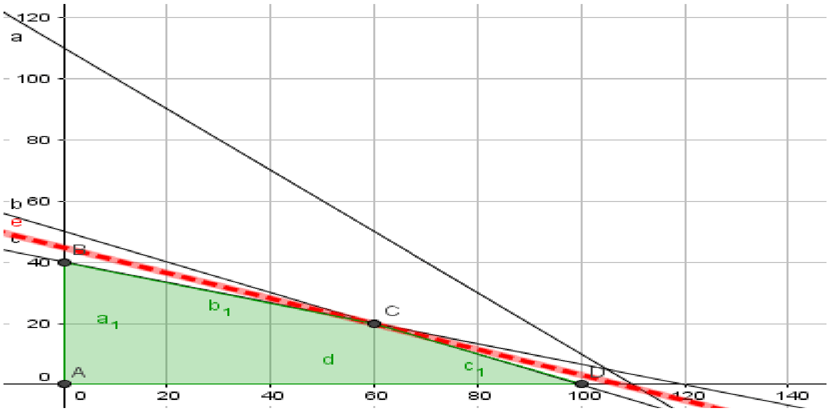
Objective Function:

Maximize $50X_1 + 120X_2$

Constraints:

- $X_1 + X_2 \leq 110$
- $100X_1 + 200X_2 \leq 10.000$
- $10X_1 + 30X_2 \leq 1.200$
- $X_1, X_2 \geq 0$

Where the restrictions are associated with the maximum availability of hectares for planting, available budget, man-hours in the planting period and non-negativity, respectively.The following graph shows the representation of the winemaking company problem. The **shaded area** corresponds with the domain of feasible solutions, where the **optimal** **basic feasible solution** is reached at vertex C, where the budget and man-days restraints are active. Thus solving said equation system the coordinate of the optimal solution is found where $X_1 = 60$ and $X_2 = 20$ (hectares). The optimal value is $V(P) = 50(60) + 120(20) = 5.400$ (dollars).



Exercise Problems

8.A store sells two types of toys, A and B. The store owner pays $8 and $14 for each one unit of toy A and B respectively. One unit of toys A yields a profit of $2 while a unit of toys B yields a profit of $3. The store owner estimates that no more than 2000 toys will be sold every month and he does not plan to invest more than $20,000 in inventory of these toys. How many units of each type of toys should be stocked in order to maximize his monthly total profit profit?

Solution:
Objective function
Maximize z=2x1+3x2

Subject to constraints
8x1 + 14x2 ≤ 20000 (2500,0)(0,1429)
X1 +x2 ≤ 2000 (2000,0) (0,2000)

9 .A company produces two types of tables, T1 and T2. It takes 2 hours to produce the parts of one unit of T1, 1 hour to assemble and 2 hours to polish.It takes 4 hours to produce the parts of one unit of T2, 2.5 hour to assemble and 1.5 hours to polish.

Per month, 7000 hours are available for producing the parts, 4000 hours for assembling the parts and 5500 hours for polishing the tables. The profit per unit of T1 is $90 and per unit of T2 is $110. How many of each type of tables should be produced in order to maximize the total monthly profit?

Solution
Objective function
Maximize z = 90 x1 + 110 x2
Constraints
2x1 + 4x2 ≤ 7000 (3500,0) (0,1750)
X1 +2.5x2 ≤ 4000 (4000,0) (0, 1600)
2x1 + 1.5 x2 ≤ 5500 (2750,0) (0,3667)
X1 , x2 ≥ 0

Vertex points Maximize z = 90 x1 + 110 x2

A (0,0)	z= o
B (0,1600)	z= 1,76000
C (1500,1000)	z= 2,45000
D (2300,600)	z= 2,73000
E (2750,o)	z= 2,47500

10.A farmer plans to mix two types of food to make a mix of low cost feed for the animals in his farm. A bag of food A costs $10 and contains 40 units of proteins, 20 units of minerals and 10 units of vitamins. A bag of food B costs $12 and contains 30 units of proteins, 20 units of minerals and 30 units of vitamins. How many bags of food A and B should the consumed by the animals each day in order to meet the minimum daily requirements of 150 units of proteins, 90 units of minerals and 60 units of vitamins at a minimum cost?

Ans: Minimize 10x1 + 12x2
40x1 + 30x2 ≥ 150
20x1 + 20x2 ≥ 90
10 x1 + 30x2 ≥ 60
X1, x2 ≥ 0

Optimal value
Minimum Z = 42 (3,1)

Special Cases in Graphical Method

11. Multiple Optimal Solution

Solve by using graphical method

Max $Z = 4x_1$
$+ 3x_2$
Subject to

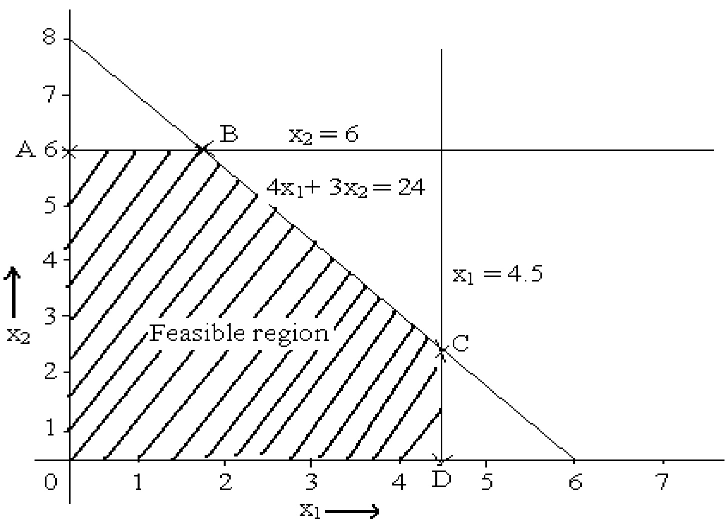
$4x_1 + 3x_2 \leq 24$
$x_1 \leq 4.5$
$x_2 \leq 6$
$x_1 \geq 0$, $x_2 \geq 0$

Solution

The first constraint $4x_1 + 3x_2 \leq 24$, written in a form of
equation $4x_1 + 3x_2 = 24$
Put $x_1 = 0$, then
$x_2 = 8$ Put x_2
$= 0$, then $x_1 = 6$
The coordinates are (0, 8) and (6, 0)

The second constraint $x_1 \leq 4.5$, written in a form of equation
$x_1 = 4.5$

The third constraint $x_2 \leq 6$, written in a form of equation
$x_2 = 6$



The corner points of feasible region are A, B, C and D. So the coordinates for the corner points are

A (0, 6)
B (1.5, 6) (Solve the two equations $4x_1 + 3x_2 = 24$ and $x_2 = 6$ to get the coordinates) C (4.5, 2) (Solve the two equations $4x_1 + 3x_2 = 24$ and $x_1 = 4.5$ to get the coordinates) D (4.5, 0)

We know that Max $Z = 4x_1$
$+ 3x_2$ At A (0, 6)
$Z = 4(0) + 3(6) = 18$

At B (1.5, 6)
$Z = 4(1.5) + 3(6) = 24$

At C (4.5, 2)
$Z = 4(4.5) + 3(2) = 24$

At D (4.5, 0)
$Z = 4(4.5) + 3(0) = 18$

Max Z = 24, which is achieved at both B and C corner points. It can be achieved not only at B and C but every point between B and C. Hence the given problem has multiple optimal solutions.

12.No Optimal Solution/InFeasible solution

Solve graphically

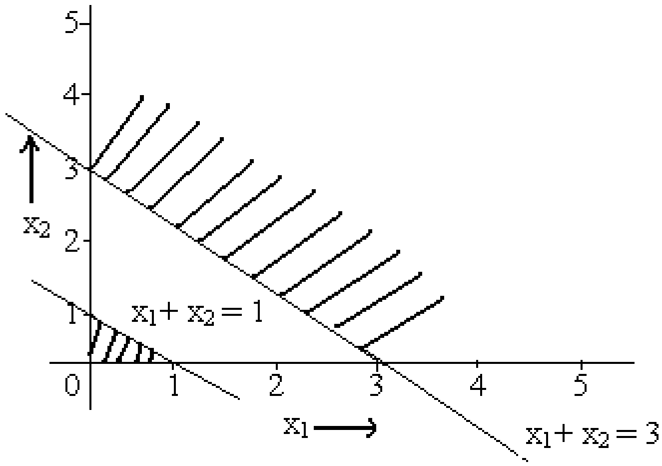
Max $Z = 3x_1 + 2x_2$

Subject to
$x_1 + x_2 \leq 1$
$x_1 + x_2 \geq 3$
$x_1 \geq 0$, $x_2 \geq 0$

Solution

The first constraint $x_1 + x_2 \leq 1$, written in a form of equation
$x_1 + x_2 = 1$
Put $x_1 = 0$, then
$x_2 = 1$ Put x_2
$=0$, then $x_1 = 1$
The coordinates are (0, 1) and (1, 0)

The first constraint $x_1 + x_2 \geq 3$, written in a form of equation
$x_1 + x_2 = 3$
Put $x_1 = 0$, then
$x_2 = 3$ Put x_2
$=0$, then $x_1 = 3$
The coordinates are (0, 3) and (3, 0)



There is no common feasible region generated by two constraints together i.e. we cannot identify even a single point satisfying the constraints. Hence there is no optimal solution.

13 Unbounded Solution

Example

Solve by graphical method

$\text{Max } Z = 3x_1 + 5x_2$
Subject to
$2x_1 + x_2 \geq 7$
$x_1 + x_2 \geq 6$
$x_1 + 3x_2 \geq 9$
$x_1 \geq 0 , x_2 \geq 0$

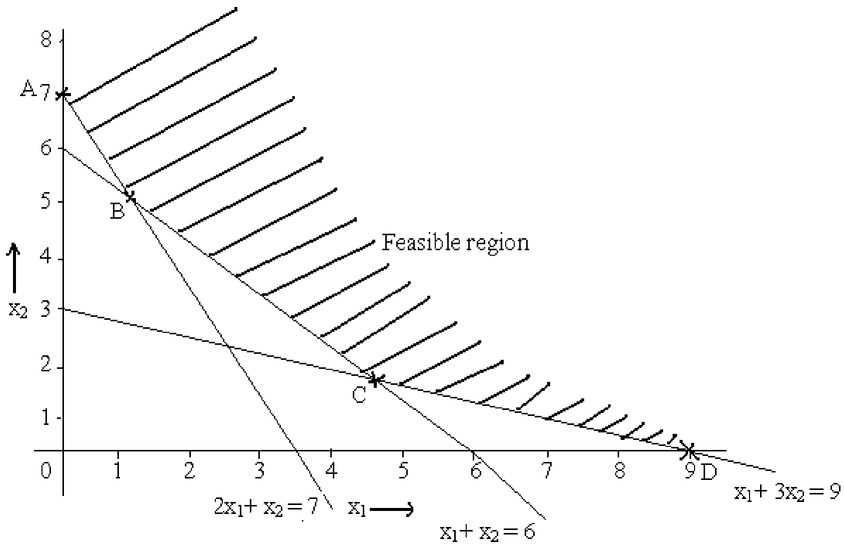
Solution

The first constraint $2x_1 + x_2 \geq 7$, written in a form of equation
$2x_1 + x_2 = 7$
Put $x_1 = 0$, then x_2
$= 7$ Put $x_2 = 0$,
then $x_1 = 3.5$
The coordinates are (0, 7) and (3.5, 0)

The second constraint $x_1 + x_2 \geq 6$, written in a form of equation
$x_1 + x_2 = 6$
Put $x_1 = 0$, then
$x_2 = 6$ Put x_2
$= 0$, then $x_1 = 6$
The coordinates are (0, 6) and (6, 0)

The third constraint $x_1 + 3x_2 \geq 9$, written in a form of equation

$x_1 + 3x_2 = 9$
Put $x_1 = 0$, then
$x_2 = 3$ Put x_2
$= 0$, then $x_1 = 9$
The coordinates are (0, 3) and (9, 0)



The corner points of feasible region are A, B, C and D. So the coordinates for the corner points are

A (0, 7)
B (1, 5) (Solve the two equations $2x_1 + x_2 = 7$ and $x_1 + x_2 = 6$ to get the coordinates)
C (4.5, 1.5) (Solve the two equations $x_1 + x_2 = 6$ and $x_1 + 3x_2 = 9$ to get the coordinates) D (9, 0)

We know that Max $Z = 3x_1 + 5x_2$ At A (0, 7)
$Z = 3(0) + 5(7) = 35$

At B (1, 5)
$Z = 3(1) + 5(5) = 28$

At C (4.5, 1.5)
$Z = 3(4.5) + 5(1.5) = 21$

At D (9, 0)
$Z = 3(9) + 5(0) = 27$
The values of objective function at corner points are 35, 28, 21 and 27. But there exists infinite number of points in the feasible region which is unbounded. The value of objective function will be more than the value of these four corner points i.e. the maximum value of the objective function occurs at a point at ∞. Hence the given problem has unbounded solution.