PROBABILITY

Probability Terms and Definition

- Experiment: Any phenomenon like rolling a dice, tossing a coin, drawing a card from a well-shuffled deck, etc.
- Outcome: The Result of any event; like number appearing on a dice, side of a coin, drawn out card, etc.
- Sample Space: The set of all possible outcomes.
- Event: Any combination of possible outcomes or the subset of sample space; like getting an even number on rolled dice, getting a head/tail on a flipped coin, drawing out a king/queen/ace of any suit.

Some of the important probability terms are discussed here:

Term	**Definition**	**Example**
Sample Space	The set of all the possible outcomes to occur in any trial	1. Tossing a coin, Sample Space (S) = {H,T} 2. Rolling a die, Sample Space (S) = {1,2,3,4,5,6}
Sample Point	It is one of the possible results	In a deck of Cards: • 4 of hearts is a sample point. • the queen of clubs is a sample point.
Experiment or Trial	A series of actions where the outcomes are always uncertain.	The tossing of a coin, Selecting a card from a deck of cards, throwing a dice.
Event	It is a single outcome of an experiment.	Getting a Heads while tossing a coin is an event.

Term	Definition	Example
Outcome	Possible result of a trial/experiment	T (tail) is a possible outcome when a coin is tossed.
Complimentary event	The non-happening events. The complement of an event A is the event, not A (or A’)	Standard 52-card deck, A = Draw a heart, then A’ = Don’t draw a heart
Impossible Event	The event cannot happen	In tossing a coin, impossible to get both head and tail at the same time

Mutually exclusive events A and B are said to be mutually exclusive if both of them does not occur at the same time For example getting an odd number and getting an even number in throwing a dice

Probability Formulas

Probability of A = (Number of a Favorable outcomes to event A) / (Total number of outcomes)

P = n (A) / n (S)

Where P is the probability, A is the event and S is the sample space. Now, let’s looks at some very common examples.

Example 1: Probability of getting an even number on rolling a dice once.
Solution: Sample Space (S) = {1, 2, 3, 4, 5, 6}
Event (E) = {2, 4, 6}
Therefore, n (S) = 6 and n (E) = 3
Putting this in the probability formula, we get:
P(E) = 3 / 6 = 1 / 2 = 0.5
This means, that the chances of getting an even number upon rolling a dice is 0.5

Example 2: Probability of getting HEAD at least once on tossing a coin twice.
Solution: Sample Space (S) = {HH, HT, TH, TT}; where H denotes Head and T denotes Tail.
Event (E) = {HH, HT, TH}
Therefore, Therefore, n (S) = 4 and n (E) = 3
Putting this in the probability formula, we get:
P = 3 / 4 = 0.75
This means, that the chances of getting at least one HEAD on tossing a coin twice are 0.75

Odds in Favour of the Event

Odds in the favor of any event is the ratio of the number of ways that an outcome can occur to the number of ways it cannot occur. Let's look at an example.

Example 3: If **a** represents the odds in favor of getting number 4 on a single roll of dice & **b** represents the outcomes of not getting 4, then,

n (a) = Number of favorable outcomes = 1
n (b) = Number of favorable outcomes = (6 – 1) = 5
Odds in favor = 1 : 5 or 1 / 5
Probability (P) = Number of favorable outcomes/(Number of favorable outcomes + Number of unfavorable outcomes)
P = 1 / (1 + 5) = 1 / 6

Odds Against the Event

Odds against any event is the ratio of the number of ways that an outcome cannot occur to the number of ways it can occur. Let's understand it through an example.

Example 4: If **a** represents the odds against getting number 4 on a single roll of dice & **b** represents the outcomes of getting 4, then –

n (a) = Number of favorable outcomes = 1
n (b) = Number of favorable outcomes = (6 – 1) = 5
Odds in favor = 5 : 1 or 5 / 1
Probability (P) = Number of favorable outcomes / (Number of favorable outcomes +

Number of unfavorable outcomes)
P = 5 / (1 + 5) = 5 / 6

Question 1: Find the probability of 'getting 3 on rolling a die'.

Solution:

Sample Space = {1, 2, 3, 4, 5, 6}

Number of favourable event = 1

i.e. {3}

Total number of outcomes,

Thus, Probability, P = 1/6

Question 2: Draw a random card from a pack of cards. What is the probability that the card drawn is a face card?

Solution:

A standard deck has 52 cards.

Total number of outcomes = 52

Number of favourable events = 4 x 3 = 12 (considered Jack, Queen and King only)

Probability, P = Number of Favourable Outcomes/Total Number of Outcomes = 12/52= 3/13.

Question 3: A vessel contains 4 blue balls, 5 red balls and 11 white balls. If three balls are drawn from the vessel at random, what is the probability that the first ball is red, the second ball is blue, and the third ball is white?

Solution: The probability to get the first ball is red or the first event is 5/20.

Now, since we have drawn a ball for the first event to occur, then the number of possibilities left for the second event to occur is 20 – 1 = 19.

Hence, the probability of getting the second ball as blue or the second event is 4/19.

Again with the first and second event occurred, the number of possibilities left for the third event to occur is 19 – 1 = 18.

And the probability of the third ball is white or third event is 11/18.

Therefore, the probability is 5/20 x 4/19 x 11/18 = 44/1368 = 0.032.

Or we can express it as P = 3.2%.

Question 4: Two dice are rolled, find the probability that the sum is:

1. **equal to 1**
2. **equal to 4**
3. **less than 13**

Solution:

1) To find the probability that the sum is equal to 1 we have to first determine the sample space S of two dice as shown below.
S = { (1,1),(1,2),(1,3),(1,4),(1,5),(1,6)
(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)
(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)
(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)
(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)
(6,1),(6,2),(6,3),(6,4),(6,5),(6,6) }
1) Let E be the event "sum equal to 1". Since, there are no outcomes which where a sum is equal to 1, hence,
P(E) = n(E) / n(S) = 0 / 36 = 0
2) Three possible outcomes give a sum equal to 4 they are:
E = {(1,3),(2,2),(3,1)}
Hence, P(E) = n(E) / n(S) = 3 / 36 = 1 / 12
3) From the sample space, we can see all possible outcomes for the event E, which give a sum less than 13. Like:(1,1) or (1,6) or (2,6) or (6,6).
So you can see the limit of an event to occur is when both dies have number 6, i.e. (6,6).
Hence, P(E) = n(E) / n(S) = 36 / 36 = 1
P(the sum > 7) = 15 /36

The **probability** of the complementary event A' of A is given by P(A') = 1 – P(A).

Equation Of Addition and Multiplication Theorem

Notations :

1. P(A + B) or P(A∪B) = Probability of happening of A or B
   = Probability of happening of the events A or B or both
   = Probability of occurrence of at least one event A or B
2. P(AB) or P(A∩B) = Probability of happening of events A and B together.

Addition theorem on probability:

If two events A and B are mutually exclusive the probability of occurrence of either event A or B is the sum of the individual probabilities

$P(A or B) = P(A) + P(B)$.

(1) When events are not mutually exclusive:
If A and B are two events which are not mutually exclusive, then
$P(A \cup B) = P(A) + P(B) – P(A \text{ and } B)$
or $P(A + B) = P(A) + P(B) – P(AB)$
For any three events A, B, C
$P(A \cup B \cup C) = P(A) + P(B) + P(C) – P(A \cap B) – P(B \cap C) – P(C \cap A) + P(A \cap B \cap C)$
or $P(A + B + C) = P(A) + P(B) + P(C) – P(AB) – P(BC) – P(CA) + P(ABC)$

(2) When events are mutually exclusive:
If *A* and *B* are mutually exclusive events, then
$n(A \cap B) = 0 \Rightarrow P(A \cap B) = 0$
$\therefore P(A \cup B) = P(A) + P(B)$.
For any three events A, B, C which are mutually exclusive,
$P(A \cap B) = P(B \cap C) = P(C \cap A) = P(A \cap B \cap C) = 0$
$\therefore P(A \cup B \cup C) = P(A) + P(B) + P(C)$.
The probability of happening of any one of several mutually exclusive events is equal to the sum of their probabilities, *i.e.* if $A_1, A_2 \ldots\ldots\ldots A_n$ are mutually exclusive events, then
$P(A_1 + A_2 + \ldots + A_n) = P(A_1) + P(A_2) + \ldots\ldots + P(A_n)$
i.e. $P(\Sigma A_i) = \Sigma P(A_i)$.

(3) **When events are independent :**
If *A* and *B* are independent events, then $P(A \cap B) = P(A).P(B)$
$\therefore P(A \cup B) = P(A) + P(B) – P(A).P(B)$

Conditional probability

Let A and B be two events associated with a random experiment. Then, the probability of occurrence of A under the condition that B has already occurred and $P(B) \neq 0$, is called the conditional probability and it is denoted by $P(A/B)$.
Thus, $P(A/B)$ = Probability of occurrence of A, given that B has already happened.

Similarly, P(B/A) = Probability of occurrence of B, given that A has already happened.
Sometimes, P(A/B) is also used to denote the probability of occurrence of A when B occurs. Similarly, P(B/A) is used to denote the probability of occurrence of B when A occurs.

Multiplication Theorem Of Probability

If two events A and B are independent the probability of occurrence of both A and B is the product of their individual probabilities. P(A and B) = P(A).P(B)

1. If *A* and *B* are two events associated with a random experiment, then
   $P(A \cap B) = P(A).P(B/A)$, if $P(A) \neq 0$ or $P(A \cap B) = P(B).P(A/B)$, if $P(B) \neq 0$.

2. **Extension of multiplication theorem:**
   If $A_1, A_2 \ldots\ldots\ldots A_n$ are n events related to a random experiment, then
   $P(A_1 \cap A_2 \cap A_3 \cap \ldots \cap A_n) = P(A_1)\, P(A_2/A_1)$
   $P(A_3/A_1 \cap A_2) \ldots\ldots P(A_n/A_1 \cap A_2 \cap \ldots \cap A_{n-1})$,
   where $P(A_i/A_1 \cap A_2 \cap \ldots \cap A_{i-1})$, represents the conditional probability of the event , given that the events $A_1, A_2 \ldots\ldots\ldots A_{i-1}$ have already happened.

3. **Multiplication theorems for independent events:**
   If *A* and *B* are independent events associated with a random experiment, then $P(A \cap B) = P(A).P(B)$ *i.e.*, the probability of simultaneous occurrence of two independent events is equal to the product of their probabilities.

   By multiplication theorem, we have $P(A \cap B) = P(A).P(B/A)$. Since *A* and *B* are independent events, therefore $P(B/A) = P(B)$. Hence, $P(A \cap B) = P(A).P(B)$.

4. **Extension of multiplication theorem for independent events:**
   If $A_1, A_2 \ldots\ldots\ldots A_n$ are independent events associated with a random experiment, then
   $P(A_1 \cap A_2 \cap A_3 \cap \ldots \cap A_n) = P(A_1)\, P(A_2) \ldots\ldots P(A_n)$.

   Axioms on probability
   1. **Probability always lies between 0 and 1**
      $0 \leq P(A) \leq 1$
   2. **If P(A) is 1, then event A is said to be Sure event**
   3. **If P(A) is 0, then event A is said to be an impossible event**
   4. **Sum of probabilities is one, IF A is an event $P(A^c) = 1 = P(A)$**

Solved examples for You

Question: Find the probability of getting an even number greater than or equal to 4 in a dice roll.

Solution: Sample space (S) = {1, 2, 3, 4, 5, 6} and E = {4, 6}
P (E) = n (E) / n (S) = 2 / 6
P (E) = 1 / 3

Question: Find the probability of getting at least one HEAD in a double coin toss.

Solution: S = {HH, HT, TH, TT}
E = {HH, TH, HT}
P (E) = n (E) / n(S)
So, P (E) = 3 / 4

1. What is the chance that a leap year selected at random will contain 53 Sundays?

   Leap year has 366 days which is 52weeks + 2days

   Possibilities of the two days are{(s,m),(m,t),(t,w),(w,th), (th,f),(f,sa),(sa,s)}

   Therefore P(53 Sundays in a leap year)=2/7

   P(54 Sundays in a leap year) = 0

   P(52 Sundays in a leap year) =1

2. The probability that A can solve a problem in statistics is 4/5, that B can solve it is 2/3 and C can solve it is 3/7. If all of them try independently, find the probability that the problem will be solved.

   the probability that the problem will be solved means any one of the three can solve .

   Given P(A) =4/5, P(B)= 2/3,and P(C)= 3/7

   P(A or B or C) = P(A) +P(B) +P(C) – P(A and B) – P(A and C) – P(Band C) +P(A and B and C)

$= P(A) + P(B) + P(C) – P(A).\ P(B) – P(A).P(C) – P(B).\ P(C) + P(A).P(B).P(C)$

$= 4/5 + 2/3 + 3/7 - (4/5).(2/3) - (2/3 x 3/7) - (4/5 x 3/7) + (2/3 x\ 4/5 x\ 3/7)$

$= 0.8 + 0.6667 + 0.4286 - 0.53333 – 0.2857 – 0.3429 + 0.2286$

$= 0.962$

3. A ball is drawn at random from a box containing 6 red, 4 white, and 5 blue balls. What is the probability that a ball at random is a) red b) white c) blue d) not red e) red or white
   total number of balls = 6+4+5 =15
   P(red) = 6/15 =2/5, P(white) = 4/15, P(blue) = 5/15 =1/3
   P(not red) = 1-P(red) = 1- 2/5 = 3/5,
   P(not Red) = P(white or blue) =P(W) +P(B) = 4/15 + 5/15 =9/15 =3/5
   P(red or white) =P(R)+P(W) =6/15 +4/15 =10/15 =2/3

4. A candidate is selected for interview for three posts. For the first post there are 3 candidates, for the second there are 4 and the third there are 2. What are the chances of his getting at least one post?

   Let A, B and C be the event that the candidate is selected for the first post, second post and third post resp.

   P(A) = 1/3, P(B) = 1/4, P(C)= 1/2

   P(getting atleast one post) = P(A or B or C)

   P(A or B or C) $= P(A) + P(B) + P(C) – P(A).\ P(B) – P(A).P(C) – P(B).\ P(C) + P(A).P(B).P(C)$

   $= 1/3 + ¼ + ½ - 1/3.1/2 - 1/3\ .1/4 - 1/2.\ ¼\ + ½.1/3.1/4$

   $= 0.33 + 0.25 + 0.50 - 0.17 - 0.08 + 0.125 – 0.04$

   $= 0.75$

One card is drawn at random from a pack of 52 cards.

i) What is the probability it is either a king or queen?
ii) What is the probability of picking a that was red or black?

P(K or Q) =P(K) +P(Q) = 4/52 + 4/52 = 8/52 = 2/13

P(R or B) = P(R) + P(B) = 26/52 + 26/52 = 1

A bag contains 30 balls numbered from 1 to 30. One ball is drawn at random. Find the probability that the number of the ball drawn will be multiples of a) 5 or 7 b) 3 or 7

From 1 to 30

A =multiples of 5 {5,10,15,20,25,30}

B=Multiples of 7 { 7, 14, 21, 28}

C=Multiples of 3{3,6,9,12,15,18,21,24,27,30}

Prob(multiples of 5 0r 7) P(AorB) = P(A) + P(B) = 6/30 + 4/30 = 10/30 =1/3

P(B or C) = P(B) + P(C) – P(B and C) = 4/30 + 10/30 – 1/30 = 14/30 -1/30 = 13/30

A can solve 90 percent of the problems given in a book and B can solve 70 percent. What is the probability that atleast one of them will solve a problem selected at random?

P(A) = P(A can solve the problem) = 90/100 =0.9

P(B) = P(B can solve the problem) = 70/100 =0.7

P(A or B) = P(A) +P(B) -P(Aand B) = 0.9 +0.7 – 0.9 x0.7

= 0.9 + 0.7 – 0.63 = 1.6 -0.63 = 0.97

Prob the problem will be solved is 0.97

In a single throw of two dice, what is the probability of obtaining a total of atleast 10?

5. The probability that a boy will get a scholarship is 0.9 and that a girl will get a scholarship is 0.8. what is the probability that atleast one of them will get the scholarship?

   P(A) = P(boy will get a scholarship) = 0.9

   P(B) = P(girl will get the scholarship) = 0.8

   P(atleast one will get the scholarship is

   P(A or B) = P(A) +P(B) -P(Aand B) = 0.9 +0.8 – 0.9 x0.8

   = 0.9 + 0.8 – 0.72 = 1.7-0.72= 0.98

6. A product is assembled from three components X, Y and Z, the probability of these components being defective is respectively 0.01, 0.02, and 0.05. What is the probability that the assembled product will not be defective?

   Let A, B and C be the events that X, Y and Z are not defective

   P(A) = 1- P (X is defective) =1- 0.01 = 0.99

   P(B) = 1 – P (Y is defective) =1- 0.02= 0.98

   P(C) = 1 – P(Z is defective) =1- 0.05 =0.95

   Probability the product is not defective is P(A and B and C)

   P(A and B and C) = P(A) x P(B) x P(C) = 0.99 x0.98 x0.95=0.92

7. An article manufactured by a company consists of two parts A and B. In the process of manufacture of part A, 9 out of 100 are likely to be defective. Similarly. 5 out of 100 are likely to be defective in the manufacture of part B. Calculate the probability that the assembled parts will not be defective.

8. Items produced by a certain process each may have one or both of the two types of defects A and B. It is known that 20 percent of the items have type A defects and 10 percent have type B defects. Further more 6 percent are known to have both types of defects. What is the probability that a randomly selected item will be defective?

9. A and B appear for an interview for 2 vacancies in the same post. The probability of A's selection is 1/7 and that of B's selection is 1/5. What is the prob. that i) both of them will be selected, ii) only one of them will be selected, iii) none of them will be selected.

P(both of them will be selected) = P(A and B) =P(A) x P(B)

P(only one of them will be selected = P (A and Not B) +P(B and Not A)

P(None will be selected) = P(Not A and Not B)

10. A problem in statistics is given to two students A and B. the odds in favour of A solving it is 6 to 9 and against B solving it are 12 to 10. If A and B attempt, find the probability the problem will be solved?

P(A) = 6/(6+9), P(B) = 10 /(10 + 12)

11. The probability that a contractor will get a plumbing contract is 2/3 and the probability that he will not get an electric contract is 5/9. If the probability of getting atleast one contract is 4/5. What is the probability that he will get both the contracts?

P(A) =2/3 , P(getting an electric contract) = P(B) = 1- 5/9 = 4/9

P(AorB) = 4/5 find P(Aand B)

P(A or B) = P(A) + P(B) – P(A and B)

P(A and B) = P(A) + P(B) - P(A or B)

also for many problems in everyday life. It will not be an exaggeration to say that probability has become a part of our everyday life whether or not we admit or are conscious of the use of something so sophisticated. It is still a dream to forecast the future with 100 per cent certainty in any decision problem. The probability theory provides a media of coping up with uncertainty.

Highlighting the importance of probability theory, Ya-lun Chou* has beautifully pointed out that statistics, as a method of decision-making under uncertainty, is founded on probability theory, since probability is at once the language and the measure of uncertainty and the risks associated with it. Before learning statistical decision procedures, the reader must acquire an understanding of probability theory.

CALCULATION OF PROBABILITY

Before discussing the procedure for calculating probability it is necessary to define certain terms as given below :

1. Experiment and Events The term experiment refers to describe an act which can be repeated under some given conditions. Random experiments are those experiments whose results depend on chance such as tossing of a coin, throwing of dice. The results of a random experiment are called outcomes. If in an experiment all the possible outcomes are known in advance and none of the outcomes can be predicted with certainty, then such an experiment is called a random experiment and the outcomes as events or chance events. Events are generally denoted by capital letters A, B, C, etc.

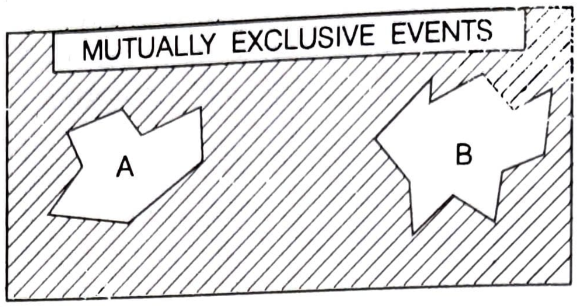
An event whose occurrence is inevitable when a certain random experiment is performed is called a certain or sure event. An event which can never occur when a certain random experiment is performed is called an impossible event. For example, in a toss of a balanced dice the occurrence of any one of the numbers 1, 2, 3, 4, 5, 6 is a sure event while occurrence of 8 is an impossible event.

An event which may or may not occur while performing a certain random experiment is known as a random event. Occurrence of 2 is a random event in the above experiment of the tossing of a dice.

2. Mutually Exclusive Events Two events are said to be mutually exclusive or incompatible when both cannot happen simultaneously in a single trial or, in other words, the occurrence of any one of them precludes the occurrence of the other. For example, if a single coin is tossed either head can be up or tail can be up, both cannot be up at the same time. Similarly, a person may be either alive or dead at a point of time—he cannot be both alive as well as dead at the same time. To take another example, if we toss a dice and observe 3, we cannot expect 5 also in the same toss of dice. Symbolically, if A and B are mutually exclusive events, $P(AB) = 0$.

The following diagram will clearly illustrate the meaning of mutually exclusive events :

* Ya-Lun Chou : *Statistical Analysis*, p. 78.



DISJOINT SETS

It may be pointed out that mutually exclusive events can always be connected by the words "either......or". Events *A*, *B*, *C* are mutually exclusive only if either *A* or *B* or *C* can occur.

3. Independent and Dependent Events Two or more events are said to be independent when the outcome of one does not affect, and is not affected by the other. For example, if a coin is tossed twice, the result of the second throw would in no way be affected by the result of the first throw. Similarly, the results obtained by throwing a dice are independent of the results obtained by drawing an ace from a pack of cards. To consider two events that are not independent, let *A* stand for a firm's spending a large amount of money on advertisement and *B* for its showing an increase in sales. Of course, advertising does not guarantee higher sales, but the probability that the firm will show an increase in sales will be higher if *A* has taken place.

Dependent events are those in which the occurrence or non-occurrence of one event in any one trial affects the probability of other events in other trials. For example, if a card is drawn from a pack of playing cards and is not replaced, this will alter the probability that the second card drawn is, say an ace, Similarly, the probability of drawing a queen from a pack of 52 cards is $\frac{4}{52}$ or $\frac{1}{13}$. But if the card drawn (queen) is not replaced in the pack, the probability of drawing again a queen is $\frac{3}{51}$ ($\because$ the pack now contains only 51 cards out of which there are 3 queens).

4. Equally Likely Events Events are said to be equally likely when one does not occur more often than the others. For example, if an unbiased coin or dice is thrown, each face may be expected to be observed approximately the same number of times in the *long run*. Similarly, the cards of a pack of playing cards are so closely alike that we expect each card to appear equally often when a large number of drawings are made with replacement. However, if the coin or the dice is biased we should not expect each face to appear exactly the same number of times.

5. Simple and Compound Events In case of simple events we consider the probability of the happening or not happening of single events. For example, we might be interested in finding out the probability of drawing a red ball from a bag containing 10 white and 6 red balls. On the other hand, in case of compound events we consider the joint occurrence of two or more events. For example, if a bag contains 10 white and 6 red balls and if two successive draws of 3 balls are made, we shall be finding out the probability of getting 3 white balls in the first draw and 3 balck balls in the second draw—we are thus dealing with a compound event.

6. Exhaustive Events Events are said to be exhaustive when their totality includes all the possible outcomes of a random experiment. For example, while tossing a dice, the possible outcomes are 1, 2, 3, 4, 5 and 6 and hence the exhaustive number of cases is 6. If two dice are thrown once, the possible outcomes are :

(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

The sample space of the experiment* *i. e.*, 36 ordered pairs (6^2). Similarly, for a throw of 3 dice exhaustive number of cases will be 216 (*i. e.*, 6^3) and for n dice they will be 6^n.

Similarly, black and red cards are examples of collectively exhaustive events in a draw from a pack of cards.

7. Complementary Events Let there be two events A and B. A is called the complementary event of B (and *vice versa*) if A and B are mutually exclusive and exhaustive. For example, when a dice is thrown, occurrence of an even number (2, 4, 6) and odd number (1, 3, 5) are complementary events.

Simultaneous occurrence of two events A and B is generally written as AB.

THEOREMS OF PROBABILITY

There are two important theorems of probability, namely :

1. The Addition Theorem; and
2. The Multiplication Theorem.

Addition Theorem

The addition theorem states that if two events A and B are *mutually exclusive* the probability of the occurrence of either A or B is the sum of the individual probability of A and B. Symbolically,

$$^{**}P(A \text{ or } B) = P(A) + P(B).$$

* The set S of all possible outcomes (or elementary events) of a given experiment is called the sample space of the experiment.

** $P(A \text{ or } B) = P(A) + P(B)$ or $P(A \cup B) = P(A) + P(B)$

where $A \cup B$ read is "A union B" denotes the union of events A and B. Knowledge of permutation and combination is extremely useful in calculating probabilities. For the sake of convenience in understanding the concept of premutation and combination an appendix is given at the end of the text.

Proof of the Theorem. If an event A can happen in a_1 ways and B in a_2 ways, then the number of ways in which either event can happen is $a_1 + a_2$. If the total number of possibilities is n, then by definition the probability of either the first or the second event happening is

$$\frac{a_1 + a_2}{n} = \frac{a_1}{n} + \frac{a_2}{n}$$

But $$\frac{a_1}{n} = P(A)$$

and $$\frac{a_2}{n} = P(B)$$

Hence $$P(A \text{ or } B) = P(A) + P(B).$$

The theorem can be extended to three or more mutually exclusive events. Thus

$$P(A \text{ or } B \text{ or } C) = P(A) + P(B) + P(C).$$

Illustration 2. One card is drawn from a standard pack of 52. What is the probability that it is either a king or a queen ? *(B. Com., Punjab Univ., 199?)*

Solution. There are 4 kings and 4 queens in a pack of 52 cards.

∴ The probability that the card drawn is a king $= \frac{4}{52}$

and the probability that the card drawn is a queen $= \frac{4}{52}$

Since the events are mutually exclusive, the probability that the card drawn is either a king or a queen

$$= \frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \frac{2}{13}$$

When events are not mutually exclusive. When events are not mutually exclusive or, in other words, it is possible for both events to occur, the addition rule must be modified. For example, what is the probability of drawing either a king or a heart from a standard pack of cards? It is obvious that the events king and heart can occur together as we can draw a king of hearts (since king and heart are not mutually exclusive events). We must deduce from the probability of drawing either a king or a heart, the chance that we can draw both of them together. Hence for finding the probability of one or more of two events that are not mutually exclusive we use the modified form of the addition theorem.

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$P(A \text{ or } B)$ = Probability of A or B happening when A or B are not mutually exclusive.

$P(A)$ = Probability of A happening

$P(B)$ = Probability of B happening

$P(AB)$ = Probability of A and B happening together

In the example taken the probability of drawing a king or a heart shall be :

$$P(\text{king or heart}) = P(\text{king}) + P(\text{heart}) + P(\text{king and heart})$$

$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52} \text{ or } \frac{16}{52} = \frac{4}{13}$$

In the case of three events,

$$P(A \text{ or } B \text{ or } C) = P(A) + P(B) + P(C) - P(AB) - P(AC) - P(BC) + P(ABC).$$

Illustration 3. The Managing Committee of Vaishalli Welfare Association formed a sub-committee of 5 persons to look into electricity problem. Profiles of the 5 persons are :

1. male age 40
2. male age 43
3. female age 38
4. female age 27
5. male age 65

If a chairperson has to be selected from this, what is the probability that he would be either female or over 30 years?

Solution. P (female or over 30) = P (female) + P (over 30) – P (*female* and over 30)

$$= \frac{2}{5} + \frac{4}{5} - \frac{1}{5} = \frac{5}{5} = 1.$$

Illustration 4. A person is known to hit the target in 3 out of 4 shots, whereas another person is known to hit the target in 2 out of 3 shots. Find the probability of the target being hit at all when they both try.

Solution. The probability that the first person hits the target $= \frac{3}{4}$

The probability that the second person hits the target $= \frac{2}{3}$

The events are not mutually exclusive because both of them may hit the target. ,

$P(AB) = P(A).\ P(B)$ since A and B are independent events.

The required probability $= (\frac{3}{4} + \frac{2}{3}) - (\frac{3}{4} \times \frac{2}{3})$

$$= \frac{17}{12} - \frac{6}{12} = \frac{11}{12}.$$

Here we have applied the theorem $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.

Illustration 5. Calculate the probability of picking a card that was a heart or a spade. Comment on your answer.

Solution. Using the addition rule,

P (heart or spade) = P (heart) + P (spade)

– P (heart and spade) $= \frac{13}{52} + \frac{13}{52} - \frac{0}{52} = \frac{26}{52} = \frac{1}{2}$

The probability that a card will be both a heart and a spade is zero since each individual card can be of one and only one suit. The intersection in this case is non-existent called the null set because it contains no outcomes since heart and spade cannot occur simultaneously in the same card.

Illustration 6. What is the probability of picking a card that was red or black?

Solution. P (*red* or *black*) = P (red) + P (black)

Since there are 26 red and 26 black cards the required probability shall be

$$\frac{26}{52} + \frac{26}{52} = \frac{52}{52} = 1{\cdot}0$$

The probability of red or black adds up to 1·0, this means that this is a certain event to happen.

Illustration 7. A bag contains 30 balls numbered from 1 to 30. One ball is drawn at random. Find the probability that the number of the ball drawn will be a multiple of (*a*) 5 or 7, and (*b*) 3 or 7.

Solution. The probability of the number being multiple of 5 is

$$P(5, 10, 15, 20, 25, 30) = \frac{6}{30}$$

The probability of the number being multiple of 7 is

$$P(7, 14, 21, 28) = \frac{4}{30}.$$

Since the events are mutually exclusive the probability of the number being a multiple of 5 or 7 will be

$$\frac{6}{30} + \frac{4}{30} = \frac{10}{30} = \frac{1}{3}$$

The probability of the number being multiple of 3 is

$$P(3, 6, 9, 12, 15, 18, 21, 24, 27, 30) = \frac{10}{30}$$

The probability of the number being multiple of 7 is

$$P(7, 14, 21, 28) = \frac{4}{30}$$

Since 21 is a multiple of 3 as well as 7, the drawing of the ball numbered 21 entails the occurrence of both the events and hence the probability of getting a number which is multiple of 3 or 7 is

$$= \frac{10}{30} + \frac{4}{30} - \frac{1}{30} = \frac{13}{30}.$$

Multiplication Theorem

This theorem states that if two events A and B are *independent*, the probability that they both will occur is equal to the product of their individual probability. Symbolically, if A and B are independent, then

$$P(A \text{ and } B) = P(A) \times P(B)$$

The theorem can be extended to three or more independent events. Thus,

$$P(A, B \text{ and } C) = P(A) \times P(B) \times P(C)$$

Proof of the Theorem. If an event A can happen in n_1 ways of which a_1 are successful and the event B can happen in n_2 ways of which a_2 are successful, we can combine each successful event in the first with each successful event in the second case. Thus, the total number of successful happenings in both cases is $a_1 \times a_2$. Similarly, the total number of possible cases is $n_1 \times n_2$.

Then by definition the probability of the occurrence of both events is

$$\frac{a_1 \times a_2}{n_1 \times n_2} = \frac{a_1}{n_1} \times \frac{a_2}{n_2}$$

But $$\frac{a_1}{n_1} = P(A)$$

and $$\frac{a_2}{n_2} = P(B).$$

$\therefore$ $$P(A \text{ and } B) = P(A) \times P(B).$$

In a similar way the theorem can be extended to three or more events.

Illustration 8. A man wants to marry a girl having qualities: white complexion—the probability of getting such a girl is one in twenty; handsome dowry—the probability of getting this is one in fifty; westernised manners and etiquettes—the probability here is one in hundred. Find out the probability of his getting married to such a girl when the possession of these three attributes is independent. *(B. Com., Punjab Univ., 199?)*

Solution. Probability of a girl with white complexion

$$= \frac{1}{20} = 0.05$$

Probability of a girl with handsome dowry

$$= \frac{1}{50} = 0.02$$

Probability of a girl with westernised manners

$$= \frac{1}{100} = 0.01$$

Since the events are independent, the probability of simultaneous occurrence of all these qualities

$$= \frac{1}{20} \times \frac{1}{50} \times \frac{1}{100} = 0.05 \times 0.02 \times 0.01 = 0.00001.$$

If we are given n independent events $A_1, A_2, A_3, \ldots\ldots, A_n$ with respective probability of occurrence as $p_1, p_2, p_3, \ldots\ldots, p_n$, then the probability of occurrence of at least one of the n events $A_1, A_2, A_{43}, \ldots\ldots, A_n$ can be determined as follows :

p (happening of at least one of the events) = 1 − p (happening of none of the events).

The following example shall illustrate the application of the above principle.

Illustration 9. A problem in statistics is given to five students A, B, C, D and E. Their chances of soiving it are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$ and $\frac{1}{6}$. What is the probability that the problem will be solved ? *(M. Com, Sukhadia Univ.; B. Com. Madras Univ. 1996 ; MBA, Kumaon Univ., 2000)*

Solution.

Probability that A fails to solve the problem is $1 - \frac{1}{2} = \frac{1}{2}$

Probability that B fails to solve the problem is $1 - \frac{1}{3} = \frac{2}{3}$

Probability that C fails to solve the problem is $1 - \frac{1}{4} = \frac{3}{4}$

Probability that D fails to solve the problem is $1 - \frac{1}{5} = \frac{4}{5}$

Probability that E fails to solve the problem is $1 - \frac{1}{6} = \frac{5}{6}$

Since the events are independent the probability that all the five students fail to solve the problem is :

$$\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \frac{5}{6} = \frac{1}{6}$$

The problem will be solved if anyone of them is able to solve it.

∴ The probability that the problem will be solved $= 1 - \frac{1}{6} = \frac{5}{6}$

CONDITIONAL PROBABILITY*

The multiplication theorem explained above is not applicable in case of dependent events. Two events A and B are said to be dependent when B can occur only when A is known to have occurred (or *vice versa*). The probability attached to such an event is called the *conditional probability* and is denoted by $P(A/B)$ or, in other words, probability of A given that B has occurred.

* When we are computing the probability of a particular event A, given information about the occurrence of another event B, this probability is referred to as conditional probability.

If two events A and B are dependent, then the conditional probability of B given A is :

$$P(B/A) = \frac{P(AB)}{P(A)}.$$

Proof. Suppose a_1 is the number of cases for the simultaneous happening of A and B out of $a_1 + a_2$ cases in which A can happen with or without happening of B.

$$\therefore \quad P(B/A) = \frac{a_1}{a_1 + a_2} = \frac{a_1/n}{(a_1 + a_2)/n} = \frac{P(AB)}{P(A)}$$

Similarly it can be shown that :

$$P(A/B) = P\frac{(AB)}{P(B)}.$$

The general rule of multiplication in its modified form in terms of conditional probability becomes :

$$P(A \text{ and } B) = P(B) \times P(A/B)$$

or

$$P(A \text{ and } B) = P(A) \times P(B/A)$$

For three events A, B and C, we have

$$P(ABC) = P(A) \times P(B/A) \times P(C/AB)$$

i.e., the probability of occurrence of A, B and C is equal to the probability of A, times the probability of B given that A has occurred, times the probability of C given that both A and B have occurred.

Illustration 10. A bag contains 5 white and 3 black balls. Two balls are drawn at random one after the other without replacement. Find the probability that both balls drawn are black.

[B. A. (H.), Econ., Delhi, 1993; B. Com., Bhartidasan Univ., 1997]

Solution. Probability of drawing a black ball in the first attempt is

$$P(A) = \frac{3}{5+3} = \frac{3}{8}$$

Probability of drawing the second black ball given that the first ball drawn is black

$$P(B/A) = \frac{2}{5+2} = \frac{2}{7}$$

∴ The probability that both balls drawn are balck is given by

$$P(AB) = P(A) \times P(B/A) = \frac{3}{8} \times \frac{2}{7} = \frac{3}{28}.$$

Illustration 11. Find the probability of drawing a queen, a king and a knave in that order from a pack of cards in three consecutive draws, the cards drawn not being replaced.

(B. Sc., Calcutta Univ.)

Solution. The probability of drawing a queen $= \frac{4}{52}$.

The probability of drawing a king after a queen has been drawn $= \frac{4}{51}$

The probability of drawing a knave given that a queen and king have been drawn $= \frac{4}{50}$

Since they are dependent events, the required probability of the compound event is :

$$\frac{4}{52} \times \frac{4}{51} \times \frac{4}{50} = \frac{64}{1,32,600} = 0.00048.$$

Any unsold copies are, however, a dead loss. A vendor has estimated the f probability distribution for the number of copies demanded.

No. of copies	15	16	17	18	19
Probability	·04	·19	·33	·26	·11

How many copies should be ordered so that his expected profit will be maximum ?

(MBA., Osman

Solution. Profit per copy = Selling Price – Purchasing Price.
= 40 Paise – 25 Paise = 15 Paise.

Expected Profit

Expected Profit = No. of copies × probability × Profit per copy.

COMPUTATION OF EXPECTED PROFIT

No. of copies	*Probability*	*Profit per copy*	*Expected Pr (paise)*
15	0·04	15	9
16	0·19	15	46
17	0·33	15	84
18	0·26	15	70
19	0·11	15	31
20	0·07	15	21

17 copies will give the maximum expected profit of 84 paise.

Illustration 21. An industrial salesman wants to know the average number of units he per sales call. He checks his past sales records and comes up with the fol probabilities :

Sales in Units	0	1	2	3	4
Probability	0·15	0·20	0·10	0·05	0·30

What is the average number of units he sells per sales call ?

Solution. The salesman wants to know the averge number of units he sells per sal This is the same thing as saying that he wants to know the expected value of each call, where a sales call is the random variable X. The expected value is calculated formula :

$$E(X) = p_1 X_1 + p_2 X_2 + p_3 X_3 + \ldots\ldots$$
$$= 0{\cdot}15\,(0) + 0{\cdot}20\,(1) + 0{\cdot}10\,(2) + 0{\cdot}05\,(3) + 0{\cdot}30\,(4) + 0{\cdot}20\,(5)$$
$$= 0 + 0{\cdot}2 + 0{\cdot}2 + 0{\cdot}15 + 1{\cdot}2 + 1{\cdot}0 = 2{\cdot}75$$

Thus he would expect to sell 2·75 or 3 units on each sales call.

MISCELLANEOUS ILLUSTRATIONS

Illustration 22. A bag contains 6 white, 4 red and 10 black balls. Two balls are dr random. Find the probability that they will both be black.

Solution. Total number of balls in the bag

$$= 6 + 4 + 10 = 20$$

Two balls can be drawn from 20 in $^{20}C_2$ ways

$$= \frac{20 \times 19}{2 \times 1} = 190 \text{ ways}$$

* The probability distribution of a random variable is the listing of the v values of the random variable with the corresponding probability ass with each value of the random variable.

And 2 balls can be drawn from 10 black balls in $^{10}C_2$ or

$$= \frac{10 \times 9}{2 \times 1} = 45 \text{ ways.}$$

∴ The probability that the two balls drawn at random are black

$$= \frac{45}{190} = 0.237$$

Illustration 23. A bag contains 8 white and 4 red balls. Five balls are drawn at random. What is the probability that 2 of them are red and 3 white?

Solution. Total number of balls in the bag = 8 + 4 = 12
Number of balls drawn = 5

5 balls can be drawn from 12 in $^{12}C_5$ ways, 2 red balls can be drawn from 4 red in $^4C_2$ ways and 3 white balls can be drawn from 8 white balls in $^8C_3$ ways.

∴ The number of favourable cases $^4C_2 \times {^8C_3}$ and the required probability is

$$p = \frac{^4C_2 \times {^8C_3}}{^{12}C_5}$$

$$= \frac{4 \times 3}{2 \times 1} \times \frac{8 \times 7 \times 6}{3 \times 2 \times 1} \times \frac{5 \times 4 \times 3 \times 2 \times 1}{12 \times 11 \times 10 \times 9 \times 8} = \frac{14}{33} = 0.424.$$

Illustration 24. Three horses *A*, *B* and *C* are in a race. A is twice as likely to win as *B* and *B* is twice as likely to win as *C*. What are the respective probability of winning?

Solution : We have $A : B : C = 4 : 2 : 1$

Probability of winning for horse $A = \frac{4}{7}$

Probability of winning for horse $B = \frac{2}{7}$

Probability of winning for horse $C = \frac{1}{7}$

Illustration 25. An investment consultant predicts that the odds against the price of a certain stock will go up during the next week are 2 : 1 and odds in favour of the price remaining the same are 1 : 3. What is the probability that the price of the stock will go down during the next week?

Solution. We have :

P (price of a certain stock not going up) $= \frac{2}{3}$

P (Price of a certain stock remaining same) $= \frac{1}{4}$

∴ The probability that the price of the stock will go down during the next week

= P (price of the stock not going up and not remaining same)

= P (price of the stock not going up × P (price of the stock not remaining same)

$$= \frac{2}{3} \times \left(1 - \frac{1}{4}\right) = \frac{2}{3} \times \frac{3}{4} = \frac{1}{2} = 0.5$$

Illustration 26. One bag contains 4 white and 2 black balls. Another contains 3 white and 5 black balls. If one ball is drawn from each bag, find the probability that (*a*) both are white, (*b*) are black, and (c) one is white and one is black.

(M. Com., Andhra Univ., Jammu Univ.)

Solution. Probability of drawing a white ball from the first bag $= \frac{4}{6}$

Probability of drawing a white ball from the second bag $= \frac{3}{8}$

Since the events are independent the probability that both the balls are white $= \frac{4}{6} \times \frac{3}{8} = \frac{1}{4}$.

(*b*) Probability of drawing a black ball from the first bag $= \frac{2}{6}$.

Probability of drawing a black ball from the second bag $= \frac{5}{8}$.

Probability that both are black $= \frac{2}{6} \times \frac{5}{8} = \frac{5}{24}$.

(*c*) That event "one is white one is black" is the same as event "either the first is white and the second is black or the first is black and the second is white."

∴ The probability that one is white and one is black

$$= \left(\frac{4}{6}\right)\left(\frac{5}{8}\right) + \left(\frac{2}{6}\right)\left(\frac{3}{8}\right)$$

$$= \frac{20}{48} + \frac{6}{48} = \frac{13}{24}.$$

Illustration 27. A bag contains 5 white and 8 red balls. Two drawings of 3 balls are made such that (*a*) the balls are replaced before the second trial, and (*b*) the balls are not replaced before the second trial. Find the probability that the first drawing will give 3 white and the second 3 red balls in each case. *(B. Com., M. D. Univ.; M.A., GND Univ., 1993; M. Com., Kerala Univ., 1998)*

Solution. (*a*) *When balls are replaced :*

Total number of balls in the bag = 5 + 8 = 13.

3 balls can be drawn from 13 in $^{13}C_3$ ways.

3 white balls can be drawn from 5 in $^{5}C_3$ ways.

3 red balls can be drawn from 8 in $^{8}C_3$ ways.

∴ The probability of drawing 3 white balls in the first trial is

$$\frac{^{5}C_3}{^{13}C_3} = \frac{5}{143}$$

and the probability of 3 red balls at the second trial is

$$= \frac{^{8}C_3}{^{13}C_3} = \frac{28}{143}$$

∴ The probability of the compound event is

$$\frac{5}{143} \times \frac{28}{143} = \frac{140}{20449} = 0{\cdot}007.$$

(*b*) *When balls are not replaced :*

At the first trial 3 white balls can be drawn in $^{5}C_3$ ways.

∴ The probability of drawing three balls at the first trial is

$$\frac{^{5}C_3}{^{13}C_3} = \frac{5}{143}$$

When the white balls have been drawn and not replaced, the bag contains 2 white and 8 red balls. Therefore, at the second trial 3 balls can be drawn from 10 in $^{10}C_3$ ways and 3 red balls can be drawn from 8 in $^{8}C_3$ ways

∴ The probability of 3 red balls in the second trial $= \frac{^{8}C_3}{^{10}C_3} = \frac{7}{15}$

∴ The probability of the compound event $= \frac{5}{143} \times \frac{7}{15} = \frac{7}{429} = 0{\cdot}016.$

Illustration 28. *A* and *B* play for a prize of Rs. 1,000. *A* is to throw a dice first and is to win if he throws 6. If he fails *B* is to throw and is to win if he throws 6 or 5. If he fails, is to throw again and to win if he throws 6, 5 or 4, and so on. Find their respective expectations.

Solution. Probability of A's winning in the 1st throw

$$= \frac{1}{6}$$

Probability of B's winning in the 2nd throw

$$= \frac{5}{6} \times \frac{2}{6} = \frac{5}{18}$$

Probability of A's winning in the 3rd throw

$$= \frac{5}{6} \times \frac{4}{6} \times \frac{3}{6} = \frac{5}{18}$$

Probability of B's winning in the 4th throw

$$= \frac{5}{6} \times \frac{4}{6} \times \frac{3}{6} \times \frac{4}{6} = \frac{5}{27}$$

Probability of A's winning in the 5th throw

$$= \frac{5}{6} \times \frac{4}{6} \times \frac{3}{6} \times \frac{2}{6} \times \frac{5}{6} = \frac{25}{324}$$

Probability of B's winning in the 6th throw

$$= \frac{5}{6} \times \frac{4}{6} \times \frac{3}{6} \times \frac{2}{6} \times \frac{1}{6} \times \frac{6}{6} = \frac{5}{324}$$

Total of A's chances of success

$$= \frac{1}{6} + \frac{5}{18} + \frac{25}{324} = \frac{169}{324}$$

Total of B's chances of success

$$= \frac{5}{18} + \frac{5}{27} + \frac{5}{324} = \frac{155}{324}$$

Their respective expectations are

$$A = \frac{169}{324} \times 1{,}000 = \text{Rs. } 521{\cdot}6$$

$$B = \frac{155}{324} \times 1{,}000 = \text{Rs. } 478{\cdot}4.$$

Illustration 29. A bag contains 2 white and 3 black balls. Four persons A, B, C and D in the order named each draw one ball and do not replace it. The person to draw a white ball receives Rs. 200. Determine their expectations.

Solution. Since only 3 black balls are contained in the bag, one person must win in the first attempt.

Probability that A wins $= \frac{2}{5}$

$\therefore$ A's expectations $= \frac{2}{5} \times 200 = \text{Rs.}80.$

Probability that A loses and B wins

$$= \frac{3}{5} \times \frac{2}{4} = \frac{3}{10}$$

$\therefore$ B's expectation $= \frac{3}{10} \times 200 = \text{Rs. } 60.$

Probability that A and B lose but C wins

$$= \left(\frac{3}{5}\right)\left(\frac{2}{4}\right)\left(\frac{2}{3}\right) = \frac{1}{5}$$

C's expectation $= \frac{1}{5} \times 200 = \text{Rs.}40$

Probability that A, B and C lose and D wins

$$= \left(\frac{3}{5}\right)\left(\frac{2}{4}\right)\left(\frac{1}{3}\right)\left(\frac{2}{2}\right) = \frac{1}{10}$$

$\therefore$ D's expectation $= \frac{1}{10} \times 200 =$ Rs. 20.

Thus the expectations of A, B, C and D respectively are Rs. 80, Rs. 60, Rs. 40 and Rs. 20.

Illustration 30. Prove that the sum of the probabilities of all possibilities in two independent events amount to certainty.

Solution. Let the probability of success and failure in the first event be p_1 and q_1 and in the second event p_2 and q_2. Then,

the chance of success in the first event and success in the second event is

$$p_1 \times p_2$$

the chance of success in the first and failure in the second event is

$$p_1 \times q_2$$

the chance of failure in the first event and success in the second event is

$$q_1 \times p_2$$

the chance of failure in the first event and failure in the second event is

$$q_1 \times q_2$$

These are all the possibilities, and the sum of these possibilities is :

$$= (p_1 \times p_2) + (p_1 \times q_2) + (q_1 \times p_2) + (q_1 \times q_2)$$

$$= p_1 (p_2 + q_2) + q_1 (p_2 + q_2)$$

But $= (p_1 + q_1)(p_2 + q_2)$

$$= 1 \times 1 = 1 \; (\because p_1 + q_1 = p_2 + q_2 = 1).$$

Illustration 31. A box contains 3 red and 7 white balls. One ball is drawn at random and in its place a ball of the other colour is put in the box. Now one ball is drawn at random from the box. Find the probability that it is red.

Solution. At the second draw red ball can be drawn in two ways if at the first draw ball drawn is red or at the first draw the ball drawn is white.

When the first ball drawn is red :

The probability of drawing a red ball $= \frac{3}{10}$

Now in the box a white ball is put in place of the red ball drawn so the box contains 2 red and 8 white balls.

Hence, the probability of drawing a red ball $= \frac{2}{10}$

When the first ball drawn is white :

The probability of drawing a white ball $= \frac{7}{10}$

Now in the box a red ball is put in place of the white ball drawn. There are thus 4 red and 6 white balls in the box.

$\therefore$ The probability of drawing a red ball $= \frac{4}{10}$.

Hence the probability of drawing a red ball

$$= \left(\frac{3}{10} \times \frac{2}{10}\right) + \left(\frac{7}{10} \times \frac{4}{10}\right)$$

$$= \frac{6}{100} + \frac{28}{100} = \frac{34}{100} = 0{\cdot}34.$$

Illustration 32. Five men in a group of 20 are graduates. If 3 men are picked out of 20 at random (*i*) what is the probability that all are graduates, and (*ii*) what is the probability of at least one being graduate ?

$$p = \frac{45}{75} = 0{\cdot}6$$

(*iv*) Two events *A* and *B* are statistically independent if

$$P(AB) = P(A) \times P(B)$$

Let *A* denote those who have TV and *B* those who are telephone subscribers. Probability of a person owning a TV

i.e., $P(A).\ p = \frac{75}{125}$ [∵ 75 persons out of a total of 125 own a TV]

Probability of a person being a telephone subscriber

$p = \frac{75}{125}$ [∵ 75 persons out of a total of 125 are telephone subscriber]

$$P(AB)p = \frac{45}{125} = \frac{9}{25}$$

$$P(A) \times P(B)p = \frac{75}{125} \times \frac{75}{125} = \frac{9}{25}$$

Hence, $P(AB)p = P(A) \times P(B)$

We, therefore, conclude that the events 'ownership of a TV' and 'telephone subscriber' are statistcally independent.

Illustration 36. Three groups of workers contain 3 men and one woman, 2 men and 2 women, and 1 man and 3 women respectively. One worker is selected at random from each group. What is the probability that the group selected consists of 1 man and 2 women? *(M. Com., Nagpur; M. Com., Chennai, 1997)*

Solution. There are three possibilities in this case :

(*i*) Man is selected from the first group and women from 2nd and 3rd groups; or
(*ii*) Man is selected from the 2nd group and women from the 1st and 3rd groups; or
(*iii*) Man is selected from the 3rd group and women from 1st and 2nd groups.

∴ The probability of selecting a group of 1 man and 2 women

$$= \left(\frac{3}{4} \times \frac{2}{4} \times \frac{3}{4}\right) + \left(\frac{2}{4} \times \frac{1}{4} \times \frac{3}{4}\right) + \left(\frac{1}{4} \times \frac{1}{4} \times \frac{2}{4}\right)$$

$$= \frac{9}{32} + \frac{3}{32} + \frac{1}{32} = \frac{13}{32}.$$

Illustration 37. What is the probability that a leap year, selected at random, will contain 53 Sundays ? *(M. Com., Agra Univ.; M. Com., Kurukshetra Univ., 1996; M. Com., M.D. Univ., 1998)*

Solution. A leap year consists of 366 days and, therefore, contains 52 complete weeks and 2 days extra. These 2 days may make the following 7 combinations :

1. Monday and Tuesday.
2. Tuesday and Wednesday
3. Wednesday and Thursday
4. Thursday and Friday
5. Friday and Saturday
6. Saturday and Sunday
7. Sunday and Monday

Of these seven equally likely cases only the last two are favourable. Hence, the required probability = 2/7.

Illustration 38. A University has to select an examiner from a list of 50 persons, 20 of them women and 30 men, 10 of them knowing Hindi and 40 not, 15 of them being teachers and the remaining 35 not. What is the probability of the University selecting a Hindi-knowing woman teacher ? *(M. Com., Jammu Univ., 1997; M. Com., M.D. Univ., 1997)*

Solution. Probability of selecting a woman $= \frac{20}{50}$

Probability of selecting a teacher $= \frac{15}{50}$

Probability of selecting a Hindi-knowing candidate $= \frac{10}{50}$

Since the events are independent the probability of the University selecting a Hindi-knowing woman teacher

$$= \frac{20}{50} \times \frac{15}{50} \times \frac{10}{50} = \frac{3}{125} = 0{\cdot}024.$$

Illustration 39. A bag contains 10 white and 6 black balls. 4 balls are successively drawn out and not replaced. What is the probability that they are alternately of different colours?

Solution. (*a*) *Beginning with white :*

The probability of drawing a white ball $= \frac{10}{16}$

The probability of drawing a black ball then $= \frac{6}{15}$

The probability of drawing a white ball then $= \frac{9}{14}$

The probability of drawing a black ball then $= \frac{5}{13}$

Therefore, the probability of the compound event

$$= \frac{10}{16} \times \frac{6}{15} \times \frac{9}{14} \times \frac{5}{13} = \frac{45}{728}$$

(*b*) *Similarly, beginning with black :*

The probability of drawing a black ball $= \frac{6}{16}$

The probability of drawing a white ball then $= \frac{10}{15}$

The probability of drawing a black ball then $= \frac{5}{14}$

The probability of drawing a white ball then $= \frac{9}{13}$

The probability of the compound event

$$= \frac{6}{16} \times \frac{10}{15} \times \frac{5}{14} \times \frac{9}{13} = \frac{45}{728}$$

The above two events are mutually exclusive. Therefore, the required probability that 4 exclusive drawn balls are alternately of different colours (without mentioning the colour with which to begin).

$$= \frac{45}{728} + \frac{45}{728} = \frac{90}{728} = 0{\cdot}124.$$

Illustration 40. (*a*) A can solve 90 per cent of the problems given in a book and *B* can solve 70 per cent. What is the probability that at lest one of them will solve a problem selected at random ?

(*B. Com. Kurukshetra 1990, M. Com., M.D., 1997*)

(*b*) In a single throw of two dice, what is the probability of obtaining a total of at least 10 ?

Solution. (*a*) Probability that *A* will not be able to solve the problem

$$= 1 - \frac{9}{10} = \frac{1}{10}$$

Probability that *B* will not be able to solve the problem

$$= 1 - \frac{7}{10} = \frac{3}{10}$$

Probability that none of them will be able to solve tha problem

$$= \frac{1}{10} \times \frac{3}{10} = \frac{3}{100}$$

Hence the probability that at least one of them will solve the problem

$$= 1 - \frac{3}{100} = \frac{97}{100}$$

(*b*) Two dice can be thrown together in $6 \times 6 = 36$ ways.

We have to find the probability of getting a sum of at least ten, *i. e.*, either ten, eleven or twelve.

The probability of getting a total 10 in a single throw of two dica (6, 4) (4, 6) (5, 5)

$$= \frac{3}{36}$$

The probability of getting a total 11 in a single throw of two dice (6, 5) (5, 6)

$$= \frac{2}{36}$$

Probability of getting a total 12 in a single throw of two dice (6, 6)

$$= \frac{1}{36}$$

Since the events are mutually exclusive the probability of obtaining a sum of at least 10 in a single throw of two dice $= \frac{3}{36} + \frac{2}{36} + \frac{1}{36} = \frac{6}{36} = \frac{1}{6}$.

Illustration 41. (*i*) A class consists of 80 students, 25 of them are girls and 55 boys, 10 of them are rich and remaining poor, 20 of them are fair complexioned. What is the probability of selecting a fair complexioned rich girl? *(M. Com., Kurukshetra, 1990)*

(*ii*) Explain why there must be a mistake in the following statement :

A quality control engineer claims that the probability for a large consignment of glass bricks containing 0, 1, 2, 3, 4, or 5 defectives are 0·11, 0·23, 0·37, 0·16, 0·09 and 0·05 respectively.

Solution. (*i*) Probability of selecting

$$\text{a fair complexioned person} = \frac{20}{80} = \frac{1}{4}$$

Probability of selecting a rich person $= \frac{10}{80} = \frac{1}{8}$

The probability of selecting a girl $= \frac{25}{80} = \frac{5}{16}$

The probability of selecting a fair complexioned rich girl

$$= \frac{1}{4} \times \frac{1}{8} \times \frac{5}{16} = \frac{5}{512} = 0{\cdot}0098$$

(*ii*) We know that probability of happening or not happening of an event can never exceed one. In the question given.

$$P(A) + P(B) + P(C) \ldots\ldots\ldots$$
$$= 0{\cdot}11 + 0{\cdot}23 + 0{\cdot}37 + 0{\cdot}16 + 0{\cdot}09 + 0{\cdot}05 = 1{\cdot}01$$

Since the total probability is greater than one, there is some mistake in the statement given.

Illustraion 42. The probability that a boy will get a scholarship is 0·9 and that a girl will get is 0·8. What is the probability that at least one of them will get the scholarship?

Solution. The probability that a boy will get a scholarship = 0·9

The probability that a girl will get a scholarship = 0·8

The probability that at least one of them will get the scholarship is

$$P(A) + P(B) - P(AB)$$
$$= {\cdot}9 + {\cdot}8 - ({\cdot}9 \times {\cdot}8)$$
$$= 1{\cdot}7 - {\cdot}72 = 0{\cdot}98.$$

Illustration 43. A fair dice is tossed twice. Find the probability of getting a 4, 5 or 6 on the first toss and a 1, 2, 3, or 4 on the second toss.

(M. Com., Madras Univ., 1988 ; Kurukshetra Univ. 1995)