16. 12. 2020 ①

UNIT - V

FUNCTION OF CHAIN RULE

$$y \to u \to x \qquad [y = f(x)]$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

Example 18:

Prove that $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ where y is a function of u and u is function of x.

Soln:

$$\frac{\Delta y}{\Delta x} = \frac{\Delta y}{\Delta u} \cdot \frac{\Delta u}{\Delta x}$$

$$\frac{dy}{dx} = \underset{\Delta x \to 0}{Lt} \frac{\Delta y}{\Delta x} = \underset{\Delta x \to 0}{Lt} \frac{\Delta y}{\Delta u} \underset{\Delta x \to 0}{Lt} \frac{\Delta u}{\Delta x}$$

$$= \underset{\Delta u \to 0}{Lt} \frac{\Delta y}{\Delta u} \underset{\Delta x \to 0}{Lt} \frac{\Delta u}{\Delta x}$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}.$$

Example :

To find the derivative of e^{7x+9}

Soln

Let $y = e^{7x+9}$

$y = e^{u}$ where $u = 7x+9$

$$\frac{dy}{du} = e^{u} \qquad \frac{du}{dx} = 7(1)+0 = 7$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= e^u \cdot 7$$

$$\frac{dy}{dx} = 7\, e^{7x+9}$$

Note :

1. If y is a function of u, u is a function of v and v is a function of x

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dv} \cdot \frac{dv}{dx} \qquad [y \to u \to v \to x]$$

2. corollary to the chain rule:

$$y = x$$

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

$$\frac{dx}{dx} = \frac{dx}{du} \cdot \frac{du}{dx}$$

$$1 = \frac{dx}{du} \cdot \frac{du}{dx}$$

$$\frac{du}{dx} = \frac{1}{\frac{dx}{du}}$$

Example 19 :

Differentiate the following with respect to x

i) $(ax+b)^n$

Soln. Let $y = (ax+b)^n$

$y = u^n$ Where $u = ax+b$

$$\frac{du}{dx} = a$$

$$\frac{dy}{du} = n u^{n-1}$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= n u^{n-1} \cdot a$$

$$\frac{dy}{dx} = na(ax+b)^{n-1}$$

ii) e^{ax+b}

Soln

Let $y = e^{ax+b}$

$y = e^u$ Where $u = ax+b$

$$\frac{du}{dx} = a$$

$$\frac{dy}{du} = e^u$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= e^u \cdot a$$

$$\frac{dy}{dx} = a e^{ax+b}$$

iii) $\log_e (ax+b)$

Soln

Let $y = \log_e (ax+b)$

$y = \log_e u$ Where $u = ax+b$

$$\frac{du}{dx} = a$$

$$\frac{dy}{du} = \frac{1}{u}$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= \frac{1}{u} \cdot a$$

$$\frac{dy}{dx} = \frac{a}{ax+b}$$

4 Example 20: Find

i) $\frac{d}{dx}(\log ax)$

Soln

Let $y = \log ax$

$y = \log u$ Where $u = ax$

$\frac{du}{dx} = a$

$$\frac{dy}{du} = \frac{1}{u}$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= \frac{1}{u} \cdot a$$

$$\frac{dy}{dx} = \frac{a}{ax}$$

ii) $\frac{d}{dx} \log(5x+7)$

Soln

Let $y = \log(5x+7)$

$y = \log u$ Where $u = 5x+7$

$\frac{du}{dx} = 5$

$$\frac{dy}{du} = \frac{1}{u}$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= \frac{1}{u} \cdot 5$$

$$\frac{dy}{dx} = \frac{5}{5x+7}$$

iii) $\frac{d}{dx} \log(\sqrt{2x+3})$

Soln. Let $y = \log \sqrt{2x+3}$

$$y = \log(2x+3)^{1/2}$$

$$= \frac{1}{2}\log(2x+3)$$

$$y = \frac{1}{2}\log u \quad \text{Where } u = 2x+3$$

$$\frac{du}{dx} = 2$$

$$\frac{dy}{du} = \frac{1}{2}\cdot\frac{1}{u}$$

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$$

$$= \frac{1}{2}\cdot\frac{1}{u}\cdot 2$$

$$= \frac{1}{\cancel{2}}\cdot\frac{1}{2x+3}\cdot\cancel{2}$$

$$\frac{dy}{dx} = \frac{1}{2x+3}$$

Example 21:

Differentiate the following with respect to x

i) $(3x^2+4x-5)^3$

Soln Let $y = (3x^2+4x-5)^3$

$$y = u^3 \quad \text{Where } u = 3x^2+4x-5$$

$$\frac{dy}{du} = 3u^2 \qquad \frac{du}{dx} = 6x+4$$

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$$

$$= 3u^2\cdot(6x+4)$$

$$= 3(3x^2+4x-5)^2(6x+4)$$

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$$= 3(3x^2+4x-5)^2 \cdot 2(3x+2)$$

$$= 6(3x+2)(3x^2+4x-5)^2$$

$$\frac{dy}{dx} = 6(3x+2)(3x^2+4x-5)^2$$

ii) x^x

Soln

Let $y = x^x$

As x as in the power, by taking $\log_e$ on both sides

$$\log_e y = \log_e x^x$$

$$= x \log_e x$$

Differentiating both sides w.r.t x

$$\log_e y = x \log_e x$$

$UV = x \log_e x$

$UV = UV' + VU'$

$\Rightarrow U = x \mid U' = 1$

$V = \log_e x \mid V' = \frac{1}{x}$

$UV = x \cdot \frac{1}{x} + \log_e x \cdot 1$

$$\frac{1}{y} \cdot \frac{dy}{dx} = x \cdot \frac{1}{x} + \log_e x \cdot 1$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = 1 + \log_e x$$

$$\frac{dy}{dx} = (1+\log_e x)\, y$$

$$\frac{d}{dx}(x^x) = (1+\log_e x)\, x^x$$

Note:

When x is in power or many factors in a product or quotient, it will be convenient to introduce $\log_e$ on both sides

Example 22:

If $y = x^{x^x}$ find $\frac{dy}{dx}$

Soln

Let $y = x^{x^x}$

Taking $\log_e$ on both sides

$$\log_e y = \log_e (x^{x^x})$$

$$= x^x \log_e x$$

Differentiate with respect to x

$$\frac{1}{y} \cdot \frac{dy}{dx} = x^x \cdot \frac{1}{x} + \log_e x \, (1 + \log_e x) x^x$$

[from example 21]

$$= x^x \left[\frac{1}{x} + (1 + \log_e x) \log_e x\right]$$

$$\frac{dy}{dx} = x^x \left[\frac{1}{x} + (1 + \log_e x) \log_e x\right] y$$

$$= x^x \left[\frac{1}{x} + (1 + \log_e x) \log_e x\right] x^{x^x}$$

Example 23:

Find the derivative of a^x where a is a constant.

Soln

Let $y = a^x$

Take $\log_e$ on both sides

$$\log_e y = \log_e a^x$$

$$\log_e y = x \log_e a$$

Differentiating with respect to x

$$\frac{1}{y} \cdot \frac{dy}{dx} = \log_e a$$

$$\frac{dy}{dx} = (\log_e a)\, y$$

$$\frac{dy}{dx} = (\log_e a)\, a^x$$

Example 24:

Find the derivative of $5^x x^5$ $\Rightarrow UV = UV' + VU'$

$u = 5^x \quad V = x^5$

Soln.

Let $y = 5^x x^5$

$y = 5^x \quad V' = 5x^4$

$\log_e y = x \log_e 5$

$$\frac{dy}{dx} = 5^x (5x^4) + x^5 (\log_e 5) 5^x$$

$\frac{1}{y} \cdot \frac{dy}{dx} = \log_e 5$

$$= 5^x x^4 (5 + x \log_e 5)$$

$\frac{dy}{dx} = (\log_e 5)\, y$

$u' = \frac{dy}{dx} = (\log_e 5)\, 5^x$

Example 25:

Differentiate the following w.r.t to x

$$y = \frac{x^{1/2} (1+x)^{1/3}}{(1+2x)^{1/4} (1+3x)^{1/5}}$$

Soln.

Taking $\log_e$ on both sides

$$\log_e y = \log_e \left[\frac{x^{1/2} (1+x)^{1/3}}{(1+2x)^{1/4} (1+3x)^{1/5}} \right]$$

$$= \frac{\log_e [x^{1/2} (1+x)^{1/3}]}{\log_e [(1+2x)^{1/4} (1+3x)^{1/5}]}$$

$$= \frac{\log_e x^{1/2} + \log (1+x)^{1/3}}{\log_e (1+2x)^{1/4} + \log (1+3x)^{1/5}}$$

$$= \log x^{1/2} + \log(1+x)^{1/3} - \log(x+2x)^{1/4} - \log(1+3x)^{1/5}$$

$$\log_e y = \frac{1}{2}\log x + \frac{1}{3}\log(1+x)^{\cancel{1/3}} - \frac{1}{4}\log(x+2x) - \frac{1}{5}\log(1+3x)$$

Differentiating both sides w.r.t x

$$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{1}{2}\cdot\frac{1}{x} + \frac{1}{3}\cdot\frac{1}{1+x} - \frac{1}{4}\cdot\frac{2}{x+2x} - \frac{1}{5}\cdot\frac{3}{1+3x}$$

$$= \frac{1}{2x} + \frac{1}{3(1+x)} - \frac{1}{2(x+2x)} - \frac{3}{5(1+3x)}$$

$$\frac{dy}{dx} = \left[\frac{1}{2x} + \frac{1}{3(1+x)} - \frac{1}{2(x+2x)} - \frac{3}{5(1+3x)}\right] y$$

$$= \left[\frac{1}{2x} + \frac{1}{3(1+x)} - \frac{1}{2(x+2x)} - \frac{3}{5(1+3x)}\right] \frac{x^{1/2}(1+x)^{1/3}}{(1+2x)^{1/4}(1+3x)^{1/5}}$$

Note:

$\frac{dy}{dx}$ can be found out without taking $\log_e$ on both sides of the equation.

Example 26:

Differentiate $x^2 \log_a x$ with respect to x

Soln

Let $y = x^2 \log_a x$

$$\frac{dy}{dx} = x^2 \frac{d}{dx}(\log_a x) + \log_a x \frac{d}{dx}(x^2)$$

$$= x^2 \frac{d}{dx}(\log_e x \cdot \log_a e) + \log_a x \frac{d}{dx}(x^2)$$

$$= \cancel{x^2}\cdot\frac{1}{\cancel{x}} \log_a e + \log_a x \times 2x$$

$$\frac{dy}{dx} = x\left(\log_a e + 2\log_a x\right)$$

Example 27:

Find $\frac{d}{dx}\left[\log_e\left(\frac{x^2+1}{x^2-1}\right)\right]$

Soln.

Let $y = \log_e\left(\frac{x^2+1}{x^2-1}\right)$

$$y = \log_e(x^2+1) - \log_e(x^2-1)$$

$$\frac{d}{dx}\left[\log_e\left(\frac{x^2+1}{x^2-1}\right)\right] = \frac{1}{(x^2+1)}(2x) - \frac{1}{(x^2-1)}\cdot(2x)$$

$$= \frac{2x}{x^2+1} - \frac{2x}{x^2-1}$$

$$= 2x\left(\frac{1}{x^2+1} - \frac{1}{x^2-1}\right)$$

$$= 2x\left[\frac{(x^2-1)-(x^2+1)}{(x^2+1)(x^2-1)}\right]$$

$$\frac{dy}{dx} = \frac{4x}{(x^2+1)(x^2-1)}$$

Differentiation of Implicit Functions:

Example 28: Find $\frac{dy}{dx}$ if

i) $x^2+y^2=1$

Soln

Differentiating with respect to x

$$x^2+y^2=1$$

$$2x+2y\frac{dy}{dx}=0$$

$$2y\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{2y} = -x/y$$

ii) $xy = c^2$

Soln Differentiating with respect to x

$$xy = c^2$$

$$x\frac{dy}{dx} + y = 0$$

$$x\frac{dy}{dx} = -y$$

$$\frac{dy}{dx} = \frac{-y}{x}$$

Example 29 :

Find $\frac{dy}{dx}$ if $x^3 - 8xy + 3y^2 = 14$

Soln

Differentiating both sides w.r.to x

$$x^3 - 8xy + 3y^2 = 14$$

$$3x^2 - 8\left(x\frac{dy}{dx} + y\right) + 6y \cdot \frac{dy}{dx} = 0$$

$$3x^2 - 8x\frac{dy}{dx} - 8y + 6y \cdot \frac{dy}{dx} = 0$$

$$3x^2 - 8y + (6y - 8x)\frac{dy}{dx} = 0$$

$$(6y - 8x)\frac{dy}{dx} = 8y - 3x^2$$

$$\frac{dy}{dx} = \frac{8y - 3x^2}{6y - 8x}$$

Parametric Form :

If x is a function of t and y is another function of t. both the variable x and y are function of a third variable

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

Example 30:

Find $\frac{dy}{dx}$ when $x = 4t$ and $y = 2t^2$

Soln. Given $x = 4t$ $y = 2t^2$

$$\frac{dx}{dt} = 4 \qquad \frac{dy}{dt} = 4t$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$$\frac{dy}{dx} = \frac{4t}{4} = \cancel{4}t/\cancel{4} = t$$

Given function $x = 4t$ & $y = 2t^2$

$$\frac{2y}{x} = \frac{2(2t^2)}{4t} = \frac{4t^2}{4t}$$

$$t = \frac{2y}{x}$$

$$\frac{dy}{dx} = \frac{2y}{x}$$

Value of a Derivative at specified Values of x

Example 31:

If $f(x) = \frac{x^3 - 2x^2 + 50}{x^2}$ find $f(5)$ and $f(10)$

Soln.

$$f(x) = \frac{x^3}{x^2} - \frac{2x^2}{x^2} + \frac{50}{x^2}$$

$$= x - 2 + \frac{50}{x^2}$$

Differentiating both sides,

$$\frac{50}{x^2} = x^{-2} = -2x^{-2-1} = -2x^{-3} = \frac{-2}{x^3}$$

$$f(x) = 1 + 50\left(\frac{-2}{x^3}\right).$$

$$f(x) = 1 - \frac{100}{x^3}$$

When $x = 5$

$$f(5) = 1 - \frac{100}{(5)^3} = 1 - \frac{100}{125}$$

$$= 1 - 0.8$$

$$f(5) = 0.2$$

When $x = 10$

$$f(10) = 1 - \frac{100}{(10)^3} = 1 - \frac{100}{1000}$$

$$= 1 - 0.1$$

$$f(10) = 0.9$$

Successive Differentiation

If y is a function of x, its derivative $\frac{dy}{dx}$ is some other function of x. $\frac{dy}{dx}$ is called first derivative. The derivative of $\frac{dy}{dx}$ is second derivative. $\frac{d^2y}{dx^2}$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

Example 32:

If $y = ae^{mx} + be^{-mx}$ show that

$$\frac{d^2y}{dx^2} = m^2y$$

Soln

$$y = ae^{mx} + be^{-mx}$$

$$\frac{dy}{dx} = a \cdot me^{mx} + b(-m)e^{-mx}$$

$$= m(ae^{mx} - be^{-mx})$$

$$\frac{dy}{dx} = m(ae^{mx} - be^{-mx})$$

$$\frac{d^2y}{dx^2} = m\left[a \cdot me^{mx} - b(-m)e^{-mx}\right]$$

$$= m\left[ame^{mx} + bme^{-mx}\right]$$

$$= m^2(ae^{mx} + be^{-mx})$$

$$\frac{d^2y}{dx^2} = m^2 y$$

Example 33:

Find the second derivative of $(2x-7)^4$ and its value when $x=5$

Soln.

Let $y = (2x-7)^4$

$y = u^4$ where $u = 2x-7$

$\frac{dy}{du} = 4u^3$ $\frac{du}{dx} = 2$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= 4(2x-7)^3 \cdot 2$$

$$\frac{dy}{dx} = 8(2x-7)^3$$

$$\frac{d^2y}{dx^2} = 8\left[3(2x-7)^2 \cdot (2)\right]$$

$$= 8\left[6(2x-7)^2\right]$$

$$d^2y/dx^2 = 48(2x-7)^2$$

When $x=5$

$$\frac{d^2y}{dx^2} = 48(2(5)-7)^2$$

$$= 48(10-7)^2$$

$$= 48(3)^2 = 48(9)$$

$$\frac{d^2y}{dx^2} = 432$$

Example 34:

Find the fourth derivative of $\log_e \sqrt{3x+4}$

Soln

$$y = \log_e \sqrt{3x+4}$$

$$= \log_e (3x+4)^{1/2}$$

$$y = \frac{1}{2} \log_e (3x+4)$$

$$y = \frac{1}{2} \log_e u \qquad \text{where } u = 3x+4$$

$$\frac{du}{dx} = 3$$

$$\frac{dy}{du} = \frac{1}{2} \cdot \frac{1}{u}$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= \frac{1}{2} \cdot \frac{1}{(3x+4)} \cdot 3$$

$$\frac{dy}{dx} = \frac{3}{2(3x+4)}$$

$$\frac{d^2y}{dx^2} = \frac{3}{2} \frac{d}{dx} \frac{1}{(3x+4)}$$

Let $v = (3x+4)^{-1}$ where $u = 3x+4$

$$\frac{du}{dx} = 3$$

$$v = u^{-1}$$

$$\frac{dv}{du} = -u^{-2}$$

$$\frac{dv}{dx} = \frac{dv}{du} \cdot \frac{du}{dx}$$

$$= -(3x+4)^{-2} \cdot 3$$

$$= \frac{-3}{(3x+4)^2}$$

$$\frac{d^2y}{dx^2} = \left(\frac{3}{2}\right)\left(\frac{-3}{(3x+4)^2}\right)$$

$$= \frac{-9}{2} \frac{1}{(3x+4)^2}$$

$$\frac{d^3y}{dx^2} = \frac{+9}{\cancel{2}} (\cancel{+2}) (3x-4)^{(-3)} (3)$$

$$\frac{d^3y}{dx^3} = 27(3x+4)^{-3}$$

$$\frac{d^4y}{dx^4} = (27)(-3)(3x+4)^{-4}(3)$$

$$= -243(3x+4)^{-4}$$

$$\frac{d^4y}{dx^4} = \frac{-243}{(3x+4)^4}$$

Example 35:

If $y = x e^{x^2}$ find $\frac{d^2y}{dx^2}$ and $\frac{d^3y}{dx^3}$

Soln Let $v = e^{x^2}$

$v = e^u$ where $u = x^2$

$\frac{du}{dx} = 2x$

$$\frac{dv}{du} = e^u$$

$$\frac{dv}{dx} = \frac{dv}{du} \cdot \frac{du}{dx}$$

$$= e^u \cdot 2x$$

$$\frac{d}{dx}(e^{x^2}) = 2xe^{x^2}$$

$$y = xe^{x^2}$$

$$\frac{dy}{dx} = x\frac{d}{dx}(e^{x^2}) + e^{x^2}\frac{d}{dx}(x)$$

$$= x(2xe^{x^2}) + e^{x^2} \cdot 1$$

$$= 2x^2 e^{x^2} + e^{x^2}$$

$$= (2x^2+1)e^{x^2}$$

$$\frac{d^2y}{dx^2} = (2x^2+1)\frac{d}{dx}(e^{x^2}) + e^{x^2}\frac{d}{dx}(2x^2+1)$$

$$= (2x^2+1)(2xe^{x^2}) + e^{x^2}(4x)$$

$$= (4x^3+2x)e^{x^2} + 4xe^{x^2}$$

$$\frac{d^2y}{dx^2} = (4x^3+6x)e^{x^2}$$

$$\frac{d^3y}{dx^3} = (4x^3+6x)\frac{d}{dx}(e^{x^2}) + e^{x^2}\frac{d}{dx}(4x^3+6x)$$

$$= (4x^3+6x)(2xe^{x^2}) + e^{x^2}(12x^2+6)$$

$$= (8x^4+12x^2+12x^2+6)e^{x^2}$$

$$\frac{d^3y}{dx^3} = (8x^4+12x^2+6)e^{x^2}$$

Example 36:

If $y = x + \sqrt{x^2+a^2}$ where show that

$\frac{d^2y}{dx^2} = \frac{1}{2\sqrt{2}a}$ at $x = a$.

Soln

$$y = x + \sqrt{x^2+a^2}$$

$$\frac{dy}{dx} = 1 + \frac{1}{2\sqrt{x^2+a^2}}(2x)$$

$$= \frac{x+\sqrt{x^2+a^2}}{\sqrt{x^2+a^2}}$$

$$\sqrt{x^2+a^2}\frac{dy}{dx} = y$$

$$\sqrt{x^2+a^2}\frac{d^2y}{dx^2} + \frac{dy}{dx}\frac{2x}{2\sqrt{x^2+a^2}} = \frac{dy}{dx}$$

$$\sqrt{x^2+a^2}\frac{d^2y}{dx^2} = \left(1-\frac{x}{x^2+a^2}\right)\frac{dy}{dx}$$

$$\frac{d^2y}{dx^2} = \frac{\sqrt{x^2+a^2}-x}{(x^2+a^2)(x^2+a^2)}\frac{dy}{dx}$$

When $x = a$

$$\frac{d^2y}{dx^2} = \left(\frac{\sqrt{2a^2}-a}{2a^2}\right)\cdot\frac{a+\sqrt{2}a}{\sqrt{2}a}$$

$$= \frac{2a^2-a^2}{2\sqrt{2}a^2}$$

$$= \frac{a^2}{2} \cdot \frac{\sqrt{2}}{a^3}$$

$$\frac{d^2y}{dx^2} = \frac{1}{2\sqrt{2}a}$$

Example 37:

If $y = ax^2 + bx$ Show that $x^2 \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2 = 0$

Soln

$$y = ax^2 + bx$$

$$\frac{dy}{dx} = a(2x) + b(1) = 2ax + b$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = 2a(1) = 2a$$

$$x^2 \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = x^2(2a) - 2x(2ax+b) + (2(ax^2+bx))$$

$$= 2ax^2 - 4ax^2 - 2bx + 2ax^2 + 2bx$$

$$= 0$$

Example 38:

If $y = (x + \sqrt{x^2-1})^m$ Prove that

$$(x^2-1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - m^2y = 0$$

Soln.

$$y = (x + \sqrt{x^2-1})^m$$

$$\frac{dy}{dx} = m(x + \sqrt{x^2-1})^{m-1} \frac{d}{dx}(x + \sqrt{x^2-1})$$

$$= m(x + \sqrt{x^2-1})^{m-1}\left(1 + \frac{2x}{2\sqrt{x^2-1}}\right)$$

$$= m(x + \sqrt{x^2-1})^{m-1}\left(\frac{\sqrt{x^2-1} + x}{\sqrt{x^2-1}}\right)$$

$$= \frac{m(x + \sqrt{x^2-1})^m}{x^2-1}$$

$$\sqrt{x^2-1}\frac{dy}{dx} = m(x + \sqrt{x^2-1})^m$$

Differentiating both sides w.r.t. x

$$\sqrt{x^2-1}\,\frac{d^2y}{dx^2} + \frac{2x}{2\sqrt{x^2-1}}\,\frac{dy}{dx} =$$

$$m.m\left(x+\sqrt{x^2-1}\right)^{m-1}\left(1+\frac{2x}{2\sqrt{x^2-1}}\right)$$

Multiply both side by $\sqrt{x^2-1}$

$$(x^2-1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = m^2y$$

$$(x^2-1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - m^2y = 0$$