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SUBJECT TITLE : TIME SERIES AND INDEX NUMBERS
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UNIT IV

WEIGHTED INDEX NUMBERS

The unweighted indices assign equal importance to all the items included in the index. Construction of useful index numbers requires a conscious effort to assign to each commodity a weight in accordance to its importance in the total phenomenon that the index is supposed to describe.
Weighted index is of two types:

- Weighted Aggregative index and
- Weighted Average of relatives

Weighted Aggregative index

In this method price of each commodity is weighted by the quantity either in the base year or in the current year. There are various methods of assigning weights and thus there are many methods of constructing index numbers.

Some of the important formulae used under this method are
a) Laspeyre's Index (P_{01}^{L})
b) Paasche's Index (P_{01}^{P})
c) Dorbish and Bowley's Index (P_{01}^{DB})
d) Fisher's Ideal Index (P_{01}^{F})
e) Marshall-Edgeworth Index (P_{01}^{Em})
f) Kelly's Index (P_{01}^{K})

a. Laspeyre's method
The base period quantities are taken as weights. The Index is

$$P_{01}^{L} = \frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100$$

b. Paasche's method

The current year quantities are taken as a weight. In this method, we use continuously revised weights and thus this method is not frequently used when the number of commodities is large. The Index is

$$P_{01}^{P} = \frac{\sum p_1 q_1}{\sum p_0 q_1} \times 100$$

c. Dorbish and Bowley's method

In order in take into account the impact of both the base and current year, we make use of simple arithmetic mean of Laspeyre's and Paasche's formula
The Index is

$$P_{01}^{DB} = \frac{P_{01}^{L} + P_{01}^{P}}{2}$$

$$= \frac{\frac{\sum p_1 q_0}{\sum p_0 q_0} + \frac{\sum p_1 q_1}{\sum p_0 q_1}}{2} \times 100$$

d. Fisher's Ideal Index

It is the geometric mean of Laspeyre's Index and Paasche's Index, given by:

$$P_{01}^{F} = \sqrt{P_{01}^{L} \times P_{01}^{P}}$$

$$= \sqrt{\frac{\sum p_1 q_0}{\sum p_0 q_0} \times \frac{\sum p_1 q_1}{\sum p_0 q_1}} \times 100$$

e. Marshall-Edgeworth method

In this method also both the current year as well as base year prices and quantities are considered. The Index is

$$P_{01}^{ME} = \frac{\sum p_1 (q_0 + q_1)}{\sum p_0 (q_0 + q_1)} \times 100$$

$$= \frac{\sum p_1 q_0 + \sum p_1 q_1}{\sum p_0 q_0 + \sum p_0 q_1} \times 100$$

f. Kelly's method

The Kelly's Index is

$$P_{01}^{K} = \frac{\sum p_1 q}{\sum p_0 q} \times 100, \qquad q = \frac{q_0 + q_1}{2}$$

where q refers to quantity of some period, not necessarily of the mean of the base year and current year. It is also possible to use average quantity of two or more years as weights. This method is known as fixed weight aggregative index.

1. Construct weighted aggregate index numbers of price from the following data by applying
1. Laspeyre's method
2. Paasche's method
3. Dorbish and Bowley's method
4. Fisher's ideal method
5. Marshall-Edgeworth method

Commodity	2016		2017	
	Price	Quantity	Price	Quantity
A	2	8	4	6
B	5	10	6	5
C	4	14	5	10
D	2	19	2	13

Solution:

Calculation of various indices

Commodity	2016		2017		p_1q_0	p_0q_0	p_1q_1	p_0q_1
	Price p_0	Quantity q_0	Price p_1	Quantity q_1				
A	2	8	4	6	32	16	24	12
B	5	10	6	5	60	50	30	25
C	4	14	5	10	70	56	50	40
D	2	19	2	13	38	38	26	26
					$\sum p_1q_0 = 200$	$\sum p_0q_0 = 160$	$\sum p_1q_1 = 130$	$\sum p_0q_1 = 103$

(1) Laspeyre's Index:

$$P_{01}^{L} = \frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100$$

$$= \frac{200}{160} \times 100 = 125$$

(2) Paasche's Index

$$P_{01}^{P} = \frac{\sum p_1 q_1}{\sum p_0 q_1} \times 100$$

= (130 / 103) ×100 = 126.21

(3) Dorbish and Bowley's Index

$$P_{01}^{DB} = \frac{P_{01}^{L} + P_{01}^{P}}{2} = \frac{125 + 126.21}{2}$$

= 125.6

(4) Fisher's Ideal Index

$$P_{01}^{F} = \sqrt{\frac{\sum p_1 q_0}{\sum p_0 q_0} \times \frac{\sum p_1 q_1}{\sum p_0 q_1}} \times 100$$

$$= \sqrt{\frac{200}{160} \times \frac{130}{103}} \times 100$$

$$= \sqrt{1.578} \times 100 = 1.2561 \times 100$$

= 125.61

(5) Marshall-Edgeworth method

$$P_{01}^{ME} = \frac{\sum p_1 q_0 + \sum p_1 q_1}{\sum p_0 q_0 + \sum p_0 q_1} \times 100$$

$$= \frac{200 + 130}{160 + 103} \times 100 = \frac{330}{263} \times 100$$

$$= 125.48$$

2. Calculate the price indices from the following data by applying (1) Laspeyre's method (2) Paasche's method and (3) Fisher ideal number by taking 2010 as the base year.

Commodity	2010		2011	
	Prices	Quantities	Prices	Quantities
A	20	10	25	13
B	50	8	60	7
C	35	7	40	6
D	25	5	35	4

Solution: Calculations

p_0	q_0	p_1	q_1	$p_0 q_0$	$p_0 q_1$	$p_1 q_0$	$p_1 q_1$
20	10	25	13	200	260	250	325
50	8	60	7	400	350	480	420
35	7	40	6	245	210	280	240
25	5	35	4	125	100	175	140
				970	920	1185	1125

(1) Laspeyre's Index

$$P_{01}^{L} = \frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100$$

$$= \frac{1185}{970} \times 100$$

$$= 122.16$$

(2) Paasche's Index

$$P_{01}^{P} = \frac{\sum p_1 q_1}{\sum p_0 q_1} \times 100$$

$$= \frac{11254}{920} \times 100$$

$$= 122.28$$

(3) Fisher's Ideal Index

$$P_{01}^{F} = \sqrt{\frac{\sum p_1 q_0}{\sum p_0 q_0} \times \frac{\sum p_1 q_1}{\sum p_0 q_1}} \times 100$$

$$= \sqrt{\frac{1185}{970} \times \frac{1125}{920}} \times 100$$

$$= \sqrt{1.2216 \times 1.2228} \times 100$$

$$= \sqrt{1.49377} \times 100$$

$= 1.2055 \times 100$

$= 120.55$

3. Calculate the Dorbish and Bowley's price index number for the following data taking 2014 as base year.

Items	2014		2015	
	Prices (per kg)	Quantities (purchased)	Prices (per kg)	Quantities (purchased)
Oil	80	3	100	4
Pulses	35	2	45	3
Sugar	25	2	30	3
Rice	50	30	54	35
Cereals	35	2	40	3

Solution:

Price Index by Dorbish and Bowley's Method

p_0	q_0	p_1	q_1	$p_0 q_0$	$p_0 q_1$	$p_1 q_0$	$p_1 q_1$
80	3	100	4	240	320	300	400
35	2	45	3	70	105	90	135
25	2	30	3	50	75	60	90
50	30	54	35	1500	1750	1620	1890
35	2	40	3	70	105	80	120
				1930	2355	2150	2635

$$P_{01}^{DB} = \frac{\frac{\sum p_1 q_0}{\sum p_0 q_0} + \frac{\sum p_1 q_1}{\sum p_0 q_1}}{2} \times 100$$

$$= \frac{1}{2}\left[\frac{2150}{1930} + \frac{2635}{2355}\right] \times 100$$

$$= \frac{1}{2}[1.1139 + 1.1188] \times 100$$

$$= \frac{1}{2}[2.2327] \times 100$$

= 1.1164 x 100 = 111.64

= 1.1164 x 100 = 111.64

4. Compute Marshall – Edgeworth price index number for the following data by taking 2016 as base year.

Items sold out in a men's wear	2016		2017	
	Prices	Quantity	Prices	Quantity
Shirts	700	150	900	175
Pants	1000	100	1200	150
Sandals	500	70	600	100
Shoes'	1500	50	1800	60
Belts	400	100	600	150
Watches	1200	300	1500	250

Solution:

Price Index by Marshall-Edgeworth Method

p_0	q_0	p_1	q_1	$p_0 q_0$	$p_0 q_1$	$p_1 q_0$	$p_1 q_1$
700	150	900	175	105000	122500	135000	156500
1000	100	1200	150	100000	150000	120000	180000
500	70	600	100	35000	50000	42000	60000
1500	50	1800	60	75000	90000	120000	108000
400	100	600	150	40000	60000	60000	90000
1200	300	1500	250	36000	300000	450000	375000
				391000	772500	927000	969500

Marshall – Edgeworth Index:

$$P_{01}^{ME} = \frac{\sum p_1q_0 + \sum p_1q_1}{\sum p_0q_0 + \sum p_0q_1} \times 100$$

$$= \frac{969500 + 927000}{772500 + 391000} \times 100$$

$$= \frac{1896500}{1163500} \times 100$$

$$= 162.99$$

5. Calculate a suitable price index form the following data.

Commodity	Quantity	Price	
		2007	2010
X	25	3	4
Y	12	5	7
Z	10	6	5

Solution:

In this problem, the quantities for both current year and base year are same. Hence, we can conclude Kelly's Index price number.

Commodity	q	p_0	p_1	p_0q	p_1q
X	25	3	4	75	100
Y	12	5	7	60	84
Z	10	6	5	60	50
				195	234

Kelly's price Index number:

$$P_{01}^{K} = \frac{\sum p_1q}{\sum p_0q} \times 100$$

$$= \frac{234}{195} \times 100$$

$$= 120$$

2. Weighted average of price relatives

The weighted average of price relatives can be computed by introducing weights into the unweighted price relatives. Here also, we may use either arithmetic mean or the geometric mean for the purpose of averaging weighted price relatives.

The weighted average price relatives using arithmetic mean:
If $p = [p_1/\ p_0] \times 100$ is the price relative index and $w = p_0q_0$ is attached to the commodity, then the weighed price relative index is

$$P_{01} = \frac{\sum\left[\frac{p_1}{p_0} \times 100\right] \times p_0q_0}{\sum p_0q_0} = P_{01} = \frac{\sum wp}{\sum w}$$

The weighted average price relatives using geometric mean:

$$P_{01} = \text{antilog}\left(\frac{\sum w \log p}{\sum w}\right)$$

1. Compute price index for the following data by applying weighted average of price relatives method using (i) Arithmetic mean and (ii) Geometric mean.

Item	p_0	q_0	p_1
Wheat	3.0	20 kg	4.0
Flour	1.5	40 kg	1.6
Milk	1.0	10 kg	1.5

Solution:
(i) Computation for the weighted average of price relatives using arithmetic mean.

Item	p_0	q_0	p_1	w	p	$\log p$	wp	$w \log p$
Wheat	3.0	20	4.0	60	133.3	2.1249	8000	127.494
Flour	1.5	40	1.6	60	106.7	2.0282	6400	121.692
Milk	1.0	10	1.5	10	150.0	2.1761	1500	21.761
				$\sum w = 130$			$\sum wp = 15900$	$\sum w \log p = 270.947$

$$P_{01} = \frac{\sum wp}{\sum w} = \frac{15,900}{130} = 122.31$$

This means that there has been a 22.31 % increase in prices over the base year.
(ii) Index number using geometric mean of price relatives is:

$$P_{01} = \text{Antilog}\ \frac{\sum w \log p}{\sum w} = \text{Antilog}\ \frac{270.947}{130}$$

= Antilog (2.084) = 121.3
This means that there has been a 21.3 % increase in prices over the base year.

3. Quantity Index Number

The quantity index number measures the changes in the level of quantities of items consumed, or produced, or distributed during a year under study with reference to another year known as the base year.

Laspeyre's quantity index:

$$Q_{01}^{L} = \frac{\sum q_1 p_0}{\sum q_0 p_0} \times 100$$

Paasche's quantity index

$$Q_{01}^{P} = \frac{\sum q_1 p_1}{\sum q_0 p_1} \times 100$$

Fisher's quantity index

$$Q_{01}^{F} = \sqrt{Q_{01}^{L} \times Q_{01}^{P}}$$

$$= \sqrt{\frac{\sum q_1 p_0}{\sum q_0 p_0} \times \frac{\sum q_1 p_1}{\sum q_0 p_1}} \times 100$$

These formulae represent the quantity index in which quantities of the different commodities are weighted by their prices.

Compute the following quantity indices from the data given below:
(i) Laspeyre's quantity index (ii) Paasche's quantity index and (iii) Fisher's quantity index

Commodity	1970		1980	
	Price	Total value	Price	Total value
A	10	80	11	110
B	15	90	9	108
C	8	96	17	340

Solution:
Since we are given the value and the prices, the quantity figures can be obtained by dividing the value by the price for each of the commodities.

Commodity	p_0	q_0	p_1	q_1	p_0q_0	p_1q_0	p_0q_1	p_1q_1
A	10	8	11	10	80	88	100	110
B	15	6	9	12	90	54	180	108
C	8	12	17	20	96	204	160	340
	Total				266	342	440	558

(i) Laspeyre's quantity index

$$Q_{01}^{L} = \frac{\sum q_1 p_0}{\sum q_0 p_0} \times 100$$

$$= \frac{440}{266} \times 100$$

$$= 165.4$$

(ii) Paasche's quantity index

$$Q_{01}^{P} = \frac{\sum q_1 p_1}{\sum q_0 p_1} \times 100$$

$$= \frac{558}{342} \times 100$$

$$= 163.15$$

(iii) Fisher's quantity index

$$Q_{01}^{F} = \sqrt{Q_{01}^{L} \times Q_{01}^{P}}$$

$$= \sqrt{\frac{\sum q_1 p_0}{\sum q_0 p_0} \times \frac{\sum q_1 p_1}{\sum q_0 p_1}} \times 100$$

$$= \sqrt{\frac{440}{266} \times \frac{558}{342}} \times 100$$

$$= 1.6428 \times 100$$

$$= 164.28$$