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SUBJECT TITLE : TIME SERIES AND INDEX NUMBERS

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UNIT V

TESTS FOR INDEX NUMBERS

Fisher has given some criteria that a good index number has to satisfy. They are called (i) Unit test (ii)Time reversal test (iii) Factor reversal test (iv) Circular test. Fisher has constructed in such a way that this index number satisfies most of these tests and hence it is called Fisher's Ideal Index number.

Unit test

This requires the formula to be independent of the units in which prices and quantities are quoted.

If rice is one of the commodities and a formula gives a result on the basis of its price per kg. and quantity in kg., the **formula** should give the same result even when its corresponding price per ton and quantity in tons are taken into account. Index number shows the relative change and if the price has doubled in the current year in comparison with the base year, it is so whether kg. or ton is the unit.

All the methods except the simple aggregative method satisfy this test.

The following example shows the different results given by the simple aggregative method although the price condition is the same. Laspeyre's, Paasche's and Fisher's formulae give the same result in spite of the difference in units.

Item	Unit	Price Quantity	Price Quantity	

		P_0	P_1	q_0	q_1	P_0q_0	p_1q_0	p_0q_1	P_1q_1
Rice	Ton	3000	4500	1	2	3000	4500	6000	9000
Cloth	Metre	100	200	4	5	400	800	500	1000
Total		Σp_0 = 3100	Σp_1 = 4700	-	-	Σp_0q_0 = 3400	Σp_1q_0 = 5300	Σp_0q1 = 6500	Σp_1q_1 = 10000

By Simple Aggregative Method

$$P_{01} = \sqrt{\frac{\sum p1}{\sum p0}\ X\ 100 = \frac{4700}{3100}\ x\ 100} \quad = 151.61$$

By Laspeyre's Formula

$$P_{01} = \frac{\sum P_1 q_0}{\sum P_0 q_0} \text{ x } 100 = \frac{5300}{3400} \text{ x } = \frac{5300}{34} = 155.88$$

By Paasche's formula,

$$P_{01} = \frac{\sum P_1 q_0}{\sum P_0 q_0} \text{ x } 100 = \frac{10000}{65000} \text{ x } 100 = \frac{2000}{13} = 153.85$$

By Fisher's formula

$$P_{01} = \sqrt{P_{\ 01^L}\ P_{\ 01^P}} = \sqrt{\frac{5300}{34}\ X\ \frac{2000}{13}} = \sqrt{\frac{5300000}{221}} = 154.86$$

The same prices and quantities are quoted below in different units:

Item	Unit	Price		Quantity					
		P_0	P_1	q_0	q_1	P_0q_0	p_1q_0	p_0q_1	P_1q_1

Rice	Kg	3	5.5	1000	2000	3000	4500	6000	9000
Cloth	Cm	1	2.0	400	500	400	800	500	1000
Total		$\Sigma p_0 = 4$	$\Sigma p_1 = 6.5$	-	-	$\Sigma p_0 q_0 =$ 3400	$\Sigma p_1 q_0 =$ 5300	$\Sigma p_0 q1 =$ 6500	$\Sigma p_1 q_1 =$ 10000

Totals except those of p_0 and p_1 remain the same and so Laspeyre's , Paasche's and Fisher's formulae give the same results as earlier.

By Simple Aggregative Method,

$$P_{01} = \frac{\Sigma P_1}{\Sigma P_0} \text{ x } 100 = \frac{6.5}{4} \text{ x } 100 = 162.50$$

Time reversal test

Fisher has pointed out that a formula for an index number should maintain time consistency by working both forward and backward with respect to time. This is called time reversal test. Fisher describes this test as follows.

"The test is that the formula for calculating an index number should be such that it gives the same ratio between one point of comparison and the other, no matter which of the two is taken as base or putting in another way the index number reckoned forward should be the reciprocal of that reckoned back ward". A good index number should satisfy the time reversal test.

This statement is expressed in the form of equation as $P_{01} \times P_{10} = 1$.

Fisher's formula, Marshall-Edgeworth formula, Kelly's formula, Simple Aggregative Method and Weighted and unweighted Geometric Means of Relatives Methods satisfy this test.

Prove that Fisher's formula satisfies time reversal test

Time reversal test is satisfied when $P_{01} \times P_{10} = 1$.(omitting the factor 100)

Using Fisher's formula

$$P_{01} = \sqrt{\frac{\sum p_1 q_0}{\sum p_0 q_0} \times \frac{\sum p_1 q_1}{\sum p_0 q_1}}$$

$$P_{10} = \sqrt{\frac{\sum p_0 q_1}{\sum p_1 q_1} \times \frac{\sum p_0 q_0}{\sum p_1 q_0}}$$

$$P_{01} \times P_{10} = \sqrt{\frac{\sum p_1 q_0}{\sum p_0 q_0} \times \frac{\sum p_1 q_1}{\sum p_0 q_1} \times \frac{\sum p_0 q_1}{\sum p_1 q_1} \times \frac{\sum p_0 q_0}{\sum p_1 q_0}}$$

Hence, $P_{01} \times P_{10} = \sqrt{1} = 1$

Factor reversal test

This test is also suggested by Fisher According to the factor reversal test, the product of price index and quantity index should be equal to the corresponding value index.

In Fisher's words "Just as each formula should permit the interchange of two times without giving inconsistent results so it ought to permit interchanging the prices and quantities without giving inconsistent results. i.e, the two results multiplied together should give the true ratio".

This requires the formula to be such that

P_{01} x $Q_{01} = \frac{\sum P_1 q_1}{\sum P_0 q_0}$, after ignoring the factor 100 in each index.

Fisher's is the only formula which satisfies this test.

Prove that Fisher's index satisfies factor reversal test

Factor reversal test is satisfied if

$$P_{01} \times Q_{01} = \frac{\sum p_1q_1}{\sum p_0q_0}$$

$$\text{Now, } P_{01} = \sqrt{\frac{\sum p_1q_0}{\sum p_0q_0} \times \frac{\sum p_1q_1}{\sum p_0q_1}}$$

$$Q_{01} = \sqrt{\frac{\sum p_0q_1}{\sum p_0q_0} \times \frac{\sum p_1q_1}{\sum p_1q_0}}$$

$$\text{Hence, } P_{01} \times Q_{01} = \sqrt{\frac{\sum p_1q_0}{\sum p_0q_0} \times \frac{\sum p_1q_1}{\sum p_0q_1} \times \frac{\sum p_0q_1}{\sum p_0q_0} \times \frac{\sum p_1q_1}{\sum p_1q_0}}$$

$$= \sqrt{\frac{\sum p_1q_1}{\sum p_0q_0} \times \frac{\sum p_1q_1}{\sum p_0q_0}}$$

$$= \frac{\sum p_1q_1}{\sum p_0q_0}$$

Thus, Fisher's Index satisfies Factor reversal test.

Circular Test

Circular test is an extension of the time reversal test. If three years 0, 1 and 2 are under consideration, this requires the formula to be such that

$$P_{01} \text{ x } P_{12} \text{ x } P_{20} = 1$$

P_{01} is the index number of the second year in comparison with the first year, P_{12} is the index number of the third year in comparison with the second and $P_{2()}$ is the index number of the first year in comparison with the third.

Fisher's formula does not satisfy this test. The simple aggregative method, G.M.of relatives method and Kelly's formula satisfy this test

The table below gives the prices of base year and current year of 5 commodities with their quantities. Use it to verify whether Fisher's ideal index satisfies time reversal test.

Commodity	Base year		Current year	
	Unit price (₹)	Quantity	Unit price (₹)	Quantity
A	4	40	5	60
B	5	50	10	70
C	8	65	12	80
D	6	20	6	90
E	7	30	10	75

Solution: Index number by Fisher's ideal index method

Commodity	p_0	q_0	p_1	q_1	p_0q_0	p_0q_1	p_1q_0	p_1q_1
A	4	40	5	60	160	240	200	300
B	5	50	10	70	250	350	500	700
C	8	65	12	80	520	640	780	960
D	6	20	6	90	120	540	120	540
E	7	30	10	75	210	525	300	750
					1260	2295	1900	3250

$$P_{01} = \sqrt{\frac{\sum q_1 p_0}{\sum q_0 p_0} \times \frac{\sum q_1 p_1}{\sum q_0 p_1}}$$

$$= \sqrt{\frac{1900}{1260} \times \frac{3250}{2295}}$$

$$P_{10} = \sqrt{\frac{\sum q_0 p_1}{\sum q_1 p_1} \times \frac{\sum q_0 p_0}{\sum q_1 p_0}}$$

$$= \sqrt{\frac{2295}{3250} \times \frac{1260}{1900}}$$

$$\text{Hence, } P_{01} \times P_{10} = \sqrt{\frac{1900}{1260} \times \frac{3250}{2295} \times \frac{2295}{3250} \times \frac{1260}{1900}}$$

$$= \sqrt{1} = 1$$

Fisher's Index number satisfies time reversal test.

Calculate the price index and quantity index for the following data by Fisher's ideal formula and verify that it satisfies the factor reversal test.

Commodity	Base year		Current year	
	Price (₹)	Quantity (`000 tonnes)	Price (₹)	Quantity (`000 tonnes)
A	40	70	40	32
B	50	84	30	80
C	60	58	25	50

Solution

Commodity	p_0	q_0	p_1	q_1	p_1q_1	p_1q_0	p_0q_0	p_0q_1
A	40	70	40	32	1280	2800	2800	1280
B	50	84	30	80	2400	2520	4200	4000
C	60	58	25	50	1250	1450	3480	3000
					5930	6770	10480	8280

Factor Reversal test: $P_{01} \times Q_{01} = \dfrac{\sum p_1q_1}{\sum p_0q_0}$

$$P_{01} = \sqrt{\frac{\sum p_1q_0}{\sum p_0q_0} \times \frac{\sum p_1q_1}{\sum p_0q_1}}$$

$$Q_{01} = \sqrt{\frac{\sum q_1p_0}{\sum q_0p_0} \times \frac{\sum q_1p_1}{\sum q_0p_1}}$$

$$P_{01} \times Q_{01} = \sqrt{\frac{\sum p_1q_0}{\sum p_0q_0} \times \frac{\sum p_1q_1}{\sum p_0q_1} \times \frac{\sum q_1p_0}{\sum q_0p_0} \times \frac{\sum q_1p_1}{\sum q_0p_1}}$$

$$= \sqrt{\frac{6770}{10480} \times \frac{5930}{8280} \times \frac{8280}{10480} \times \frac{5930}{6770}}$$

$$= \left(\sqrt{\frac{5930}{10480}}\right)^2$$

$$= \frac{5930}{10480}$$

$$= \frac{\sum p_1q_1}{\sum p_0q_0}$$

Hence, Fisher ideal index number satisfies the factor reversal test.

CHAIN BASE INDEX NUMBERS

When the data are available for more than two years, the method available besides the fixed base method for computing index numbers, is the chain base method. Under this, link relatives are calculated first. Link relative is a price (or quantity) relative with the condition that the base year is the preceding year. Whenever more than one commodity is considered, the link relatives of all the commodities are averaged (simple or weighted). In other words, the link relatives as well as their averages are index numbers in which for each year the preceding year is the base year. These averages of link relatives show the conditions of the different years in comparison with their preceding years and are found to be of great use by businessmen and industrialists. They are chained together to common base year for long term analysis using the formula,

$$\text{Chain Index} = \frac{\text{Current year link relative x Preceding year chain index}}{100}$$

As long as the base year is common, the chain base indices are likely to be same as the fixed base indices. Sometimes, we may wish to convert chain base indices (C.B.I.) to fixed base indices (F.B.I.) (where in the bases become different) or vice versa. The formula for such conversions as suggested by some authors is

$$\text{Current year F.B.I} = \frac{\text{Current year C.B.I x Preceding year F.B.I}}{100}$$

Example 8 : Construct (a) fixed base and (b) chain base index numbers from the following data relating to production of electricity.

Year	1981	1982	1983	1984	1985	1986	1987	1988
Production	25	27	30	24	28	29	31	35
Year	1989	1990	1991	1992	1993	1994	1995	1996
Production	40	41	36	32	37	38	39	40

Solution

1981	25	100	100.00	100.00
1982	27	108	108.00	108.00
1983	30	120	111.11	120.00
1984	24	96	80.00	96.00
1985	28	112	116.67	112.00
1986	29	116	103.57	116.00
1987	31	124	106.90	124.00
1988	35	140	112.90	140.00
1989	40	160	114.29	160.01
1990	41	164	102.50	164.01
1991	36	144	87.80	144.00
1992	32	128	88.89	128.00
1993	37	148	115.63	148.01
1994	38	152	102.70	152.01
1995	39	156	102.63	156.01
1996	40	160	102.56	160.00

Quantities of production are given for 16 years. The production of "every year is divided by that of 1981, i.e., 25 and is multiplied by 100 to get the fixed base quantity indices (Q_{01}) given in col (3).

For calculating link relatives (L.R.) of col. (4), quantity of every year is divided by that of its preceding year and is multiplied by 100.

Link relatives are converted into chain base- indices (Q_{01}) given in. col.(5) using the usual formula.

Prepare index numbers from the average prices of three groups of commodities given below by taking the base year as 1998 and the weights as 5,3 and 2 respectively.

Group	1998	1999	2000	2001	2002
I	50	55	52	49	55
II	4	5	3	5	6
III	10	10	11	10	9

Solution

	Price			Price Relatives (P)			WP			ΣWP Prices	Fixed base I.N
	I	**I**	**III**	**I**	**I**	**III**	**I**	**I**	**III**		
1998	50	4	10	100	100	100	500	300	200	1000	100.0
1999	52	5	10	110	125	100	550	375	200	1125	112.5
2000	52	3	22	204	75	110	520	225	220	965	96.5
2001	49	5	10	98	125	100	490	375	200	1065	106.5
2002	55	6	9	110	150	90	550	450	180	1180	118

The price of each commodity in every year is divided by its price in 1998 and is multiplied by 100 to get the price relative (P). The price relatives of the three commodities are multiplied by 5, 3 and 2 respectively to get WP values. They are added year wise (ΣWP) and the total is divided by 10 (ΣW) to get fixed base index numbers.

From the following prices of three groups of commodities for the years 1993 to 1997, find the chain base index numbers.

Groups	1993	1994	1995	1996	1997
I	4	6	8	10	12
II	16	20	24	30	36
III	8	10	16	20	24

Solution

Groups	Prices			Link Relatives (P)			Total	Mean	Chain
	I	II	III	I	II	III	(ΣP)	(ΣP I.N)	Base I.N
1993 4	4	16	8	100.00	100	100	300.00	100.00	100.00
1994 6	6	20	10	150.00	125	125	400.00	133.33	133.33
1995 8	8	24	16	133.33	120	160	413.33	137.78	183.70
1996 10	10	30	20	125.00	125	125	375.00	125.00	229.63
1997.12	12	36	24	120.00	120	120	360.00	120.00	275.56

The price of each commodity in every year is divided by its price in the preceding year and is multiplied by 100 to get the link relative(P). As no weight is given, link relatives are added year wise and the total is divided by 3. The average of each year is multiplied by the chain index number of the preceding year and is divided by 100 to get the chain index number of that year. (Refer to the formula to calculate chain base index number from the link relatives).

For the first year (1993) the link relatives and the chain base index number are taken as 100 each.

Note: If weights are given, weighted averages of the link relatives are to be calculated for all the years before converting them into chain index numbers.

COST OF LIVING INDEX

Cost of living index number shows the impact of changes in the prices of a number of commodities and services on a particular class of people in the current year in comparison with the base year. Cost of living index number is also known as consumer price index number. It is essential to assess the change in retail price and to decide the quantum of allowance to be provided to the employees to offset the change in price and to keep them at their standard of living. Though the general problems have narrowed down, each aspect still needs careful approach.

Main steps in the construction of Cost-of-Living Index Number:

1. The Purpose. At the outset, the class of people for whom the index number is intended is to be identified. The knowledge of their area of living, their ways of life, their necessities, their habits, etc. play an important role in getting good results. As far as possible the individuals of a group should have equal income.

2. The Base Year. Similar survey might have been conducted earlier. The current interest might be to study the subsequent changes. For example, the pay scales of the employees of Tamil Nadu Govt, were revised in 1994. For any future consideration of the employees, 1994 is to be taken as the base year.

3. Family Budget Enquiry. A sample survey, known as family budget enquiry, is conducted and the items to be included, their quantity, etc. are found. It is customary to have the items under the five heads (i) Food (ii) Clothing (iii) Fuel and Lighting (iv) House Rent and (v) Miscellaneous. From the families of the concerned class of people, a sample of adequate size is selected. From each such family, the details of the different items consumed, their quality and quantity are noted. Though the items come under five groups stated earlier, many sub groups are likely under each group. For example, food includes sugar, pulses, wheat, rice, etc. Miscellaneous group consists of Movie, Medicine, Education and others. It should be remembered that non-consumption monetary transactions such as payments to insurance, provident fund, etc. are not considered.

4. The Prices. The average price paid for each item is to be gathered from the shops of the region. The prices are retail prices. As mentioned earlier under general problems in the construction of index numbers, it is a difficult task to gather and to arrive at an average price of an item. The shops where many of the families buy and the most likely prices in those shops are to be noted before finding their average. It is advisable to entrust experienced and conscientious enumerators with this work. Cash prices are taken into account and not the credit prices which include interest. But black market prices are to be taken as such if the items are essential and they are not available in the open market. Discounts and rebates when allowed for all the families are accounted for.

5. **The Average**. Both arithmetic mean and geometric mean can be used, the former owing to its case of calculation and the latter owing to its suitability.

6. **The Formula**. Two formulae are available. They are given below.

i) Aggregate Expenditure Method or Weighted

Aggregative Method: In the usual notations, the

Cost of Living Index Number $= \frac{\sum p_1 q_0}{\sum p_0 q_0}$ X 100

It is the most popular method, and the formula is nothing but Laspeyre's. On the basis of base year quantities, total expenditures in current year and base year are calculated and the percentage of change is worked out.

(ii) Family Budget Method or Weighted Averages of Relatives Method.

The formula under this method as given in usual notations is

Cost of Living Index Number = $\frac{\sum WP}{\sum W}$

Weights (W) are determined on the basis of the family budget enquiry wherein the relative importance of the items within a group and the relative importance of a group to the total are known. When W is base year value (p_0q_0), both the methods become one and the same.

Instead of finding the weighted arithmetic mean of price relatives as in the above formula, weighted geometric mean may also be calculated if required, using the following formula:

Cost of Living Index Number = Antilog $\left(\frac{\sum W \log P}{\sum W}\right)$

Uses:

1. Cost of living index numbers are the indicators of changes in real wages. Money wages are changing and so are prices. Cost of living index numbers help to know whether money wages overtake the rising prices or are overpowered by them.
2. Decisions on dearness allowance are based on the cost-of-living indices.
3. They are further used for deflation of income and value in national accounts

Construct cost of living index, for 2000 taking 1999 as the base year from the following data using 'Aggregate Expenditure' Method.

Article	**Quantity in 1999 (Kg.)**	**1999**	**2000**
A	6	5.75	6.00
B	1	5.00	8.00
C	6	6.00	9.00
D	4	8.00	10.00
E	2	2.00	1.80
F	1	20.00	15.00

Solution

Article	Quantity 1999 (q_0)	Price			
		1999(p_0)	2000(p_1)	p_1q_0	p_0q_0
A	6	5.75	6.00	36.00	34.50
B	1	5.00	8.00	8.00	5.00
C	6	6.00	9.00	54.00	36.00
D	4	8.00	10.00	40.00	32.00
E	2	2.00	1.80	3.60	4.00
F	1	20.00	15.00	15.00	20.00
Total	-	-	-	$\Sigma P1q_0 =$ 156.60	$\Sigma p_0q_0 =$ 131.50

Cost of Living Index Number $= \frac{\sum P_1 q_0}{\sum P_0 q_0}$ X 100

$= \frac{156.60}{131.50}$ X 100

= 119.09

Example 12 : Calculate the cost of living index number form the following date

Item	Base Year Price	Current year price	Weight
Food	39	47	4
Fuel	8	12	1
Clothing	14	18	3
House rent	12	15	2
Miscellaneous	25	30	1

Solution

Item	P_0	P_1	Weight W	$P = \frac{P_1}{P_0}$ x 100	WP
Food	39	47	4	120.51	482.04
Fuel	8	12	1	150.00	150.00
Clothing	14	18	3	128.57	385.71
House rent	12	15	2	125.00	250.00
Miscellaneous	25	30	1	120.00	120.00

Cost of Living Index Number $= \frac{\sum WP}{\sum W} = \frac{1387.75}{11} = 126.16$

Example 13 : Using geometric mean, calculate the cost of living index number for the year 2000.

Commodity	Prince (1990)	Price (2000)	Weight
Food	60	108	40
Clothing	50	984	17
Fuel and Lighting	40	65	13
House Rent	125	225	27
Miscellaneous	120	240	3

Solution

Commodity	**P_0**	**P_1**	**W**	**$P = \frac{P_1}{P_0}$ x 100**	**LogP**	**WlogP**
Food	60	108	40	180.0	2.2553	90.2120
Clothing	50	984	17	188.0	2.2742	38.6614
Fuel and Lighting	40	65	13	162.5	2.2909	28.7417
House Rent	125	225	27	180.0	2.2553	60.89.31
Miscellaneous	120	240	3	200.0	2.3010	6.9030
Total	-	-	ΣW=100	-	-	Σ(W logP) =225.4112

Cost of Living Index Number $= \text{Antilog}\left(\frac{\sum W \text{ logP}}{\sum W}\right)$

$= \text{Antilog}\left(\frac{225.4112}{100}\right)$

$=$ Antilog 2.2541

$=$ **179.51**