DEPARTMENT OF STATISTICS,
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SUBJECT TITLE : TIME SERIES AND INDEX NUMBERS
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UNIT II

SEASONAL VARIATIONS

Seasonal variations are regular and periodic variations having a period of one year duration. Some of the examples which show seasonal variations are production of cold drinks, which are high during summer months and low during winter season. Sales of sarees in a cloth store which are high during festival season and low during other periods.

The reason for determining seasonal variations in a time series is to isolate it and to study its effect on the size of the variable in the index form which is usually referred as seasonal index.

Measurement of seasonal variations:

The study of seasonal variation has great importance for business enterprises to plan the production schedule in an efficient way so as to enable them to supply to the public demands according to seasons.

There are different devices to measure the seasonal variations. These are

- Method of simple averages.
- Ratio to trend method
- Ratio to moving average method
- Link relative method.

Method of simple averages

This is the simplest of all the methods of measuring seasonality. This method is based on the additive modal of the time series. That is the observed values of the series is expressed by $Y_t = T_t + S_t + C_t + I_t$ and in this method it is assumed that trend and cyclical variations are absent.

The method consists of the following steps.

1. The data are arranged season-wise in chronological order.
2. For each season, the total of seasonal values are found and called seasonal total.
3. Each seasonal total is divided by number of years and seasonal average is obtained.
4. The total and the average of seasonal averages are found. The average is called grand average.
5. Seasonal Index of every season is calculated as follows

$$Seasonal\ index = \frac{seasonal\ average}{grand\ average}\ x\ 100$$

Note: total of the seasonal indices = 100 x no. of seasons

1. Compute the Average Seasonal movement for the following series:

Year	I	II	III	IV		
1998	3.5	3.9	3.4	3.6		
1999	3.5	4.1	3.7	4.0		
2000	3.5	3.9	3.7	4.2		
2001	4.0	4.6	3.8	4.5		
2002	4.1	4.4	4.2	4.5		
Seasonal Total	18.6	20.9	18.8	20.8	Total	Grand Average
easonal Average	3.72	4.18	3.76	4.16	15.82	3.96
Seasonal Index	93.9 94	105.6 106	94.9 95	105.0 105	399.5	

$$Seasonal\ index = \frac{seasonal\ average}{grand\ average}\ x\ 100$$

For Quarter I, S.I = 3.72/3.96 X100 = 93.9
For, Quarter II S.I = 4.18/3.96X100 = 105.6
For Quarter III, S.I = 3.76/3.96 X100 = 94.9
For Quarter IV, S.I = 4.16/3.96 X100 = 105.1

2. Assuming no trend in the series, calculate seasonal indices for the following data:

	Quarter					
Year	I	II	III	IV		
2004	78	66	84	80		
2005	76	74	82	78		
2006	72	68	80	70		
2007	74	70	84	74		
2008	76	74	86	82		

$$Seasonal\ index = \frac{seasonal\ average}{grand\ average}\ x\ 100$$

For Quarter I, S.I = 75.2/76.4 X100 = 98.4
For, Quarter II S.I = 70.4/76.4X100 = 92.2
For Quarter III, S.I = 83.2/76.4 X100 = 108.9
For Quarter IV, S.I = 76.8/76.4 X100 = 100.5

Advantages and Disadvantages:

Method of simple average is easy and simple to execute.

This method is based on the basic assumption that the data do not contain any trend and cyclic components. Since most of the economic and business time series have trends and as such this method though simple is not of much practical utility.

2. Method of Moving Average.

Trend values are calculated by the method of moving averages. Period of moving average is equal to the number of seasons. Seasonal effects are subsequently calculated under additive and multiplicative models.

(a) Difference from Moving Average.

This method is based on additive model. The steps are as follows:

(i) Seasonal values are written in chronological order and trend values are calculated by the method of moving averages.
Period of Moving Average = No. of Seasons.

(ii) Short-term fluctuations (^Actual values - Trend) are calculated.

(iii) They are tabulated season-wise in chronological order.

(iv) The total and the average of the short - term fluctuations are found for each season.

(v) The average of those averages is found. It is the correction term.

(vi) From each seasonal average obtained in step (iv) the correction term is subtracted to get the seasonal fluctuation.

Note: The total of the seasonal fluctuations will be zero.

Example:

Determine the seasonal fluctuation in time series given below, indicating clearly the procedure followed.

Year	Quarter I	Quarter II	Quarter III	Quarter IV
1951	30	81	62	119
1952	33	104	86	171
1953	42	133	99	221
1954	56	172	129	335
1955	67	201	136	302

(I.C.W.A. Inter., J 66)

Year and Quarter		Value(Y)	4 Season moving total	2 Period moving total	Trend (Y_t)	Short-term fluctuation ($Y-Y_t$)
1951	I	30		-	-	-
	II	81	292	-	-	-
	III	62	295	587	73.375	-11.375
	IV	119	318	613	76.625	42.375
1952	I	33	342	660	82.500	-49.500
	II	104	394	736	92.000	12.000
	III	86	403	797	99.625	-13.625
	IV	171	432	835	104.375	66.625
1953	I	42	445	877	109.625	-67.625
	II	133	495	940	117.500	15.500
	III	99	509	1004	125.500	-26.500
	IV	221	548	1057	132.125	88.875
1954	I	56		1126	140.750	-84.750
			578			
	II	172	692	1270	158.750	13.250
	III	129	703	1395	174.375	-45.375
	IV	335	732	1435	179.375	155.625
1955	I	67	739	1471	183.875	-116.875
	II	201	706	1445	180.625	20.375
	III	136		-	-	-
	IV	302		-	-	-

Short term fluctuation

Year	I	II	III	IV
1951	-	-	-11.375	42.375
1952	-49.500	12.000	-13.625	66.625
1953	-67.625	15.500	-26.500	88.875
1954	-84.750	13.250	-45.375	155.625
1955	-116.875	20.375		-
Total	-318.750	61.125	-96.875	353.500
Average	-79.688	15.281	-24.219	88.375
Seasonal Fluctuation	-79.625	15.344	-24.156	88.438

Total of the averages is -0.251 and so the correction term is - 0.251/4 = -0.063.

Note: Total of the seasonal fluctuations = 0.001 $\cong$ 0

(b) Ratio - to - Moving Average.

This method assumes multiplicative model. The steps are as follows:

(i) Seasonal values are written in chronological order and trend values are calculated by the method of moving averages.
Period of Moving Average = No. of seasons

(ii) Trend values . i.e. $\frac{Actual\ value}{Trend\ value}\ X\ 100\ are\ calculated$

(iii) The percentages are tabulated season wise, totaled and averaged.

(iv) The total and the average of the seasonal averages are found. The average is called grand averaged.

(v) Correction factor is calculated as follows
Correction Factor = $\frac{100}{Grand\ Average}$

(vi) Seasonal averages are multiplied by the correction factor to get the seasonal indices.

Note: Total of the seasonal indices = 100 x No.of seasons.

Example:

Find the Seasonal Index form the following table by ratio to moving average method.

Seasons	1990	1991	1992	1993	1994
I Quarter	40	42	41	45	44
II Quarter	35	37	35	36	38
III Quarter	38	39	38	36	38
IV Quarter	40	38	42	41	42

Solution

Year and arter		Value (Y)	4 Season moving total	2 Period moving total	Trend (Y_t)	Actual value as % of Trend $\frac{Y}{Y_t} X\ 100$
1990	I	40		-	-	-
	II	35	153	-	-	-
	III	38	155	308	38.50	98.70
	IV	40	157	312	39.00	102.56
1991 I		42	158	315	39.38	106.65
	II	37	156	314	39.25	94.27
	III	39	155	311	38.88	100.31
	IV	38	153	308	38.50	98.70
1992	I	41	152	305	38.13	107.53
	II	35	156	308	38.50	90.91
	III	38	160	316	39.50	96.20
	IV	42	161	321	40.13	104.66
1993	I	45	159	320	40.00	112.50
	II	36	158	317	39.63	90.84
	III	36	157	315	39.38	91.42
IV		41	159	316	39.50	103.80
1994	I	44	161	320	40.00	110.00
	II	38		323	40.38	94.11
	III	38		-	-	-
	IV	42				

Year	Percentages of the values to Trend					
	QI	QII	QIII	QIV		
1990	-	-	98.70	102.56		
1991	106.65	94.27	100.31	98.70		
1992	107.53	90.91	96.20	104.66		
1993	112.50	90.84	91.42	103.80		
1994	110.00	94.11				
Seasonal total	436.68	370.13	385.62	409.72	Total	Grand Average

Seasonal	109.17	92.53	96.66	102.43	400.79	100.1975
Seasonal Index	108.95	92.35	96.47	102.23	400.00	-

Note:

1. Total of the seasonal indices = 400.00

2. In step (ii), trend is eliminated from the given series. Subsequently seasonal indices are calculated as in the method of simple averages.

3. Correction Factor = 100/100.1975.

Merits:

1. It is claimed to be the best method. While averaging, the effect of cyclical variations is eliminated. It is found to agree with the nature of many series.

2. It estimates and eliminates trend. It is better than the method of simple averages which assumes that the trend is absent.

3. The fluctuations of the ratios are less under it than under Ratio-to-Trend method.

4. It is flexible in the sense that different methods are available for additive and multiplicative models.

5. It is the most popular method. Its popularity is due to its easier calculations and satisfactory estimates.

Demerits:

1. It is easier than Ratio-to-Trend and Link Relatives Methods. It is more difficult than the method of Simple Averages.
2. While calculating trend, the moving averages (trend) of a few seasons in the beginning and equal number of seasons in the end are not available.

Ratio-to-Trend Method.

This is similar to Ratio- to - Moving Average method. The difference is only in the method of calculation of trend. Trend is calculated by the method of least squares in this method. The steps are as follows:

i) The average of the actual values per season is found for each year. Based on all such averages, the values of trend of various years are obtained by the method of least squares. Those are the trend values of the mid periods of the respective years. Using the change in trend per annum and the change in trend per season (and the change in trend per half a season when required), the trend values of all the seasons are calculated.

ii) Ratio-to-Trend of each season is obtained by

$$\frac{Actual\ value}{Trend}\ X\ 100$$

iii) Such ratios are in percentages. They are tabulated season wise in chronological order.

iv) The total and the average of each season are found.

v) The average of those seasonal averages is found and called 'Grand Average'.

Correction Factor = $\frac{100}{Grand\ Average}$

vi) Seasonal averages are multiplied by the correction factor to get the seasonal indices.

Note:

1. Total of seasonal indices = 100 x No.of seasons.
2. In step (ii), trend is eliminated from the given series. Subsequently, seasonal indices are calculated as in the method of simple averages.

Example:

Find the seasonal variations by the ratio to trend method from the data given below:

Year	Quarter l	Quarter II	Quarter lll	Quarterly
1972	39	20	60	85
1973	45	23	62	90
1974	44	25	69	92
1975	53	30	70	97
1976	60	32	76	100

(B.Com. Osmania, O 77

Solution

Year X	Total	Average Y=y	x= X-1974	xy	X^2	Trend Y_t
1972	204	51.0	-2	-102.0	4	50.70
1973	220	55.0	**-1**	- 55.0	**1**	54.65
1974	230	57.5	0	0	0	58.60
1975	250	62.5	**1**	62.5	1	62.55
1976	268	67.0	2	134.0	4	66.50
Total	-	$\Sigma y = 293.0$	-	$\Sigma xy = 39.5$	$\Sigma x^2 = 10$	$\Sigma Y_t = 293.00$

Mid year $\bar{X}$ = 1974

Trend of the mid year, $a = \frac{\sum y}{N} = \frac{293}{5} = 58.60$

Actual change in trend per season $= \frac{\sum xy}{\sum x^2} = \frac{39.5}{10} = 3.95$

Change in trend per half a season $= \frac{3.95}{8} = 0.49$

Linear trend equation $Y_t = a + b\ (X - \bar{X}$

$Y_t = 58.60 + 3.95\ (X - 1974)$

i.e "Trend of 2.5th season of 1972.

X = 1972 $Y_t = 58.60 + 3.95\ (1972 - 1974)$

$= 50.70$

Trend of 3rd Season = 50.70 = 0.49 = 51.19

Trend of other sesasons is found such that the difference = 0.99

1972	49.21	50.20(50.70)	51.19	52.18
1973	53.17	54.16	55.15	56.14
1974	57.13	58.12	59.11	60.10
1975	61.09	62.08	63.07	64.06
1976	65.05	66.04	67.03	68.02

Ratio - to - Trend Values

1972	79.25	39.84	117.21	162.90
1973	84.63	.42.47	112.42	160.31
1974	77.02	43.01	116.73	153.08
1975	86.76	48.32	110.99	151.42
1376	92.24	48.46	113.38	147.02

Seasonal Total	419.90	222.10	570.73	774.73	Total	Grand Average
Seasonal Average	83.98	44.42	114.15	154.95	397.50	99.375
Seasonal Index	84.51	44.70	114.87	155.92	400.00	

Merits:

1. Unlike in the methods of moving average, there is no loss of data and ratio to trend can be calculated for every given season. This advantage is considerable when the period covered by the time series is very short.
2. It is simple to understand.

Demerits:

1. It is not as easy to calculate as the method of simple averages.

2. It tries to eliminate the cyclical and irregular components by averaging for each season. Whenever there are large cyclical swings, it fails to iron out them. The results by the methods of moving averages are closer to the actual data than those by the ratio to trend method.

4. Link Relative Method. The following are the steps in the calculation of Seasonal indices.

i) The seasonal values are arragned in chronological order and the link relative of every season is calculated using the formula

$$\text{Link relative (L.R.)} = \frac{Value\ of\ Current\ Season}{Value\ of\ previous\ season}\ X\ 100$$

For the very first season, there is no link relative.

ii) Link relatives are tabulated season wise in chronological order.

iii) The total and the average for each season are found

iv) The averages are converted into chain relatives. For the first season, it is assumed as100. For others, the formula is

$$\text{Chain relative (C.R.)} = \frac{\text{Average Link relative of the current season X Chain relative of the previous season}}{100}$$

v) Using the above formula, the chain relative of the first season taking the last season as its previous season is calculated.

i.e., C.R. for the first seasons =

$$\frac{\text{Average L. R of the season X C. R. of the last season}}{100}$$

d, 2a, 3d, ... are the correction factors for second, third, fourthseasons. They are subtracted from the respective chain relatives.

(iv) The average of the corrected chain relative is calculated.

(v) The seasonal indices are calculated.

$$\text{Seasonal index} = \frac{\text{Corrected chain relative of the season}}{Average\ of\ corrected\ vhain\ relatives}\ X\ 100$$

Example:

The data below give the average quarterly prices of a commodity for five years. Calculate seasonal indices by the method of link relatives.

	Years				
Quarters	**1**	**2**	**3**	**4**	**5**
1	30	35	31	31	34
2	26	28	29	31	36
3	22	22	28	25	26

4	31	36	32	35	33

(C.A. Foundation, M 2K)

Solution

	Link Relatives			
Years	Q1	Q2	Q3	Q4
1		86.67	84.62	140.91
2	112.90	80.00	78.57	163.64
3	86.11	93.55	96.55	114.29
4	96.88	100.00	80.65	140.00
5	97.14	105.88	72.22	126.92
Total	393.03	466.10	412.61	685.76
Average	98.26	93.22	82.52	137.15
Chain Relative	100.00	93.22	76.93	105.51
Corrected Chain Relative	1000.00	92.30	75.09	102.75
Seasional Index	108.07	99.74	81.15	111.04

Merits:

1. Method of link relatives is more simple and easier than both ratio to moving average and ratio to trend methods.

2. Link relative of the first season of the first year can not be calculated. Moving averages of the first two quarters of the first year and last two quarters of the last year are not available when there are four seasons. Consequently, loss of data is less in the method of link relatives than in the methods of moving averages.

Demerit:

Correction of chain indices is to eliminate the linear trend. Non-linear trend and cyclical components are not eliminated.

Cyclical and Irregular Variations.

Measurement of cyclical variations:

The various methods used for measuring cyclical variations are

- Residual method
- Reference cycle analysis method
- Direct method
- Harmonic analysis method

2. Method of Moving Average.

Trend values are calculated by the method of moving averages. Period of moving average is equal to the number of seasons. Seasonal effects are subsequently calculated under additive and multiplicative models.

(a) Difference from Moving Average.

This method is based on additive model. The steps are as follows:

(vii) Seasonal values are written in chronological order and trend values are calculated by the method of moving averages.
Period of Moving Average = No. of Seasons.

(viii) Short-term fluctuations (^Actual values - Trend) are calculated.

(ix) They are tabulated season-wise in chronological order.

(x) The total and the average of the short - term fluctuations are found for each season.

(xi) The average of those averages is found. It is the correction term.

(xii) From each seasonal average obtained in step (iv) the correction term is subtracted to get the seasonal fluctuation.

Note: The total of the seasonal fluctuations will be zero.

Example:

Determine the seasonal fluctuation in time series given below, indicating clearly the procedure followed.

Year	Quarter I	Quarter II	Quarter III	Quarter IV
1951	30	81	62	119
1952	33	104	86	171
1953	42	133	99	221
1954	56	172	129	335
1955	67	201	136	302

(I.C.W.A. Inter., J 66)

Year and Quarter		Value(Y)	4 Season moving total	2 Period moving total	Trend (Y_t)	Short-term fluctuation ($Y-Y_t$)
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	II	81	292	-	-	-
	III	62	295	587	73.375	-11.375
	IV	119	318	613	76.625	42.375
1952	I	33	342	660	82.500	-49.500
	II	104	394	736	92.000	12.000
	III	86	403	797	99.625	-13.625
	IV	171	432	835	104.375	66.625
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	II	133	495	940	117.500	15.500
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	IV	221	548	1057	132.125	88.875
1954	I	56		1126	140.750	-84.750
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	II	172	692	1270	158.750	13.250
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1955	I	67	739	1471	183.875	-116.875
	II	201	706	1445	180.625	20.375
	III	136		-	-	-
	IV	302		-	-	-

Short term fluctuation

Year	I	II	III	IV
1951	-	-	-11.375	42.375
1952	-49.500	12.000	-13.625	66.625
1953	-67.625	15.500	-26.500	88.875
1954	-84.750	13.250	-45.375	155.625
1955	-116.875	20.375		-
Total	-318.750	61.125	-96.875	353.500
Average	-79.688	15.281	-24.219	88.375
Seasonal Fluctuation	-79.625	15.344	-24.156	88.438

Total of the averages is -0.251 and so the correction term is - 0.251/4 = -0.063.

Note: Total of the seasonal fluctuations = 0.001 $\cong$ 0

(b) Ratio - to - Moving Average.

This method assumes multiplicative model. The steps are as follows:

(vi) Seasonal values are written in chronological order and trend values are calculated by the method of moving averages.

Period of Moving Average = No. of seasons

(vii) Trend values . i.e. $\frac{Actual\ value}{Trend\ value}\ X\ 100\ are\ calculated$

(viii) The percentages are tabulated season wise, totaled and averaged.

(ix) The total and the average of the seasonal averages are found. The average is called grand averaged.

(x) Correction factor is calculated as follows

Correction Factor = $\frac{100}{Grand\ Average}$

(vi) Seasonal averages are multiplied by the correction factor to get the seasonal indices.

Note: Total of the seasonal indices = 100 x No.of seasons.

Example:

Find the Seasonal Index form the following table by ratio to moving average method.

Seasons	1990	1991	1992	1993	1994
I Quarter	40	42	41	45	44
II Quarter	35	37	35	36	38
III Quarter	38	39	38	36	38
IV Quarter	40	38	42	41	42

Solution

Year and arter		Value (Y)	4 Season moving total	2 Period moving total	Trend (Y_t)	Actual value as % of Trend $\frac{Y}{Y_t} X\ 100$
1990	I	40		-	-	-
	II	35	153	-	-	-
	III	38	155	308	38.50	98.70
	IV	40	157	312	39.00	102.56
1991 I		42	158	315	39.38	106.65
	II	37	156	314	39.25	94.27
	III	39	155	311	38.88	100.31
	IV	38	153	308	38.50	98.70
1992	I	41	152	305	38.13	107.53
	II	35	156	308	38.50	90.91
	III	38	160	316	39.50	96.20
	IV	42	161	321	40.13	104.66
1993	I	45	159	320	40.00	112.50
	II	36	158	317	39.63	90.84
	III	36	157	315	39.38	91.42
IV		41	159	316	39.50	103.80
1994	I	44	161	320	40.00	110.00
	II	38		323	40.38	94.11
	III	38		-	-	-
	IV	42				

Year	Percentages of the values to Trend					
	QI	QII	QIII	QIV		
1990	-	-	98.70	102.56		
1991	106.65	94.27	100.31	98.70		
1992	107.53	90.91	96.20	104.66		
1993	112.50	90.84	91.42	103.80		
1994	110.00	94.11				
Seasonal total	436.68	370.13	385.62	409.72	Total	Grand Average

Seasonal	109.17	92.53	96.66	102.43	400.79	100.1975
Seasonal Index	108.95	92.35	96.47	102.23	400.00	-

Note:

1. Total of the seasonal indices = 400.00

2. In step (ii), trend is eliminated from the given series. Subsequently seasonal indices are calculated as in the method of simple averages.

3. Correction Factor = 100/100.1975.

Merits:

1. It is claimed to be the best method. While averaging, the effect of cyclical variations is eliminated. It is found to agree with the nature of many series.

6. It estimates and eliminates trend. It is better than the method of simple averages which assumes that the trend is absent.

7. The fluctuations of the ratios are less under it than under Ratio-to-Trend method.

8. It is flexible in the sense that different methods are available for additive and multiplicative models.

9. It is the most popular method. Its popularity is due to its easier calculations and satisfactory estimates.

Demerits:

3. It is easier than Ratio-to-Trend and Link Relatives Methods. It is more difficult than the method of Simple Averages.

4. While calculating trend, the moving averages (trend) of a few seasons in the beginning and equal number of seasons in the end are not available.

Ratio-to-Trend Method.

This is similar to Ratio- to - Moving Average method. The difference is only in the method of calculation of trend. Trend is calculated by the method of least squares in this method. The steps are as follows:

i) The average of the actual values per season is found for each year. Based on all such averages, the values of trend of various years are obtained by the method of least squares. Those are the trend values of the mid periods of the respective years. Using the change in trend per annum and the change in trend per season (and the change in trend per half a season when required), the trend values of all the seasons are calculated.

ii) Ratio-to-Trend of each season is obtained by

$$\frac{Actual\ value}{Trend}\ X\ 100$$

iii) Such ratios are in percentages. They are tabulated season wise in chronological order.

iv) The total and the average of each season are found.

v) The average of those seasonal averages is found and called 'Grand Average'.

Correction Factor = $\frac{100}{Grand\ Average}$

vi) Seasonal averages are multiplied by the correction factor to get the seasonal indices.

Note:

1. Total of seasonal indices = 100 x No.of seasons.
2. In step (ii), trend is eliminated from the given series. Subsequently, seasonal indices are calculated as in the method of simple averages.

Example:

Find the seasonal variations by the ratio to trend method from the data given below:

Year	Quarter l	Quarter II	Quarter lll	Quarterly
1972	39	20	60	85
1973	45	23	62	90
1974	44	25	69	92
1975	53	30	70	97
1976	60	32	76	100

Solution

Year X	Total	Average Y=y	x= X-1974	xy	X^2	Trend Y_t
1972	204	51.0	-2	-102.0	4	50.70
1973	220	55.0	**-1**	- 55.0	**1**	54.65
1974	230	57.5	0	0	0	58.60
1975	250	62.5	**1**	62.5	1	62.55
1976	268	67.0	2	134.0	4	66.50
Total	-	$\Sigma y = 293.0$	-	$\Sigma xy = 39.5$	$\Sigma x^2 = 10$	$\Sigma Y_t = 293.00$

Mid year $\bar{X}$ = 1974

Trend of the mid year, a = $\frac{\sum y}{N} = \frac{293}{5} = 58.60$

Actual change in trend per season = $\frac{\sum xy}{\sum x^2} = \frac{39.5}{10} = 3.95$

Change in trend per half a season = $\frac{3.95}{8} = 0.49$

Linear trend equation $Y_t = a + b\ (X - \bar{X}$

Y_t =58.60 + 3.95 (X - 1974)

i.e "Trend of 2.5th season of 1972.

X = 1972 Y_t = 58.60 + 3.95 (1972 - 1974)

= 50.70

Trend of 3rd Season = 50.70 = 0.49 = 51.19

Trend of other seasons is found such that the difference = 0.99

1972	49.21	50.20(50.70)	51.19	52.18
1973	53.17	54.16	55.15	56.14
1974	57.13	58.12	59.11	60.10
1975	61.09	62.08	63.07	64.06
1976	65.05	66.04	67.03	68.02

Ratio - to - Trend Values

1972	79.25	39.84	117.21	162.90
1973	84.63	.42.47	112.42	160.31
1974	77.02	43.01	116.73	153.08
1975	86.76	48.32	110.99	151.42
1376	92.24	48.46	113.38	147.02

Seasonal Total	419.90	222.10	570.73	774.73	Total	Grand Average
Seasonal Average	83.98	44.42	114.15	154.95	397.50	99.375
Seasonal Index	84.51	44.70	114.87	155.92	400.00	

Merits:

1. Unlike in the methods of moving average, there is no loss of data and ratio to trend can be calculated for every given season. This advantage is considerable when the period covered by the time series is very short.

2. It is simple to understand.

Demerits:

1. It is not as easy to calculate as the method of simple averages.
2. It tries to eliminate the cyclical and irregular components by averaging for each season. Whenever there are large cyclical swings, it fails to iron out them. The results by the methods of moving averages are closer to the actual data than those by the ratio to trend method.

5. Link Relative Method. The following are the steps in the calculation of Seasonal indices.

ii) The seasonal values are arragned in chronological order and the link relative of every season is calculated using the formula

$$\text{Link relative (L.R.)} = \frac{Value\ of\ Current\ Season}{Value\ of\ previous\ season}\ X\ 100$$

For the very first season, there is no link relative.

ii) Link relatives are tabulated season wise in chronological order.

iii) The total and the average for each season are found

iv) The averages are converted into chain relatives. For the first season, it is assumed as100. For others, the formula is

$$\text{Chain relative (C.R.)} = \frac{\text{Average Link relative of the current season X Chain relative of the previous season}}{100}$$

vi) Using the above formula, the chain relative of the first season taking the last season as its previous season is calculated.

i.e., C.R. for the first seasons =

$$\frac{\text{Average L. R of the season X C. R. of the last season}}{100}$$

d, 2a, 3d, ... are the correction factors for second, third, fourthseasons.

They are subtracted from the respective chain relatives.

(vi) The average of the corrected chain relative is calculated.

(vii) The seasonal indices are calculated.

$$\text{Seasonal index} = \frac{\text{Corrected chain relative of the season}}{Average\ of\ corrected\ vhain\ relatives}\ X\ 100$$

Example:

The data below give the average quarterly prices of a commodity for five years. Calculate seasonal indices by the method of link relatives.

	Years				
Quarters	**1**	**2**	**3**	**4**	**5**

1	30	35	31	31	34
2	26	28	29	31	36
3	22	22	28	25	26
4	31	36	32	35	33

Solution

	Link Relatives			
Years	Q1	Q2	Q3	Q4
1		86.67	84.62	140.91
2	112.90	80.00	78.57	163.64
3	86.11	93.55	96.55	114.29
4	96.88	100.00	80.65	140.00
5	97.14	105.88	72.22	126.92
Total	393.03	466.10	412.61	685.76
Average	98.26	93.22	82.52	137.15
Chain Relative	100.00	93.22	76.93	105.51
Corrected Chain Relative	1000.00	92.30	75.09	102.75
Seasional Index	108.07	99.74	81.15	111.04

Merits:

1. Method of link relatives is more simple and easier than both ratio to moving average and ratio to trend methods.

2. Link relative of the first season of the first year can not be calculated. Moving averages of the first two quarters of the first year and last two quarters of the last year are not available when there are four seasons. Consequently, loss of data is less in the method of link relatives than in the methods of moving averages.

Demerit:

Correction of chain indices is to eliminate the linear trend. Non-linear trend and cyclical components are not eliminated.

Cyclical Variations or Oscillatory Variation

This is a short term variation occurs for a period of more than one year. The rhythmic movements in a time series with a period of oscillation(repeated again and again in same manner) more than one year is called a cyclical variation and the period is called a cycle. The time series related to business and economics show some kind of cyclical variations.

One of the best examples for cyclical variations is „Business Cycle". In this cycle there are four well defined periods or phases.

- Boom
- Decline
- Depression
- Improvement

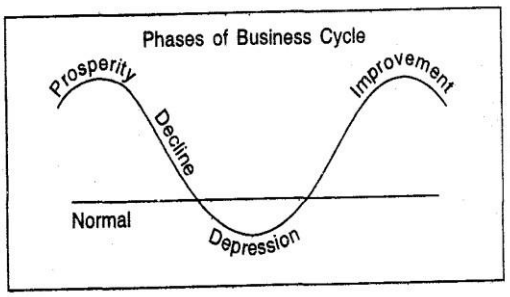

Figure 2. Phases of Business Cycle

Measurement of cyclical variations:

The various methods used for measuring cyclical variations are

- Residual method
- Reference cycle analysis method
- Direct method
- Harmonic analysis method

Irregular Variation

It is also called Erratic, Accidental or Random Variations. The three variations trend, seasonal and cyclical variations are called as regular variations, but almost all the time series including the regular variation contain another variation called as random variation. This type of fluctuations occurs in random way or irregular ways which are unforeseen, unpredictable and due to some irregular circumstances which are beyond the control of human being such as earth quakes, wars, floods, famines, lockouts, etc. These factors affect the time series in the irregular ways. These irregular variations are not so significant like other fluctuations