DESCRIPTIVE STATISICS
UNIT II
MEASURES OF CENTRAL TENDENCY
An average or measure of central tendency gives a single representative value for a set of usually unequal values. This value is the point around which all the values cluster. So, the measure of central tendency is also called a measure of central location.
Definition
An average is a value which is a representative of a set of data
Various important measures of central tendency are
i) Arithmetic mean
ii) Geometric mean — Mathematical Averages
iii) Harmonic mean
iv) Median and Quartiles
v) Mode — Positional Averages

Objectives or Functions of an average

i. Averages provide a quick understanding of complex data.
ii. Averages enable comparison
iii. Average facilitate sampling techniques.
iv. Averages pave the way for further statistical analysis.
v. Averages establish the relationship between variables.

Characteristics or desirable properties of an average

i. It should be simple to understand and easy to calculate.
ii. An average should be rigidly defined.
iii. It should be based on all items.
iv. It should not be unduly affected by extreme values.
v. It should lend itself for algebraic manipulation.
vi. It should have sampling stability,

ARITHMETIC MEAN
Definition
Arithmetic mean is the total (sum) of all values divided by the number of observations.
Calculation of Arithmetic mean
Raw data
When the observed values are given individually such as $x_1, x_2, x_3 \ldots x_n$ the arithmetic mean is given by

$$\text{Arithmetic mean } \bar{X} = \frac{\text{Total of all values}}{\text{Number of the observations}}$$

$$= \frac{x_1 + x_2 + x_3 + \ldots + x_n}{n}$$

$$= \frac{\Sigma x_i}{n}$$

Given $\bar{X}$ =1600 and n=5 find the total. $\Sigma x_i = n * \bar{X} = 5 * 1600 = 8000$

1. Calculate the arithmetic mean for the following
   1600, 1590, 1560, 1610, 1640, 10

$$\text{Arithmetic mean, } \bar{X} = \frac{1600 + 1590 + 1560 + 1610 + 1640 + 10}{6}$$

$$= \frac{8010}{6} = 1335$$

Calculate Arithmetic mean

S.No.	1	2	3	4	5	6	7	8	9	10
Sales in 1000's(x)	34	55	45	62	48	57	28	57	62	78

$$\text{Arithmetic mean, } \bar{X} = \frac{34+ 55 + 45 + 62 + 48 +57+28+57+62+78}{10}$$

$$= \frac{526}{10} = 52.6 \text{ (average sales)}$$

Discrete data

Let $x_i, x_2, x3 \ldots. x_n$ be the n values of the variable x with corresponding frequency $f_i, f_2, f_3 \ldots. f_n$. then

$$\text{the arithmetic mean } \bar{X} = \frac{x_1.f_1 + x_2. f_2 + x_3.f_3 + \ldots. + x_n . f_n}{f_1 + f_2 + f_3 + \ldots. + f_n}$$

$$= \frac{\Sigma x_i f_i}{\Sigma f_i}$$

Calculate the arithmetic mean

x	f	xf
2	4	2x4= 8
4	6	24
6	10	60
8	12	96
10	8	80
12	7	84
14	3	42
	$\Sigma f_i =$ 50	$\Sigma x_i f_i$ = 394

,

$$\text{Arithmetic mean, } \bar{X} = \frac{\Sigma x_i f_i}{\Sigma f_i} = \frac{394}{50} = 78.8$$

Continuous data

Let $m_i, m_2, m_3 \ldots m_n$ be the mid values of the class interval of the variable x with corresponding frequency $f_i, f_2, f_3 \ldots f_n$. then

$$\text{the arithmetic mean } \bar{X} = \frac{m_1.f_1 + m_2.f_2 + m_3.f_3 + \ldots + m_n.f_n}{f_1 + f_2 + f_3 + \ldots + f_n}$$

$$= \frac{\Sigma m_i f_i}{\Sigma f_i}$$

Calculate the arithmetic mean

Class interval(x)	m	f	mf
20-40	(20+40)/2 = 30	4	120
40-60	50	6	300
60-80	70	10	700
80-100	90	12	1080
100-120	110	8	880
		$\Sigma f_i =$ 40	$\Sigma m_i f_i$ = 3080

$$\text{Arithmetic mean, } \bar{X} = \frac{\Sigma m_i f_i}{\Sigma f_i} = \frac{3080}{40} = 77.0$$

Merits and demerits of Arithmetic mean :

Merits:

1. It is rigidly defined.
2. It is easy to understand and easy to calculate.
3. If the number of items is sufficiently large, it is more accurate and more reliable.
4. It is a calculated value and is not based on its position in the series.
5. It is possible to calculate even if some of the details of the data are lacking.
6. Of all averages, it is affected least by fluctuations of sampling.
7. It provides a good basis for comparison.

Demerits:

1. It cannot be obtained by inspection nor located through a frequency graph.
2. It cannot be in the study of qualitative phenomena not capable of

numerical measurement i.e. Intelligence, beauty, honesty etc.,

3. It can ignore any single item only at the risk of losing its accuracy.
4. It is affected very much by extreme values.
5. It cannot be calculated for open-end classes.
6. It may lead to fallacious conclusions, if the details of the data from which it iscomputed are not given.

MEDIAN

It is the value which divides the data into two equal parts.

Fifty percent of the observations will be less than median value and 50% of the values will be more than the median value.

Calculation

Raw data

Median = value of (n+1)/2 th observation after the values are arranged in ascending order of magnitude.

For example, the median of 20,30,35,64,23,46,78,34,20

Arranging the data in ascending order

20,20,23,30,34,35,46,64,78

Md = value of $(9+1)/2 = 5^{th}$ observation

= 34

Suppose the given number of observations is even then median will be the average of two central values

For example, if the data is the median of 20,30,35,64,23,46,78,34,20,56

Arranging the data in ascending order

20,20,23,30,34,35,46,56,64,78

Md = value of $(10+1)/2 = 5.5^{th}$ observation

= (value of 5^{th} observation + value of 6^{th} observation)/2

= (34+35)/2 =34.5

Discrete data

Md = value of x corresponding to the cumulative frequency just greater than or equal to N/2

1. See the data is in ascending order
2. Find the c.f.
3. Calculate N/2
4. In C.f. column see the value greater than or equal to N/2
5. Md= value of x corresponding to this c.f.

Find the median

x	2	4	6	8	10	12	14
f	4	6	10	12	8	7	3

x	f	c.f
2	4	4
4	6	10
6	10	20
8	12	32
10	8	40
12	7	47
14	3	50
	$\Sigma f_{i} = 50$	

N/2= 50/2= 25.5
Md=8

Continuous data

Md = L + {(N/2 – c.f) x c/f}
L lower limit of the median class
c class interval of the median class
f frequency of the median class
c.f. cumulative frequency of the class preceding the median class
N= Σf_i

Md class is the class corresponding to the c.f. just greater than or equal to N/2

Class interval(x)	f
20-40	4
40-60	6
60-80	10
80-100	12
100-120	8
	$\Sigma f_{i} = 40$

Class interval(x)	f	c.f
20-40	4	4
40-60	6	10
60-80	10	20
80-100	12	32
100-120	8	40
	$\Sigma f_{i} = 40$	

N/2= 40/2 =20
Median class is 60-80
L=60, c=80-60=20, f= 10, c.f= 10

Md = L + {(N/2 – c.f) x c/f}
= 60 + {(20 -10)x (20/10)}
= 60 + {10x2}
=60+20 = 80

marks	No. of students	C.f.
10-25	6	6
25-40	20	26
40-55	44	70
55-70	26	96
70-85	3	99
85-100	1	100
	$\Sigma f_i = 100$	

N/2= 100/2=50
Median class is 40-55
L= 40, f = 44, c = 55 – 40 =15, c.f. = 26

Md = L + {(N/2 – c.f) x c/f}
= 40 + {[50 – 26] x 15/44}
=40 + {(24 x15)/44}
=40 + [360/44]
= 40+8.18 = 48.18

Merits of Median:

1. Median is not influenced by extreme values because it is a positional average.
2. Median can be calculated in case of distribution with open end intervals.
3. Median can be located even if the data are incomplete.
4. Median can be located even for qualitative factors such as ability, honesty etc.

Demerits of Median:

1. A slight change in the series may bring drastic change in median value.
2. In case of even number of items or continuous series, median is an estimated value other than any value in the series.
3. It is not suitable for further mathematical treatment except its use in mean deviation.
4. It is not taken into account all the observations.

QUARTILES

It is the value which divides the data into FOUR equal parts.
There are three quartiles.

Q_1 the first quartile or the lower quartile divides the data in such a way that 25 percent of the observations will be less than Q_1 value and 75% of the values will be more than the Q_1 value.

Q_3 the Third quartile or upper quartile divides the data in such a way that 75 percent of the observations will be less than Q_3 value and 25% of the values will be more than the Q_3 value
The second quartile is nothing but the median.

Fifty percent of the observations will be less than median value and 50% of the values will be more than the median value.

Calculation

Raw data

Median = value of (n+1)/2 th observation after the values are arranged in ascending order of magnitude.

Q_1 = value of (n+1)/4 th observation after the values are arranged in ascending order of magnitude.

Q_3 = value of 3(n+1)/4 th observation after the values are arranged in ascending order of magnitude.

For example, the median of 20,30,35,64,23,46,78,34,20

Arranging the data in ascending order

20,20,23,30,34,35,46,64,78

Md = value of (9+1)/2 =5^{th} observation

= 34

Find Q_1 and Q_3, 20,30,35,64,23,46,78,34,20

Arranging the data in ascending order

20,20,23,30,34,35,46,64,78

Q_1 = value of (9+1)/4 =2.5^{th} observation

= value of 2^{nd} observation + 0.5(3^{rd} value-2^{nd} value)

= 20+0.5(23 -20) =20 + 0.5 x 3 = 20+ 1.5 = 21.5

Q_3 = value of 3(9+1)/4 =7.5th observation

= 7^{th} observation + 0.5 (8^{th}value – 7^{th} value)

= 46 + 0.5 (64-46) =46 +(0.5 x 18) = 46 + 9 = 55

Suppose the given number of observations is even then median will be the average of two central values

For example, if the data is the median of 20,30,35,64,23,46,78,34,20,56

Arranging the data in ascending order

20,20,23,30,34,35,46,56,64,78

Md = value of (10+1)/2 =5.5^{th} observation

= (value of 5^{th} observation + value of 6^{th} observation)/2

= (34+35)/2 =34.5

Find Q_1 and Q_3 20,30,35,64,23,46,78,34,20,56

Solution

Arranging the data in ascending order

20,20,23,30,34,35,46,56,64,78

Q_1 = value of (10+1)/4 =2.75th observation
= 2nd value + 0.75 (3rd value – 2nd value)
= 20 + 0.75 (23-20)
= 20 + (0.75 x3) = 20 +2.25 =22.25

Q_3= value of 3 (n+1\4)th observation
= (3 x2.75 =8.25th) observation
= 8th value + 0.25 (9th value - 8th value)
= 56 + 0.25 (64-56)
=56 + 0.25 (8) = 56 + 2 = 58

Discrete data

Md = value of x corresponding to the cumulative frequency just greater than or equal to N/2

Q_1= value of x corresponding to the cumulative frequency just greater than or equal to N/4

Q_3= value of x corresponding to the cumulative frequency just greater than or equal to 3N/4

1. See the data is in ascending order
2. Find the c.f.
3. Calculate N/2
4. In C.f. column see the value greater than or equal to N/2
5. Md= value of x corresponding to this c.f.

find the median and the quartiles

Find the Quartiles

x	f
2	4
4	6
6	10
8	12
10	8
12	7
14	3
	$\Sigma f_i =$ 50

x	f	c.f.
2	4	4
4	6	10
6	10	20
8	12	32
10	8	40
12	7	47

14	3	50
	$\Sigma f_i = 50$	

$N/2 = 50/2 = 25$
Therefore Md=8

$N/4 = 50/4 = 12.5$
Q_1= value of x corresponding to the cumulative frequency just greater than or equal to N/4 = 20
$Q_1 = 6$

$3 N/4 = 37.5$
Q_3= value of x corresponding to the cumulative frequency just greater than or equal to 3N/4
Q_3= value of x corresponding to the cumulative frequency just greater than 37.5 i.e.,40

$Q_3 = 10$

Continuous data

$Md = L + \{(N/2 – c.f) \times c/f\}$
L lower limit of the median class
c class interval of the median class
f frequency of the median class
c.f. cumulative frequency of the class preceding the median class
$N= \Sigma f_i$

$Q_1 = L_1 + \{(N/4 – c.f_1) \times c_1/f_1\}$
L_1 lower limit of the Q_1 class
c_1 class interval of the Q_1 class
f_1 frequency of the Q_1 class
$c.f._1$ cumulative frequency of the class preceding the Q_1 class
$N= \Sigma f_i$

Q_1 class is the class corresponding to the c.f. just greater than or equal to N/4

$Q_3 = L_3 + \{(3N/4 – c.f_3) \times c_3/f_3\}$
L_3 lower limit of the Q_3 class
c_3 class interval of the Q_3 class
f_3 frequency of the Q_3 class
$c.f._3$ cumulative frequency of the class preceding the Q_3 class
$N= \Sigma f_i$

Q_3 class is the class corresponding to the c.f. just greater than or equal to 3N/4

Calculate the Quartiles and median

Class interval(x)	f
20-40	4
40-60	6
60-80	10
80-100	12
100-120	8
	$\Sigma f_i = 40$

Class interval(x)	f	c.f
20-40	4	4
40-60	6	10
60-80	10	20
80-100	12	32
100-120	8	40
	$\Sigma f_i = 40$	

N/2= 40/2 =20
Median class is 60-80
L=60, c=80-60=20, f= 10, c.f= 10

Md = L + {(N/2 – c.f) x c/f}
= 60 + {(20 -10) x (20/10)}
= 60 + {10x2}
=60+20 = 80

marks	No. of students	C.f.
10-25	6	6
25-40	20	26
40-55	44	70
55-70	26	96
70-85	3	99
85-100	1	100
	$\Sigma f_i = 100$	

$N/2 = 100/2 = 50$
Median class is 40-55
$L = 40$, $f = 44$, $c = 55 – 40 = 15$, c.f. = 26
$Md = L + \{(N/2 – c.f) \times c/f\}$
$= 40 + \{[50 – 26] \times 15/44\}$
$= 40 + \{(24 \times 15)/44\}$
$= 40 + [360/44]$
$= 40 + 8.18 = 48.18$

$Q_1 = L_1 + \{(N/4 – c.f_1) \times c_1/f_1\}$

Q_1 class 25-40, $L_1 = 25$, $c_1 = 40 – 25 = 15$, $f_1 = 20$, $c.f_1 = 6$

$Q_1 = 25 + \{(25 - 6)(15/20)\}$
$= 25 + \{19 \times 15/20\}$
$= 25 + 19 \times 0.75$
$= 25 + 14.25 = 39.25$

$Q_3 = L_3 + \{(3N/4 – c.f_3) \times c_3/f_3\}$
$3N/4 = 3 \times 25 = 75$

Q_3 class is 55-70
$L_3 = 55$, $c_3 = 70 - 55 = 15$, $f_3 = 26$, $c.f_3 = 70$

$Q_3 = L_3 + \{(3N/4 – c.f_3) \times c_3/f_3\}$
$= 55 + \{(75 – 70) \times 15/26\}$
$= 55 + \{5 \times 0.57\}$
$= 55 + 2.88$
$= 57.88.$

MODE

Mode is the value of x which is repeated more often or more frequently

Raw data

Mode is found by observation. The number of times each value occurs ids noted and the value which is repeated maximum number of times is the mode.

Find mode 20,30,35,64,23,46,78,34,20,56
Mode is 20 as it is repeated twice while other values are repeated only once.

Case i) Unimodal – only one mode
For example in the series 40,30,20,17,18,32,29,23,17,17,24,24,12 mode is 17,
Case ii) Bimodal – two modes
For example in the series 40,30,20,17,18,32,29,23,17,17,24,24,12,24,23

mode1 =17, mode 2 is 24,
case iii)
in the series 40,34,45,45,34,40 there is no mode or mode is ill-defined

Discrete data

Mode = value of x corresponding to the highest frequency

Calculate the mode for the following data

x	f
2	4
4	6
6	10
8	12
10	8
12	7
14	3
	$\Sigma f_i =$ 50

Mode = value of x corresponding to the highest frequency 12
Mode =8

Continuous data

Mode = $l + [\ \{(f_1-f_0)/(2f_1- f_0-f_2)\ \}xc]$

Where f_1 is the frequency of the modal class
f_0 is the frequency of the class preceding the modal class
f_2 is the frequency of the class succeeding the modal class
c is the class interval of the modal class
l is the lower limit of the modal class

Modal class is the class corresponding to the highest frequency

Calculate the mode

Marks	10-25	25-40	40-55	55-70	70-85	85-100
No. of students	6	20	44	25	3	1

Solution

marks	No. of students
10-25	6

25-40	20 f_0
40-55	44 f_1
55-70	26 f_2
70-85	3
85-100	1
	$\Sigma f_i = 100$

Modal class is 40 -55

$L = 40$, $f_1 = 44$, $f_0 = 20$, $f_2 = 26$, $c = 55 - 40 = 15$.

Mode = $l + [\{(f_1 - f_0)/(2f_1 - f_0 - f_2)\} \times c]$

$= 40 + [\{(44 - 20)/(2 \times 44 - 20 - 26)\} \times 15]$

$= 40 + \{[24/(88-46)] \times 15\}]$

$= 40 + [(24/42) \times 15]$

$= 40 + [0.5714 \times 15]$

$= 40 + 8.57$

$= 48.57$

Relationship between mean, median and mode

Mode = 3median - 2 mean

Merits of Mode:

1. It is easy to calculate and in some cases it can be located mere inspection
2. Mode is not at all affected by extreme values.
3. It can be calculated for open-end classes.
4. It is usually an actual value of an important part of the series.
5. In some circumstances it is the best representative of data.

Demerits of mode:

1. It is not based on all observations.
2. It is not capable of further mathematical treatment.
3. The Mode is ill-defined, generally, it is not possible to find mode in some cases.
4. As compared with the mean, mode is affected to a great extent, by sampling fluctuations.
5. It is unsuitable in cases where the relative importance of items has to be considered.

GEOMETRIC MEAN

Definition: Geometric mean of n observations is the **n^{th} root of product of n observations.**
If $x_1, x_2, x_3 \ldots x_n$ be the n observations the G.M is $(x_1 * x_2 * x_3 * \ldots * x_n)^{(1/n)}$
For example, the G.M. of 2,4,8 is $(2x4x8)^{(1/3)} = (64)^{(1/3)} = 4$.

But in practice we use log to find G.M.

RAW DATA

If $x_1, x_2, x_3 \ldots x_n$ be the n observations

G.M. = $(x_1 * x_2 * x_3 * \ldots * x_n)^{(1/n)}$

Taking log on both sides

Log (G.M.) = (1/n) [log x_1 + log x_2+ log x_3+… + log x_n]

= (1/n) Σ [log x_i]

G.M. = Antilog { (1/n) Σ [log x_i] }

Find the geometric mean for the following x: 3,6,24,48

x	Log x
3	0.4771
6	0.7782
24	1.3802
48	1.6812
	Σ [log x_i] =4.3167

G.M. = Antilog {(1/n) Σ [log x_i]}
= Antilog {(1/4) x 4.3167}
= Antilog {1.0792}
= 12.00

Discrete data:

Let $x_1, x_2, x_3 \ldots . x_n$ be the n values of the variable x with corresponding frequency $f_1, f_2, f_3 \ldots . f_n$. then

G.M. = Antilog($\frac{\Sigma [f \log x]}{\Sigma f}$)

Find the geometric mean for the data given below

x	10	15	25	40	50
f	4	6	10	7	3

Solution

x	f	log x	f log x
10	4	1.0000	**4.0000**
15	6	1.1761	**7.0566**
25	10	1. 3979	**13.9790**
40	7	1.6021	**11.2147**
50	3	1.6990	**5.0970**
	30		Σ [f log x]=41.3473

G.M. = Antilog($\frac{\Sigma\,[f \log x]}{\Sigma f}$) =A.L. [41.3473/30] = A.L.(1.3782) = 23.89

Continuous data:
Let m_1, m_2, m_3…. m_n be the midpoints of the n classes of the variable x with corresponding frequency f_1, f_2, f_3…. f_n. then
G.M. = Antilog($\frac{\Sigma\,[f \log m]}{\Sigma f}$)

Compute the geometric mean

Marks (x)	0-10	10-20	20-30	30-40	40-50
No. of students(f)	5	7	15	25	8

Solution

x	f	m	log m	f log m
0-10	5	5	**0.6990**	**3.4950**
10-20	7	15	**1.1761**	**8.2327**
20-30	15	25	**1.3979**	**20.9685**
30-40	25	35	**1.5441**	**38.6025**
40-50	8	45	**1.6532**	**13.2256**
	Σf =60			**84.5243**

G.M. = Antilog($\frac{\Sigma\,[f \log m]}{\Sigma f}$) =A.L. [84.5243/60] =A.L. [1.4087] = 25.63

Merits of Geometric mean :

1. It is rigidly defined
2. It is based on all items
3. It is very suitable for averaging ratios, rates and percentages
4. It is capable of further mathematical treatment.
5. Unlike AM, it is not affected much by the presence of extreme values

Demerits of Geometric mean:

1. It cannot be used when the values are negative or if any of the observations is zero
2. It is difficult to calculate particularly when the items are very large or when there is a frequency distribution.
3. It brings out the property of the ratio of the change and not the absolute differenceof change as the case in arithmetic mean.
4. The GM may not be the actual value of the serie

HARMONIC MEAN

Definition:

Harmonic mean is the reciprocal of the arithmetic mean of the reciprocal of observation.

8, 10,40,26

Reciprocals: 8 is 1/8, 10is 1/10, 40 is 1/40 ,26 is 1/26

A,M. of 1/8, 1/10,1/40 and 1/26 is (1/8 + 1/10+1/40+1/26)/4

H.M. = 4/(1/8 + 1/10+1/40+1/26)

RAW DATA

If $x_1, x_2, x_3 \ldots x_n$ be the n observations

$$H.M. = \frac{n}{\Sigma(1/x)}$$

Find the harmonic mean for the following x: 3,6,24,48

x	1/x
3	0.3333
6	0.1667
24	0.0417
48	0.0208
	0.5625

$$H.M. = \frac{n}{\Sigma(1/x)} = \frac{4}{(0.5625)} = 7.11$$

Discrete data:

Let $x_i, x_2, x_3 \ldots . x_n$ be the n values of the variable x with corresponding frequency $f_i, f_2, f_3 \ldots . f_n$. then

$$H.M. = [\frac{\Sigma f}{\Sigma(f/x)}]$$

Find the harmonic mean for the data given below

x	10	15	25	40	50
f	4	6	10	7	3

Solution

x	f	f/x
10	4	**0.4000**
15	6	**0.4000**
25	10	**0.4000**
40	7	**0.1750**
50	3	**0.0600**
	30	**1.4350**

$$H.M. = [\frac{\Sigma f}{\Sigma(f/x)}] = 30/(1.4350) = 20.$$

Continuous data:

Let $m_1, m_2, m_3 \ldots m_n$ be the n values of the variable x with corresponding frequency $f_1, f_2, f_3 \ldots f_n$. then

H.M. = $\left[\frac{\Sigma f}{\Sigma(f/m)} \right]$

Compute the Harmonic mean

Marks (x)	0-10	10-20	20-30	30-40	40-50
No. of students(f)	5	7	15	25	8

Solution

x	f	m	f / m
0-10	5	5	1.0000
10-20	7	15	0.4667
20-30	15	25	0.6000
30-40	25	35	0.7143
40-50	8	45	0.1778
	Σf =60		2.9588

H.M. = $\left[\frac{\Sigma f}{\Sigma(f/m)} \right]$= 60/ 2.9588 = 20.28

Merits of H.M :

1. It is rigidly defined.
2. It is defined on all observations.
3. It is amenable to further algebraic treatment.
4. It is the most suitable average when it is desired to give greater weight to smaller observations and less weight to the larger ones.

Demerits of H.M :

2. It is not easily understood.
3. It is difficult to compute.
4. It is only a summary figure and may not be the actual item in the series

It gives greater importance to small items and is therefore, useful only when small items have to be given greater weightage

Weighted averages

The relative importance given to the values is the weights W

Weighted arithmetic mean

$\bar{x}_w = \frac{\Sigma\,[xw]}{\Sigma w}$ x is the variable and w is the weights

Find the weighted arithmetic mean for the following data

X	8	12	25	13	45
w	2	5	1	2	3

Solution

X	**w**	**xw**
8	**2**	**16**
12	**5**	**60**
25	**1**	**25**
13	**2**	**26**
45	**3**	**135**
	13	**262**

$\bar{\boldsymbol{x}}_{\mathrm{w}} = \frac{\Sigma\,[xw]}{\Sigma w}$ = 262 /13 =20.15

Weighted G.M. = Antilog($\frac{\Sigma\,[w \log x]}{\Sigma w}$)

Calculate weighted geometric mean

commodity	A	B	C	D
Weight	1	6	3	2
Price	5	17	30	42

Solution

x	w	Log x	w logx
5	1	0.6990	0.6990
17	6	1.2304	7.3824
30	3	1.4771	4.4313
42	2	1.6232	3.2464
	12		15.7591

Weighted G.M. = Antilog($\frac{\Sigma\,[w \log x]}{\Sigma w}$)

= Antilog($\frac{[15.7591]}{12}$) =A.L. (1.3133) = 20.57

Weighted H.M. = [$\frac{\Sigma\, w}{\Sigma(w/x)}$]

1.An aeroplane flies around a square the sides of which measures 100 km each, it covers the first side at an average speed of 100 km. /hr. the second side at 200km/hr and the third with 300 kms/hr and the fourth side at 400 kms./hr. Use the correct mean to find the average speed round the square.

The average speed round the entire square is the harmonic mean of 100, 200, 300, 400.

H.M. = $\left[\frac{\text{n}}{\Sigma(1/x)} \right] = \frac{4}{\frac{1}{100}+\frac{1}{200}+\frac{1}{300}+\frac{1}{400}} = \frac{4}{0.0100+0.0050+0.0033+0.0025}$

H.M. = 4 / 0.0208 = 192 kms, /hr

2. You can take a trip which entails travelling 900 kms, by train at an average speed of 60km. /hr., 3000 kms by ship at an average speed of 25km./hr., 400 kms by plane at 350km./hr. and finally 15 kms by taxi at an average speed of 25km./hr, what is the average speed for the entire distance.

Mode of travel	Distance travelled(w)	Speed (x)	w/x
Train	900	60	15.0000
Ship	3000	25	120.0000
Plane	400	350	1.1429
taxi	15	25	0.6000
	4315		136.7429

Weighted H.M. is the best average to find the average speed

Weighted H.M. = $\left[\frac{\Sigma \text{ w}}{\Sigma(w/x)} \right] = \left[\frac{4315}{136.7429} \right] = 31.56$

The average speed of the entire distance is 31.56 kms. /hr.