Year	Subject Title	Sem	Sub Code
2018–19 Onwards	II B.Sc Psychology - STATISTICS - II	IV	

Objective: To impart the basic knowledge of Statistical tools and their applications in Psychology.

UNIT I

Probability Distribution – Binomial, Poisson and Normal Distributions – Properties and Applications (without Proof) – Simple Problems.

UNIT II

Sampling – Advantages and Disadvantages – Simple Random Sampling – Stratified Random Sampling – Systematic Sampling – (Concept Only) – Sampling Distribution – Standard Error – Tests of Significance – Type I and Type II Errors – Large Sample Tests for Single Mean and Two Means. Tests for single proportion and difference of two proportions.

UNIT III

Small Sample Tests – Test for Single Mean and Two Means – Paired 't' Test Chi-Square Test for Independence of Attributes. Association of Attributes – Contingency Tables – Methods of Studying Association – Yule's Coefficient of Association

UNIT IV

Measurement and scaling techniques- Categorical variables-Data types-Metric, Interval and Ratio data. Non-Metric data- Nominal, ordinal data. Scales of measurement -Comparative scale, paired Comparison scale, rank order scale, constant sum scale, Non-comparative scale- continuous rating scale, Itemized rating scale- Likert scale, Guttmann scale

UNIT V

Non – Parametric Tests– Introduction advantages and disadvantages. Run test, Sign test, Median test, Mann-Whitney U test(one sample only) Kolmogrov Smirnov test(two samples).

Text Books:

1. R.S.N. Pillai and V. Bagavathi - Statistics – Theory and Practice, S.Chand & Sons Company Ltd, New Delhi.
2. S.C.Gupta and V.K.Kapoor - Fundamentals of Applied Statistics, Sultan Chand & Sons, New Delhi, 11th revised Edition, June 2012.
3. J.P Verma and Mohammed Ghufran- Statistics for Psychology, Tata Mcgraw Hill Education (P)Ltd. New Delhi.

Standard distributions

Introduction:

In this chapter we will discuss theoretical discrete distributions. When the values of the variables are distributed according to some definite probability law which can be expressed mathematically, such distributions are called theoretical discrete distributions. The important theoretical discrete distributions are

(i) Binomial
(ii) Poisson
(iii) Normal

Binomial Distribution:

Binomial dist. was discovered by James Bernoulli (1654-1705) in the year 1700 and was first published posthumously in 1713, (eight years after his death)

Let the random experiment be performed repeatedly and, let each trial has two possible outcomes generally called as let the occurrence of an event in a trial be called a success and its non-occurrence a failure

Ex., tossing of a coin (head or tail), Performance of a student in an examination (pass or fail)

Consider a set of n independent trials (n being finite), in which the prob. p of success in any trial is constant for each trial. Then $q = 1-p$, is the prob. of failure in any trial.

The prob. of x successes and consequently $(n-x)$ failures in n independent trials, in a specified order (say) SSFSFFFS ... FSF (where S represents success and ... by the compound prob, then, by the expression

$P(SSFSFFFS \ldots FSF) = P(S)\,P(S)\,P(F)\,P(S) \ldots P(S)\,P(F)$

$= p \cdot p \cdot q \cdot p \ldots p \cdot q$

$= \underbrace{p \cdot p \ldots p}_{x \text{ factors}}\ \underbrace{q \cdot q \ldots q}_{n-x \text{ factors}} = p^x q^{n-x}$

But x successes in n trials can occur in $\binom{n}{x}$ ways and the prob, for each of these ways is $p^x q^{n-x}$. Hence the prob, of x successes in n trials in any order whatsoever is given by the addition thm, of prob, by the expression

$$\binom{n}{x} p^x \cdot q^{n-x}$$

The prob, dist, of the number of successes so obtained is called Binomial probability dist.,

Definition: Let x be the number of success in n repeated Bernoulli indep., trials with prob., p of success for each trial.

A r.v x is said to follow binomial dist., if it assumes only non-negative values and its prob., mass func., is given by,

$$P(X=x) = p(x) = \begin{cases} \binom{n}{x} p^x q^{n-x} \ ; & x = 0,1,2 \ldots n \ ; \ q = 1-p \\ 0, & \text{otherwise} \end{cases}$$

The two indep., constants n and p in the dist., are known as the parameters of the dist.,

Note: Binomial dist., is a discrete dist, as x can take only the integer values viz., $0, 1, \ldots n$. Any variable which follows binomial dist., is known as binomial variate.

Characteristics of Binomial Distribution:

1. Binomial distribution is a discrete distribution in which the random variable X (the number of success) assumes the values 0,1, 2, ….n, where n is finite.
2. Mean = np, variance = npq and standard deviation $\sigma = \sqrt{npq}$,

   Coefficient of skewness $= \frac{q-p}{\sqrt{npq}}$,

   Coefficient of kurtosis $= \frac{1 - 6pq}{npq}$, clearly each of the probabilities is non-negative and sum of all probabilities is 1 ($p < 1$, $q < 1$ and $p + q = 1$, $q = 1 - p$).
3. The mode of the binomial distribution is that value of the variable which occurs with the largest probability. It may have either one or two modes.
4. If two independent random variables X and Y follow binomial distribution with parameter (n_1, p) and (n_2, p) respectively, then their sum (X+Y) also follows Binomial distribution with parameter $(n_1 + n_2, p)$

(4)

5. If n independent trials are repeated N times, N sets of n trials are obtained and the expected frequency of x success is $N(nC_x\ p^x\ q^{n-x})$. The expected frequencies of 0,1,2... n success are the successive terms of the binomial distribution of $N(p+q)^n$

Example 1:

Comment on the following: " The mean of a binomial distribution is 5 and its variance is 9"

Solution:

The parameters of the binomial distribution are n and p

We have mean $\Rightarrow np = 5$

Variance $\Rightarrow npq = 9$

$$\therefore q = \frac{npq}{np} = \frac{9}{5}$$

$$q = \frac{9}{5} > 1$$

Which is not admissible since q cannot exceed unity. Hence the given statement is wrong.

Example 2:

Eight coins are tossed simultaneously. Find the probability of getting atleast six heads.

Solution:

Here number of trials, n = 8, p denotes the probability of getting a head.

$$\therefore p = \frac{1}{2} \text{ and } q = \frac{1}{2}$$

If the random variable X denotes the number of heads, then the probability of a success in n trials is given by

$$P(X = x) = nc_x\ p^x\ q^{n-x},\quad x = 0, 1, 2, ..., n$$

$$= 8C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{8-x} = 8C_x \left(\frac{1}{2}\right)^8$$

$$= \frac{1}{2^8}\ 8C_x$$

Probability of getting atleast six heads is given by

$$P(x \geq 6) = P(x = 6) + P(x = 7) + P(x = 8)$$

$$= \frac{1}{2^8}\, 8C_6 + \frac{1}{2^8}\, 8C_7 + \frac{1}{2^8}\, 8C_8$$

$$= \frac{1}{2^8}\, [\, 8C_6 + 8C_7 + 8C_8]$$

$$= \frac{1}{2^8}\, [\, 28 + 8 + 1] \quad = \frac{37}{256}$$

Example 3:

Ten coins are tossed simultaneously. Find the probability of getting (i) atleast seven heads (ii) exactly seven heads (iii) atmost seven heads

Solution:

p = Probability of getting a head $= \frac{1}{2}$

q = Probability of not getting a head $= \frac{1}{2}$

The probability of getting x heads throwing 10 coins simultaneously is given by

$$P(X = x) = nC_x\, p^x\, q^{n-x}. \quad , \quad x = 0, 1, 2, \ldots, n$$

$$= 10C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{10-x} = \frac{1}{2^{10}}\, 10C_x$$

i) Probability of getting atleast seven heads

$$P(x \geq 7) = P(x = 7) + P(x = 8) + P(x = 9) + P(x = 10)$$

$$= \frac{1}{2^{10}}\, [\, 10C_7 + 10C_8 + 10C_9 + 10C_{10}]$$

$$= \frac{1}{1024}\, [\, 120 + 45 + 10 + 1] = \frac{176}{1024}$$

ii) Probability of getting exactly 7 heads

$$P(\, x = 7) = \frac{1}{2^{10}}\, 10C_7 = \frac{1}{2^{10}}(120)$$

$$= \frac{120}{1024}$$

iii) Probability of getting atmost 7 heads

$$P(x \leq 7) = 1 - P(x > 7)$$
$$= 1 - \{ P(x = 8) + P(x = 9) + P(x = 10)\}$$
$$= 1 - \frac{1}{2^{10}} \{10C_8 + 10C_9 + 10C_{10}\}$$
$$= 1 - \frac{1}{2^{10}} [45 + 10 + 1]$$
$$= 1 - \frac{56}{1024}$$
$$= \frac{968}{1024}$$

Example 4:

20 wrist watches in a box of 100 are defective. If 10 watches are selected at random, find the probability that (i) 10 are defective (ii) 10 are good (iii) at least one watch is defective (iv) at most 3 are defective.

Solution:

20 out of 100 wrist watches are defective

Probability of defec.ive wrist watch , $p = \frac{20}{100} = \frac{1}{5}$

$$\therefore q = 1 - p = \frac{4}{5}$$

Since 10 watches are selected at random, n =10

$$P(X = x) = nC_x\, p^x\, q^{n-x} \quad , \quad x = 0, 1, 2, \ldots, 10$$
$$= 10C_x \left(\frac{1}{5}\right)^x \left(\frac{4}{5}\right)^{10-x}$$

) Probability of selecting 10 defective watches

$$P(x = 10) = 10C_{10} \left(\frac{1}{5}\right)^{10} \left(\frac{4}{5}\right)^0$$
$$= 1 . \frac{1}{5^{10}} . 1 = \frac{1}{5^{10}}$$

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ii) Probability of selecting 10 good watches (i.e. no defective)

$$P(x = 0) = 10C_0 \left(\frac{1}{5}\right)^0 \left(\frac{4}{5}\right)^{10}$$

$$= 1.1. \left(\frac{4}{5}\right)^{10} = \left(\frac{4}{5}\right)^{10}$$

iii) Probability of selecting at least one defective watch

$$P(x \geq 1) = 1 - P(x < 1)$$

$$= 1 - P(x = 0)$$

$$= 1 - 10C_0 \left(\frac{1}{5}\right)^0 \left(\frac{4}{5}\right)^{10}$$

$$= 1 - \left(\frac{4}{5}\right)^{10}$$

iv) Probability of selecting at most 3 defective watches

$$P(x \leq 3) = P(x = 0) + P(x = 1) + P(x = 2) + P(x = 3)$$

$$= 10C_0\left(\frac{1}{5}\right)^0 \left(\frac{4}{5}\right)^{10} + 10C_1\left(\frac{1}{5}\right)^1 \left(\frac{4}{5}\right)^9 + 10C_2\left(\frac{1}{5}\right)^2 \left(\frac{4}{5}\right)^8$$

$$+ 10C_3 \left(\frac{1}{5}\right)^3 \left(\frac{4}{5}\right)^7$$

$$= 1.1. \left(\frac{4}{5}\right)^{10} + 10\left(\frac{1}{5}\right)^1 \left(\frac{4}{5}\right)^9 + \frac{10.9}{1.2} \left(\frac{1}{5}\right)^2 \left(\frac{4}{5}\right)^8$$

$$+ \frac{10.9.8}{1.2.3} \left(\frac{1}{5}\right)^3 \left(\frac{4}{5}\right)^7$$

$$= 1. (0.107) + 10 (0.026) + 45 (0.0062) + 120 (0.0016)$$

$$= 0.859 \text{ (approx)}$$

Example 5:

With the usual notation find p for binomial random variable X if n = 6 and 9P(X = 4) = P(X = 2)

Solution:

The probability mass function of binomial random variable X is given by

$$P(X = x) = nC_x\, p^x\, q^{n-x}, \quad x = 0, 1, 2, \ldots, n$$

Examples: (8)

For a Binomial dist., with parameters $n=5$, $p=0.3$ find the prob. of getting (i) atleast 3 successes (ii) atmost 3 successes iii) exactly 3 failures.

Solution:

The binomial dist., is $(0.7+0.3)^5$.

(i) Prob. of atleast 3 successes.

$= P(X=3) + P(X=4) + P(X=5)$

$= 5_{C_3}(0.3)^3(0.7)^2 + 5_{C_4}(0.3)^4(0.7) + (0.3)^5$

$= 0.1631$

(ii) Prob. of atmost 3 successes

$= P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$= (0.7)^5 + 5_{C_1}(0.7)^4(0.3)^1 + 5_{C_2}(0.7)^3(0.3)^2 + 5_{C_3}(0.7)^2(0.3)^3$

$= 0.9692$

(iii) Proba. for exactly 3 fail,

= the prob. for exactly 2 successes

$= P(X=2)$

$= 5_{C_2}(0.7)^2(0.3)^3$

$= 0.3087$

In a throw of a die, 5 or 6 is considered a success. Find the mean no. of successes and S.D in eight throws of a die

Solution:

$n = 8$

p = prob. of success $= \frac{2}{6} = \frac{1}{3}$.

$\therefore q = 1 - p = 1 - \frac{1}{3} = \frac{2}{3}$.

Mean $= np = 8 \times 1/3 = 8/3$

S.D $= \sqrt{npq} = \sqrt{8 \times 1/3 \times 2/3} = 4/3$.

Find the bin. dist, for which the [illegible] 4 and variance is 3 (9)

Solution:

We know that, for binomial dist

Mean $= np$

Variance $= npq$

Given Mean $= 4$

(ie) $np = 4$ ——— (1)

Variance $npq = 3$ ——— (2)

$4q = 3 \Rightarrow q = 3/4$

$\therefore p = 1 - q = 1 - 3/4 = 1/4$

Subs, $p = 1/4$ in (1) we get,

$n = \frac{4}{p} = 16$

The required bino. dist, is

$p(x) = 16c_x \left(\frac{1}{4}\right)^x \left(\frac{3}{4}\right)^{n-x}$

6 dice are thrown 729 times How many times do you expect atleast 3 dice to show a five or a six?

Solution:

p = prob, of getting 5 or 6 with one die

$= \frac{2}{6} = \frac{1}{3}$

$\therefore q = 1 - p = 1 - \frac{1}{3} = \frac{2}{3}$

P (atleast three dice showing five or six)

$= P(x \geq 3)$

$= P(3) + P(4) + P(5) + P(6)$

$= \frac{160 + 60 + 12 + 1}{3^6} = \frac{233}{3^6}$

For 729 times the expected no, of times atleast 3 dice showing five or six

Poisson distribution:

The poisson dist, was first discovered by a French Mathematician S D Poisson in 1837. The approximation of binomial when n is large and p is close to zero is called the poisson dist,.

Definition:

A r.v x is said to follow a poisson dist., if it assumes only non-negative values and its prob., mass fun., is given by,

$$p(X=x) = p(x) = \frac{e^{-\lambda}\lambda^x}{x!}\ ; \quad x = 0,1,2,\dots n,\dots$$

Here λ is known as the parameter of the dist., and $\lambda > 0$

Note:

(i) The bino., dist, is characterised by two parameters p.n while the poisson dist, is chara by a single parameter λ.

(ii) The sample space for the bino., dist, is $\{0,1,2 \dots n\}$ while for the poisson dist, is $\{0,1,2,\dots n,\dots\}$

(iii) Total prob, is one.

Proof:

(i.e) $\Sigma p(x) = 1$

$$\sum_0^\infty \frac{e^{-\lambda}.\lambda^x}{x!} = e^{-\lambda}\left(1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \dots\right)$$

$$= e^{-\lambda}.e^{\lambda}$$

$$=1$$

(iv) Poisson dt., is a limiting case of binomial dist, under the foll, condition (i) n is Large (ii) p is very small (iii) $np = \lambda$.

Applications of a poisson dist,:

(i) It is used in quality control to count the no, of defects in an item.

(ii) No. of deaths due in the cases

(iii) No. of mistakes committed by a typist per page

Assumption:

(i) The variable is discrete.

(ii) The events can only be either a success or failure.

(iii) The no. of trials is finite & large.

(iv) The prob. of success p is so small that q is almost equal to unity

(v) The trials are indep. to each other

Moments:

Raw moments:

$$E(X) = \text{Mean} = \mu_1' = \Sigma x\, p(x) = \sum_0^\infty x \frac{e^{-\lambda}\lambda^x}{x!}$$

$$= e^{-\lambda} \sum_0^\infty \frac{\lambda^x \cdot x}{x!}$$

$$= e^{-\lambda} \sum_1^\infty \frac{x \cdot \lambda^{x-1} \cdot \lambda}{x(x-1)!}$$

$$= e^{-\lambda}\, \lambda \sum_1^\infty \frac{\lambda^{x-1}}{(x-1)!}$$

$$= \lambda e^{-\lambda}\left(1 + \lambda + \frac{\lambda^2}{2!} + \dots\right)$$

$$= \lambda \cdot e^{-\lambda} \cdot e^{\lambda} = \lambda$$

$$\mu_1' = \lambda$$

$$E(X^2) = \mu_2' = \sum_0^\infty x^2 p_x = \sum_0^\infty [x(x-1) + x]\, p(x)$$

$$= \Sigma [x(x-1) + x] \frac{e^{-\lambda}\lambda^x}{x!}$$

1!

Some examples of Poisson variates are :

1. The number of blinds born in a town in a particular year.
2. Number of mistakes committed in a typed page.
3. The number of students scoring very high marks in all subjects
4. The number of plane accidents in a particular week.
5. The number of defective screws in a box of 100 manufactured by a reputed company.
6. Number of suicides reported in a particular day.

probability of success p of an event is very small and that of failure q is very high almost equal to 1 and n is very large.

3. The parameter: The parameter of the Poisson distribution is m. If the value of m is known, all the probabilities of the Poisson distribution can be ascertained.
4. Values of Constant: Mean = m = variance; so that standard deviation = $\sqrt{m}$

   Poisson distribution may have either one or two modes.
5. Additive Property: If X and Y are two independent Poisson distribution with parameter m_1 and m_2 respectively. Then (X+Y) also follows the Poisson distribution with parameter $(m_1 + m_2)$
6. As an approximation to binomial distribution: Poisson distribution can be taken as a limiting form of Binomial distribution when n is large and p is very small in such a way that product np = m remains constant.
7. Assumptions: The Poisson distribution is based on the following assumptions.
   i) The occurrence or non- occurrence of an event does not influence the occurrence or non-occurrence of any other event

ii) The probability of success for a short time interval or a small region of space is proportional to the length of the time interval or space as the case may be.

iii) The probability of the happening of more than one event is a very small interval is negligible.

Example 8:

Suppose on an average 1 house in 1000 in a certain district has a fire during a year. If there are 2000 houses in that district, what is the probability that exactly 5 houses will have a fire during the year? [given that $e^{-2} = 0.13534$]

Mean, $\bar{x} = np$, $n = 2000$ and $p = \dfrac{1}{1000}$

$$= 2000 \times \frac{1}{1000}$$

$$m = 2$$

The Poisson distribution is

$$P(X=x) = \frac{e^{-m} m^x}{x!}$$

$$\therefore P(X=5) = \frac{e^{-2} 2^5}{5!}$$

$$= \frac{(0.13534) \times 32}{120}$$

$$= 0.036$$

(Note: The values of e^{-m} are given in Appendix)

Example 9:

In a Poisson distribution $3P(X=2) = P(X=4)$ Find the parameter 'm'.

Solution:

Poisson distribution is given by $P(X=x) = \dfrac{e^{-m} m^x}{x!}$

Given that $3P(x=2) = P(x=4)$

$$3.\ \frac{e^{-m} m^2}{2!} = \frac{e^{-m} m^4}{4!}$$

$$m^2 = \frac{3 \times 4!}{2!}$$

$$\therefore \quad m = \pm 6$$

Since mean is always positive $\therefore$ m = 6

Example 10:

If 2% of electric bulbs manufactured by a certain company are defective. Find the probability that in a sample of 200 bulbs i) less than 2 bulbs ii) more than 3 bulbs are defective.[$e^{-4} = 0.0183$]

Solution:

The probability of a defective bulb = $p = \frac{2}{100} = 0.02$

Given that n = 200 since p is small and n is large

We use the Poisson distribution

mean, $m = np = 200 \times 0.02 = 4$

Now, Poisson Probability function, $P(X = x) = \frac{e^{-m} m^x}{x!}$

i) Probability of less than 2 bulbs are defective

$$= P(X<2)$$

$$= P(x = 0) + P(x = 1)$$

$$= \frac{e^{-4} 4^0}{0!} + \frac{e^{-4} 4^1}{1!}$$

$$= e^{-4} + e^{-4}(4)$$

$$= e^{-4}(1 + 4) = 0.0183 \times 5$$

$$= 0.0915$$

ii) Probability of getting more than 3 defective bulbs

$$P(x > 3) = 1 - P(x \leq 3)$$

$$= 1 - \{P(x = 0) + P(x = 1) + P(x=2) + P(x=3)\}$$

$$= 1 - e^{-4}\{1 + 4 + \frac{4^2}{2!} + \frac{4^3}{3!}\}$$

$$= 1 - \{0.0183 \times (1 + 4 + 8 + 10.67)\}$$

$$= 0.567$$

In the preceding sections we have discussed the discrete distributions, the Binomial and Poisson distribution.

In this section we deal with the most important continuous distribution, known as normal probability distribution or simply normal distribution. It is important for the reason that it plays a vital role in the theoretical and applied statistics.

The normal distribution was first discovered by DeMoivre (English Mathematician) in 1733 as limiting case of binomial distribution. Later it was applied in natural and social science by Laplace (French Mathematician) in 1777. The normal distribution is also known as Gaussian distribution in honour of Karl Friedrich Gauss(1809).

3.3.1 Definition:

A continuous random variable X is said to follow normal distribution with mean μ and standard deviation σ, if its probability density function

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad ;-\infty < x < \infty \ , \ -\infty < \mu < \infty, \ \sigma > 0.$$

(17)

Note:

The mean μ and standard deviation σ are called th parameters of Normal distribution. The normal distribution expressed by $X \sim N(\mu, \sigma^2)$

3.3.2 Condition of Normal Distribution:

i) Normal distribution is a limiting form of the binomial distribution under the following conditions.

a) n, the number of trials is indefinitely large ie., $n \rightarrow \infty$ and

b) Neither p nor q is very small.

ii) Normal distribution can also be obtained as a limiting form of Poisson distribution with parameter $m \rightarrow \infty$

iii) Constants of normal distribution are mean $= \mu$, variation $= \sigma^2$ Standard deviation $= \sigma$.

3.3.3 Normal probability curve:

The curve representing the normal distribution is called the

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3.3.4 Properties of normal distribution:

1. The normal curve is bell shaped and is symmetric at $x = \mu$.
2. Mean, median, and mode of the distribution are coincide i.e., Mean = Median = Mode = μ
3. It has only one mode at $x = \mu$ (i.e., unimodal)
4. Since the curve is symmetrical, Skewness = $\beta_1 = 0$ and Kurtosis = $\beta_2 = 3$.
5. The points of inflection are at $x = \mu \pm \sigma$
6. The maximum ordinate occurs at $x = \mu$ and its value is $= \frac{1}{\sigma\sqrt{2\pi}}$
7. The x axis is an asymptote to the curve (i.e. the curve continues to approach but never touches the x axis)
8. The first and third quartiles are equidistant from median.
9. The mean deviation about mean is $0.8\,\sigma$
10. Quartile deviation = $0.6745\,\sigma$
11. If X and Y are independent normal variates with mean μ_1 and μ_2, and variance σ_1^2 and σ_2^2 respectively then their sum $(X + Y)$ is also a normal variate with mean $(\mu_1 + \mu_2)$ and variance $(\sigma_1^2 + \sigma_2^2)$
12. Area Property $P(\mu - \sigma < x < \mu + \sigma) = 0.6826$
    $P(\mu - 2\sigma < x < \mu + 2\sigma) = 0.9544$
    $P(\mu - 3\sigma < x < \mu + 3\sigma) = 0.9973$

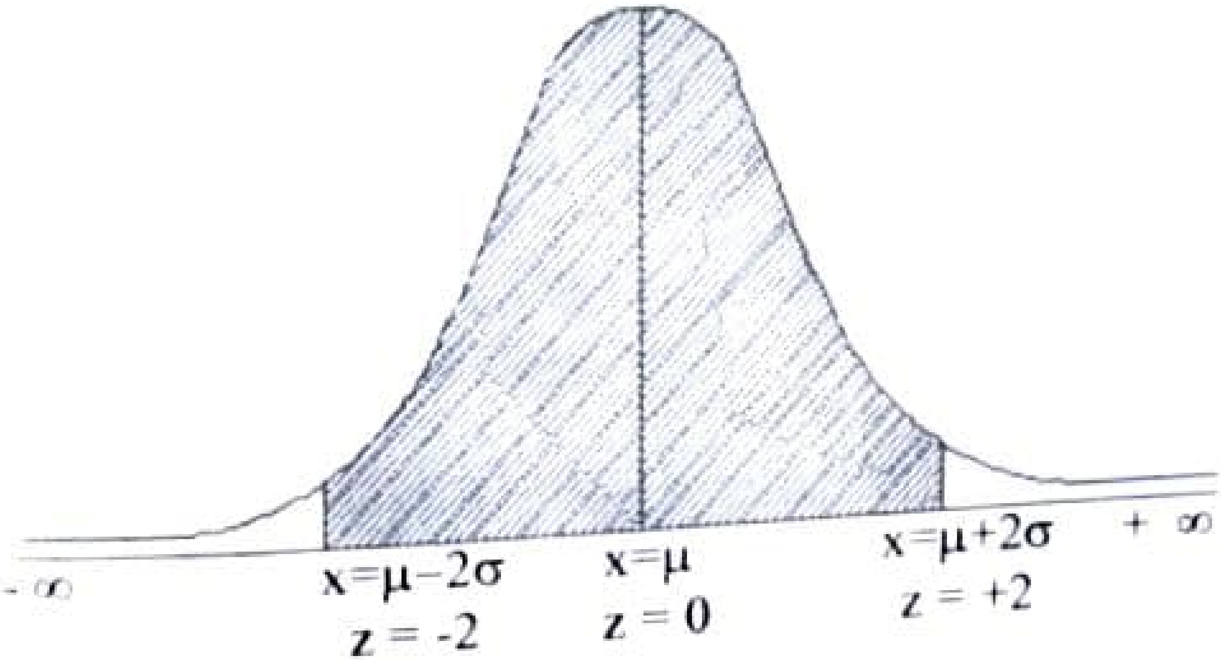
3.3.5 Standard Normal distribution:

Let X be random variable which follows normal distribution with mean μ and variance σ^2. The standard normal variate is defined as $Z = \frac{X-\mu}{\sigma}$ which follows standard normal distribution with mean 0 and standard deviation 1 i.e., $Z \sim N(0,1)$. The standard normal distribution is given by $\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$; $-\infty < z < \infty$

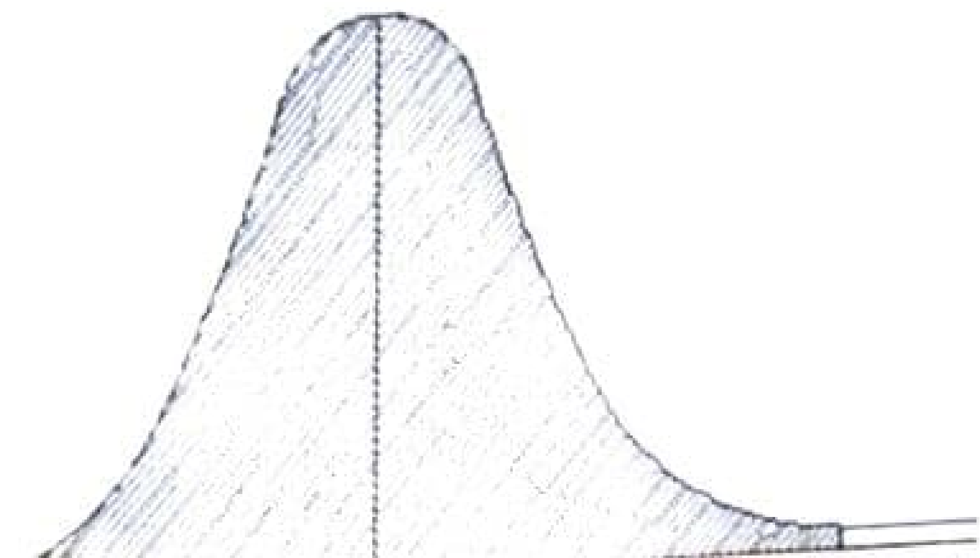
The advantage of the above function is that it doesn't contain any parameter. This enable us to compute the area under the normal probability curve.

$$P(\mu - \sigma < x < \mu+\sigma) = P(-1 \le z \le 1)$$
$$= 2P(0 < z < 1)$$
$$= 2\,(0.3413) \quad \text{(from the area table)}$$
$$= 0.6826$$

$$P(\mu - 2\sigma < x < \mu+2\sigma) = P(-2 < z < 2)$$
$$= 2P(0 < z < 2)$$
$$= 2\,(0.4772) = 0.9544$$



$$P(\mu - 3\sigma < x < \mu + 3\sigma) = P(-3 < z < 3)$$
$$= 2P(0 < z < 3)$$
$$= 2\,(0.49865) = 0.9973$$



...probability that a normal variate x lies outside the range $\mu \pm 3\sigma$ is given by

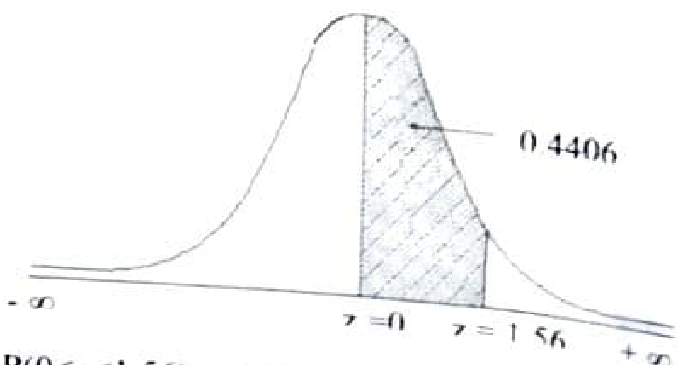
$$P(|x - \mu| > 3\sigma) = P(|z| > 3)$$
$$= 1 - P(-3 \le z \le 3)$$
$$= 1 - 0.9773 = 0.0027$$

Thus we expect that the values in a normal probability curve will lie between the range $\mu \pm 3\sigma$, though theoretically it range from $-\infty$ to ∞.

Example 15:

Find the probability that the standard normal variate lies between 0 and 1.56

Solution:

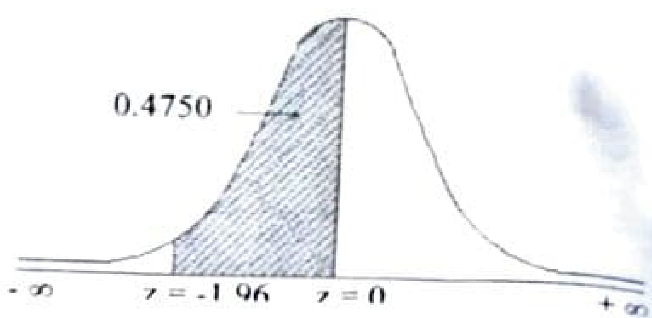


$P(0<z<1.56) =$ Area between $z = 0$ and $z = 1.56$
$= 0.4406$ (from table)

Example 16:

Find the area of the standard normal variate from -1.96 to 0.

Solution:



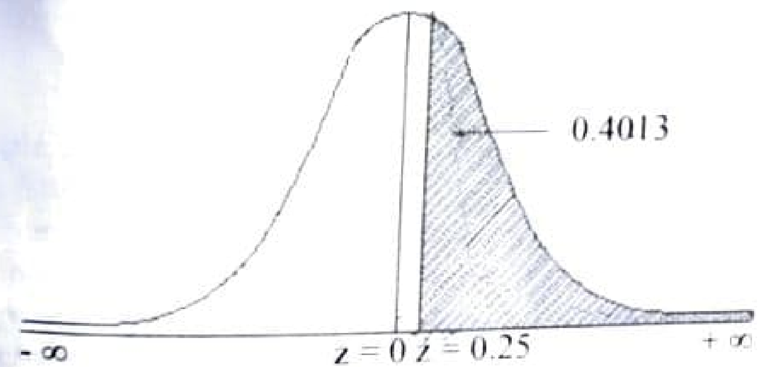
Area between $z = 0$ & $z = 1.96$ is same as the area $z = -1.96$ to $z = 0$

$P(-1.96 < z < 0) = P(0 < z < 1.96)$ (by symmetry)
$= 0.4750$ (from the table)

Example 17:

Find the area to the right of $z = 0.25$

Solution:

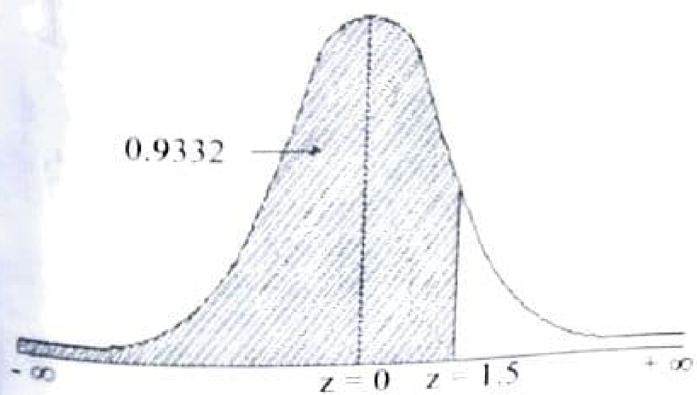


$P(z > 0.25) = P(0 < z < \infty) - P(0 < z < 0.25)$
$= 0.5000 - 0.0987$ (from the table) $= 0.4013$

Example 18:

Find the area to the left of $z = 1.5$

Solution:



$P(z < 1.5) = P(-\infty < z < 0) + P(0 < z < 1.5)$
$= 0.5 + 0.4332$ (from the table)
$= 0.9332$