



UNIT – II

Measures of Central Tendency: Mean – Median-Mode –Geometric mean-Harmonic mean (only definition) - Measures of dispersion: Range-Quartile deviation-Mean deviation-Standard deviation-Lorenz curve

Measures of Central Tendency

***Mean, Median and Mode
for Ungrouped Data
Basic Statistics***



Measures of Central Tendency

Measures of central tendency are numerical descriptive measures which indicate or locate the center of a distribution or data set.

In layman's term, a measure of central tendency is an **AVERAGE**. It is a single number of value which can be considered typical in a set of data as a whole.

For example, in a class of 40 students, the average height would be the typical height of the members of this class as a whole.



"Add the numbers, divide by how many numbers you've added and there you have it—the average amount of minutes you sleep in class each day."

MEAN

The **MEAN** of a set of values or measurements is the sum of all the measurements divided by the number of measurements in the set.

Among the three measures of central tendency, the mean is the most popular and widely used. It is sometimes called the **arithmetic mean**.

If we compute the mean of the population, we call it the parametric or population mean, denoted by μ (read "mu").

If we get the mean of the sample, we call it the sample mean and it is denoted by \bar{x} (read "x bar").

Mean for Ungrouped Data

For ungrouped or raw data, the mean has the following formula.

$$\bar{x} = \frac{\sum x}{n}$$

where

\bar{x} = mean

$\sum x$ = sum of the measurements or values

n = number of measurements

Example 1:

Ms. Sulit collects the data on the ages of Mathematics teachers in Santa Rosa School, and her study yields the following:

38 35 28 36 35 33 40

Solution:

$$\bar{x} = \frac{38 + 35 + 28 + 36 + 35 + 33 + 40}{7}$$
$$= 35$$

Based on the computed mean, 35 is the average age of Mathematics teachers in SRS.



Your turn!

Mang John is a meat vendor. The following are his sales for the past six days. Compute his daily mean sales.

| | |
|-----------|---------|
| Tuesday | P 5 800 |
| Wednesday | 8 600 |
| Thursday | 6 500 |
| Friday | 4 300 |
| Saturday | 12 500 |
| Sunday | 13 400 |

Solution:

$$\bar{x} = \frac{5800 + 8600 + 6500 + 4300 + 12500 + 13400}{6}$$
$$= 51,100$$

The average daily sales of Mang John is P51,100.



Weighted Mean

Weighted mean is the mean of a set of values wherein each value or measurement has a different weight or degree of importance. The following is its formula:

$$\bar{x} = \frac{\sum xw}{\sum w}$$

where

\bar{x} = mean

x = measurement or value

w = number of measurements

Example

Below are Amaya's subjects and the corresponding number of units and grades she got for the previous grading period. Compute her grade point average.

| Subject | Units | Grade |
|----------------|-------|-------|
| Filipino | .9 | 86 |
| English | 1.5 | 85 |
| Mathematics | 1.5 | 88 |
| Science | 1.8 | 87 |
| Social Studies | .9 | 86 |
| TLE | 1.2 | 83 |
| MAPEH | 1.2 | 87 |

$$\begin{aligned}\bar{x} &= \frac{86(.9) + 85(1.5) + 88(1.5) + 87(1.8) + 86(.9) + 83(1.2) + 87(1.2)}{.9 + 1.5 + 1.5 + 1.8 + .9 + 1.2 + 1.2} \\ &= 86.1\end{aligned}$$

Amaya's average grade is 86.1



Your turn!

James obtained the following grades in his five subjects for the second grading period. Compute his grade point average.

| Subject | Units | Grade |
|-----------|-------|-------|
| Math | 1.5 | 90 |
| English | 1.5 | 86 |
| Science | 1.8 | 88 |
| Filipino | 0.9 | 87 |
| MAKABAYAN | 1.5 | 87 |

Solution:

$$\bar{X} = \frac{90 * 1.5 + 86 * 1.5 + 88 * 1.8 + 87 * .9 + 87 * 1.5}{1.5 + 1.5 + 1.8 + .9 + 1.5}$$
$$= 87.67$$

James general average is 87.67



Properties of Mean

1. Mean can be calculated for any set of numerical data, so it always exists.
2. A set of numerical data has one and only one mean.
3. Mean is the most reliable measure of central tendency since it takes into account every item in the set of data.
4. It is greatly affected by extreme or deviant values (*outliers*)
5. It is used only if the data are interval or ratio.

MEDIAN

The **MEDIAN**, denoted M_d , is the middle value of the sample when the data are ranked in order according to size.

16 17 18 19 20 21 22

↑
Median

16 17 18 19 20 21 22 23

↑
Median

$$\frac{19 + 20}{2} = 19.5$$

o **Median :** (middle)

- o The "Median" of a data set is dependent on whether the number of elements in the data set is odd or even.
- o First reorder the data set from the smallest to the largest
- o Mark off high and low values until you reach the **middle**.
- o If there 2 middles, add them and **divide** by 2.

o **Examples : Even Number of Elements**

o Data Set = 2, 5, 9, 3, 5, 4

o Reordered = 2, 3, 4, 5, 5, 9
 ^ ^

$$\text{Median} = (4 + 5) / 2 = 4.5$$

Properties of Median

1. Median is the score or class in the distribution wherein 50% of the score fall below it and another 50% lie.
2. Median is not affected by extreme or deviant values.
3. Median is appropriate to use when there are extreme or deviant values.
4. Median is used when the data are ordinal.
5. Median exists in both quantitative or qualitative data.

MODE

The **MODE**, denoted M_o , is the value which occurs most frequently in a set of measurements or values. In other words, it is the most popular value in a given set.

Examples:

Find the Mode.

1. The ages of five students are: 17, 18, 23, 20, and 19
2. The following are the descriptive evaluations of 5 teachers: VS, S, VS, VS, O
3. The grades of five students are : 4.0, 3.5, 4.0, 3.5, and 1.0
4. The weights of five boys in pounds are: 117, 218, 233, 120, and 117

o **Mode : (most often)**

- o The "Mode" for a data set is the element that occurs the most often.
- o It is not uncommon for a data set to have more than one mode.
- o This happens when two or more elements occur with equal frequency in the data set.

o **Example :**

o **Data Set** = 2, 5, 9, 3, 5, 4, 7

o **Mode** = 5

o **Example:**

o **Data Set** = 2, 5, 2, 3, 5, 4, 7

o **Modes** = 2 and 5

Properties

1. It is used when you want to find the value which occurs most often.
2. It is a quick approximation of the average.
3. It is an inspection average.
4. It is the most unreliable among the three measures of central tendency because its value is undefined in some observations.



o **Range :**

- o The "Range" for a data set is the difference between the largest value and smallest value contained in the data set.
- o First **reorder** the data set from smallest to largest then **subtract** the first element from the last element.

Of all the measures of variability, the **range** is the **easiest and quickest way** to determine. It is simply the **difference of the highest (H) and the lowest (L) scores** in a set of data under consideration.

Formula for the Range:

$$r = H - L$$

where: r = range

H = highest score

L = lowest score

Example:

The scores of Maria in her math quizzes are as follows: 12, 25, 27, 29, 36, 38, 40, 43, 50, and 62. Find its range.

Solution:

Highest score (H) = 62 Lowest score (L) = 12

$$\begin{aligned}r &= H - L \\ &= 62 - 12 \\ &= 50\end{aligned}$$

Therefore, the range is 50.

The range

Here are the high jump scores for two girls in metres.

| | | | | | |
|--------|------|------|------|------|------|
| Joanna | 1.62 | 1.41 | 1.35 | 1.20 | 1.15 |
| Kirsty | 1.59 | 1.45 | 1.41 | 1.30 | 1.30 |

Find the range for each girl's results and use this to find out who is consistently better.

$$\text{Joanna's range} = 1.62 - 1.15 = \mathbf{0.47}$$

$$\text{Kirsty's range} = 1.59 - 1.30 = \mathbf{0.29}$$



| | | | | | |
|--------|------|------|------|------|------|
| Joanna | 1.62 | 1.41 | 1.35 | 1.20 | 1.15 |
| Kirsty | 1.59 | 1.45 | 1.41 | 1.30 | 1.30 |

Calculate the mean and the range for each girl.

| | Joanna | Kirsty |
|-------|--------|--------|
| Mean | 1.35 m | 1.41 m |
| Range | 0.47 m | 0.29 m |

Use these results to decide which one you would enter into the athletics competition and why.

9, 7, 9, 3, 7, 7, 7, 9, 5

mean: 7

median: 7

range: 6



Advantages

- Gives **quick approximation** of the variability of data.
- It is **not** very sophisticated/complicated.
- Used when the **mode** is preferred measure of central tendency. (i.e./ when you have **nominal/titular** level data.)
- It is the simplest measure of variability/dispersion.

Disadvantages

- It is **limited/partial**, if extreme scores are not representative of the sample, but are included among the scores.
- It is **not** very informative, because it is based only on the most extreme scores.
- It is severely affected by extreme scores in your data distribution.

The background is a dark blue gradient. In the four corners, there are decorative white line-art elements resembling circuit traces or neural network connections. These elements consist of straight lines of varying lengths and angles, ending in small white circles. The top-left and bottom-left corners have more complex, branching structures, while the top-right and bottom-right corners have simpler, more linear structures.

Geometric mean

GEOMETRIC MEAN

In Mathematics, the geometric mean is a type of mean or average, which indicates the central tendency or typical value of a set of numbers by using the product of their values (as opposed to the arithmetic mean which uses their sum).

Formulas

- For grouped data:

$$G = \text{Anti} \left(\sum \frac{\log x}{n} \right)$$

- For Grouped data:

$$G = \text{anti} \left(\sum \frac{f \log x}{n} \right)$$

Question 1: Find the geometric mean of the following values:

15, 12, 13, 19, 10

Solution:

Given data, 15, 12, 13, 19, 10

$$n=5, \sum \log x=5.648$$

$$\text{As } G = \text{Anti}\left(\frac{\sum \log x}{n}\right)$$

$$G = \text{Anti}\left(\frac{5.648}{5}\right)$$

$$G = \text{Anti}(1.129)$$

$$G = 13.48$$

| x | $\text{Log } x$ |
|-------|-----------------|
| 15 | 1.1761 |
| 12 | 1.0792 |
| 13 | 1.1139 |
| 19 | 1.2788 |
| 10 | 1.0000 |
| Total | 5.648 |

The geometric mean is a type of average, usually used for growth rates, like population growth or interest rates. While the arithmetic mean **adds** items, the geometric mean **multiplies** items. Also, you can only get the geometric mean for positive numbers.

Merits of Geometric Mean:

- It is based on all the observations
- It is rigidly defined
- It is capable of further algebraic treatment
- It is less affected by the extreme values
- It is suitable for averaging ratios, percentages and rates.

Limitations of Geometric Mean:

- It is difficult to understand
- The geometric mean cannot be computed if any item in the series is negative or zero.
- The GM may not be the actual value of the series
- It brings out the property of the ratio of the change and not the absolute difference of change as the case in arithmetic mean.

CALCULATION

Formula:

$$\mathbf{G.M.} = \sqrt[n]{X_1 * X_2 * X_3 * \dots * X_n} = (X_1 * X_2 * X_3 * \dots * X_n)^{1/n}$$

For example,

GM of two numbers 4 and 9 is $\sqrt{4 * 9} = \sqrt{36} = 6$

GM of three numbers 1, 4 and 128 is $\sqrt[3]{1 * 4 * 128} = \sqrt[3]{512} = 8$

• The above method can be applied if there are two or three items. But if n is a very large number, the problem of finding the n th root is a tedious work. Therefore, we make use of logarithms. The above formula reduced to its logarithmic form is

• Geometric mean = Antilog of $\frac{\log X_1 + \log X_2 + \log X_3 + \dots + \log X_n}{N}$

[or] Geometric Mean = Antilog of \log / N

CALCULATION OF GEOMETRIC MEAN – INDIVIDUAL SERIES

STEP: 1. FIND OUT THE LOGARITHM OF EACH VALUE OR THE SIZE OF ITEM FROM THE LOG TABLE : $\text{LOG } X$

2. ADD ALL THE VALUES OF LOG - $\sum \text{LOG } X$

3. THE SUM OF ($\sum \text{LOG}$) IS DIVIDED BY THE NUMBER OF ITEMS. $\sum \text{LOG } X / N$

4. FIND OUT THE ANTILOG OF THE QUOTIENT (FROM STEP 3). THIS IS THE GEOMETRIC MEAN OF THE DATA

The background is a blue gradient with decorative white circuit-like lines in the corners. The lines consist of straight segments and small circles, resembling a stylized PCB or network diagram.

HARMONIC MEAN

- **Harmonic mean** (formerly sometimes called the **subcontrary mean**) is one of several kinds of average.
- The harmonic mean is a very specific type of average.
- It's generally used when dealing with averages of units, like speed or other rates and ratios.

What is the harmonic mean of 1,5,8,10?

Here,

N=4

$$H = \frac{n}{\sum_{i=1}^n \frac{1}{x_i}}$$

$$H = 4 / (1/1) + (1/5) + (1/8) + (1/10)$$

$$H = 4 / 1.425$$

$$H = 2.80$$

Rahul drives a car at 20 mph for the first hour and 30 mph for the second. What's his average speed?

We need the harmonic
mean:

$$= 2 / (1/20 + 1/30)$$

$$= 2(0.05 + 0.033)$$

$$= 2 / 0.083$$

$$= 24.09624 \text{ mph.}$$

Harmonic Mean

- Harmonic mean is quotient of "number of the given values" and "sum of the reciprocals of the given values".
- For Ungrouped Data

$$H.M \text{ of } X = \bar{X} = \frac{n}{\sum\left(\frac{1}{x}\right)}$$

- For grouped Data

$$H.M \text{ of } X = \bar{X} = \frac{\sum f}{\sum\left(\frac{f}{x}\right)}$$

Calculate the harmonic mean of the numbers: 13.2, 14.2, 14.8, 15.2 and 16.1

Solution:

The harmonic mean is calculated as below:

AS

$$H.M \text{ of } X = \bar{X} = \frac{n}{\sum\left(\frac{1}{x}\right)}$$

$$H.M \text{ of } X = \bar{X} = \frac{5}{0.3417} = 14.63$$

| X | $\frac{1}{X}$ |
|----------|---------------------------------|
| 13.2 | 0.0758 |
| 14.2 | 0.0704 |
| 14.8 | 0.0676 |
| 15.2 | 0.0658 |
| 16.1 | 0.0621 |
| Total | $\sum\frac{1}{X} = 0.3147$ |

Example: Calculate the harmonic mean for the given below:

| Marks | 30-39 | 40-49 | 50-59 | 60-69 | 70-79 | 80-89 | 90-99 |
|-------|-------|-------|-------|-------|-------|-------|-------|
| F | 2 | 3 | 11 | 20 | 32 | 25 | 7 |

Solution: Now
We'll find H.M as:

$$\bar{x} = \frac{\sum f}{\sum \left(\frac{f}{x}\right)} = \frac{100}{1.4368} = 69.60$$

| Marks | x | f | $\frac{f}{x}$ |
|--------------|------|------------|---------------|
| 30-39 | 34.5 | 2 | 0.0580 |
| 40-49 | 44.5 | 3 | 0.0674 |
| 50-59 | 54.5 | 11 | 0.2018 |
| 60-69 | 64.5 | 20 | 0.3101 |
| 70-79 | 74.5 | 32 | 0.4295 |
| 80-89 | 84.5 | 25 | 0.2959 |
| 90-99 | 94.5 | 7 | 0.0741 |
| Total | | 4.5 | 0.4295 |

MEASURE OF DISPERSION:

(**RANGE**, QUARTILE DEVIATION, MEAN DEVIATION,
STANDARD DEVIATION)

QUARTILE DEVIATION

The measure of dispersion is provided by quartile-deviation or semi - inter -quartile range. The formula to find quartile-deviation is given by

$$Q_d = \frac{Q_3 - Q_1}{2}$$

Coefficient of Quartile Deviation

A relative measure of dispersion using quartiles is given by coefficient of quartile-deviation which is

$$\text{Coefficient of quartile deviation} = \frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100$$

Properties of Quartile Deviation

- 1) Quartile-deviation provides the best measure of dispersion for open-end classification.
- 2) It is less affected due to sampling fluctuations.
- 3) Like other measures of dispersion, quartile-deviation remains unaffected due to a change of origin but is affected in the same ratio due to change in scale.

i.e. if $y = a + bx$, a and b being constants,

$$\text{then QD of } y = |b| \times \text{QD of } x$$

Solved Problems

Problem 1 :

Following are the marks of the 10 students :

56, 48, 65, 35, 42, 75, 82, 60, 55, 50

Find QD and also its coefficient.

Solution :

After arranging the marks in an ascending order, we get

35, 42, 48, 50, 55, 56, 60, 65, 75, 82

First quartile (Q_1):

$$\begin{aligned} &= (n+1)/4 \text{ th observation} \\ &= (10+1)/4 \text{ th observation} \\ &= 2.75^{\text{th}} \text{ observation} \end{aligned}$$

= 2nd observation + 0.75 × difference between the 3rd and the 2nd observation

$$\begin{aligned} &= 42 + 0.75 (48 - 42) \\ &= 42 + 0.75 \times 6 \\ &= 42 + 4.5 \end{aligned}$$

$$Q_1 = 46.5$$

Third quartile (Q_3) :

$$\begin{aligned} &= 3(n+1)/4 \text{ th observation} \\ &= 3(10+1)/4 \text{ th observation} \\ &= 8.25^{\text{th}} \text{ observation} \end{aligned}$$

= 8th observation + 0.25 × difference between the 9th and the 8th observation

$$= 65 + 0.25 (75 - 65)$$

$$\begin{aligned} &= 65 + 0.25 \times 10 \\ &= 65 + 2.5 \end{aligned}$$

$$Q_3 = 67.5$$

The formula to find QD is given by

$$\begin{aligned} \text{QD} &= (Q_3 - Q_1) / 2 \\ \text{QD} &= (67.50 - 46.50) / 2 \end{aligned}$$

$$\begin{aligned} \text{QD} &= 21 / 2 \\ \text{QD} &= 10.5 \end{aligned}$$

The formula to find coefficient of QD is given by

$$\begin{aligned} &= [(Q_3 - Q_1) / (Q_3 + Q_1)] \times 100 \\ &= [(67.50 - 46.50) / (67.50 + 46.50)] \times 100 \\ &= [21 / 114] \times 100 \end{aligned}$$

$$\text{Coefficient of QD} = 18.42$$

Problem 2 :

If the QD of x is 6 and $3x + 6y = 20$, what is the QD of y ?

Solution :

Let us write the given equation $3x + 6y = 20$ as $y = ax + b$,
So, we get

$$y = (20/6) + (-3/6)x$$

When ' x ' and ' y ' are related as $y = a + bx$, then

$$\text{QD of 'y'} = |b| \times \text{QD of 'x'}$$

$$\text{QD of 'y'} = |-3/6| \times 6$$

$$\text{QD of 'y'} = 3$$

MEAN DEVIATION

WHAT IS MEAN DEVIATION?

THE **MEAN DEVIATION** IS THE FIRST MEASURE OF DISPERSION THAT WE WILL USE THAT ACTUALLY USES EACH DATA VALUE IN ITS COMPUTATION. IT IS THE MEAN OF THE DISTANCES BETWEEN EACH VALUE AND THE MEAN. IT GIVES US AN IDEA OF HOW SPREAD OUT FROM THE CENTER THE SET OF VALUES IS.

**FORMULA FOR
MEAN DEVIATION:
(UNGROUPED DATA)**

$$MD = \frac{\sum |x - \mu|}{n}$$

WHERE,

μ = MEAN

x = EACH VALUE

n = NUMBER OF VALUES

EACH VALUE FROM THEIR MEAN.

THREE STEPS ON FINDING THE MEAN:

1) FIND THE MEAN OF ALL VALUES.

2) FIND THE DISTANCE OF EACH VALUE FROM THAT MEAN.

3) FIND THE MEAN OF THOSE DISTANCES.

Example :

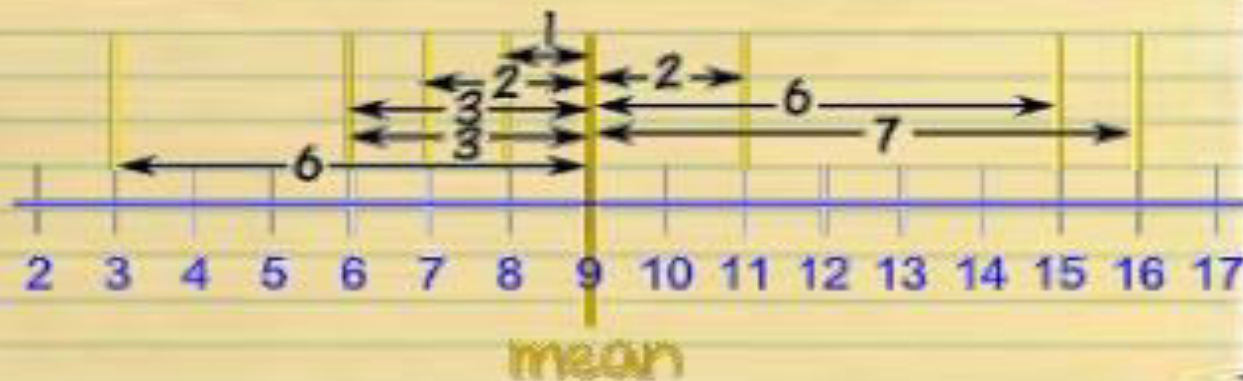
Find the mean deviation of 3, 6, 6, 7, 8, 11, 16 and 15.

Step I : Find the mean.

$$\text{Mean} = \frac{3+6+6+7+8+11+16+15}{8} = \frac{72}{8} = 9$$

STEP 2: FIND THE DISTANCE OF EACH VALUE FROM THE MEAN.

| Value | Distance from 9 |
|-------|-----------------|
| 3 | 6 |
| 6 | 3 |
| 6 | 3 |
| 7 | 2 |
| 8 | 1 |
| 11 | 2 |
| 15 | 6 |
| 16 | 7 |



$$6+3+3+2+1 = 2+6+7$$
$$15 = 15$$

It tells us how far, on average, all values are from the middle.

Step 3: Find the mean of those distances.

$$MD = \frac{6+3+3+2+1+2+6+7}{8} = \frac{30}{8} = 3.75$$

So, the mean = 9 and the mean deviation is 3.75.

EXERCISES :

1) A BOOKLET HAS 12 PAGES WITH THE FOLLOWING NUMBERS OF WORDS:

271, 354, 296, 301, 333, 326, 285,
298, 327, 316, 287 AND 314

WHAT IS THE MEAN DEVIATION OF THE NUMBER OF WORDS PER PAGE?

**STEP 1:
FIND THE MEAN.**

RAW DATA:

**271, 354, 296, 301, 333, 326, 285,
298, 327, 316, 287, 314**

$$\mu = \frac{271 + 354 + 296 + 301 + 333 + 326 + 285 + 298 + 327 + 316 + 287 + 314}{12}$$

$$\frac{3708}{12} = 309$$

STEP 2: FIND THE ABSOLUTE DEVIATIONS.

| x | $ x - \mu $ |
|-----|-------------|
| 271 | 38 |
| 354 | 45 |
| 296 | 13 |
| 301 | 8 |
| 333 | 24 |
| 326 | 17 |
| 285 | 24 |
| 298 | 11 |
| 327 | 18 |
| 316 | 7 |
| 287 | 22 |
| 314 | 5 |

$$\sum |x - \mu| = 232$$

STEP 3: FIND THE MEAN DEVIATION

$$M.D. = \frac{\sum |x - \mu|}{N} = \frac{232}{12}$$

19.33

**FORMULA FOR
MEAN DEVIATION:
(GROUPED DATA)**

$$MD = \frac{\sum f |x - \mu|}{\sum f}$$

WHERE,

μ = mean

x = each value

f = frequency

THREE STEPS ON FINDING THE MEAN:

1) FIND THE MEAN BY USING THE FORMULA

$$\mu = \frac{\sum fx}{\sum f}$$

2) SOLVE FOR $|x - \mu|$ AND MULTIPLY IT TO THE FREQUENCY OF EACH CLASS. FIND THE

$$\sum f|x - \mu|$$

3) DIVIDE THE ANSWER OF $\sum f|x - \mu|$ TO THE $\sum f$

STEP 1: FIND THE MEAN BY USING THE GIVEN FORMULA:

$$\mu = \frac{\sum fx}{\sum f}$$

| x | f | fx |
|---|---------------|----------------|
| 0 | 4 | 0 |
| 1 | 12 | 12 |
| 2 | 8 | 16 |
| 3 | 2 | 6 |
| 4 | 1 | 4 |
| 5 | 2 | 10 |
| 6 | 1 | 6 |
| | $\sum f = 30$ | $\sum fx = 54$ |

SO,

$$\mu = \frac{\sum fx}{\sum f} = \frac{54}{30} =$$

1.8

STEP 2: COMPLETE THE TABLE.

| x | f | fx | $ x - \mu $ | $f x - \mu $ |
|---|-----------------|------------------|-------------|----------------------------|
| 0 | 4 | 0 | 1.8 | 7.2 |
| 1 | 12 | 12 | 0.8 | 9.6 |
| 2 | 8 | 16 | 0.2 | 1.6 |
| 3 | 2 | 6 | 1.2 | 2.4 |
| 4 | 1 | 4 | 2.2 | 3.3 |
| 5 | 2 | 10 | 3.2 | 6.4 |
| 6 | 1 | 6 | 4.2 | 4.2 |
| | $\Sigma f = 30$ | $\Sigma fx = 54$ | | $\Sigma f x - \mu = 33.6$ |

STEP 3: DIVIDE THE ANSWER OF

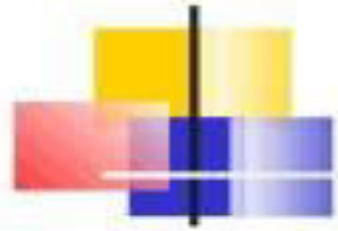
$\frac{\sum f|x-\mu|}{\sum f}$ **TO THE**
SUMMATION OF $\sum f$

$$\frac{\sum f|x-\mu|}{\sum f} = \frac{33.6}{30} = 1.12$$

MEAN DEVIATION =

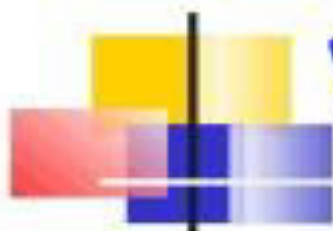
1.12

VARIANCE



Variance

Variance is the average squared deviation from the mean of a set of data. It is used to find the **standard deviation**.



Variance

1. Find the **mean** of the data.

Hint - mean is the average so add up the values and divide by the number of items.

2. Subtract the mean from each value - the result is called the **deviation from the mean**.
3. Square each deviation of the mean.
4. Find the sum of the squares.
5. Divide the total by the number of items.



Variance Formula

The **variance** formula includes the Sigma Notation, ~~what it represents~~ the sum of all the items to the right of Sigma.

$$\frac{\sum (x - \mu)^2}{n}$$

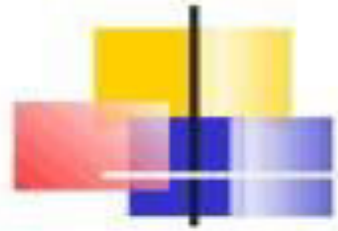
Mean is represented by μ and n is the number of items.

The background is a blue gradient with decorative white circuit-like lines in the corners. The lines consist of straight segments and small circles, resembling a network or data flow diagram.

STANDARD DEVIATION

Standard Deviation

- The concept of standard deviation was first introduced by **Karl Pearson** in 1893.
- Karl Pearson after observing all these things has given us a more scientific formula for calculating or measuring dispersion. While calculating **SD** we **take deviations of individual observations from their AM and then each squares. The sum of the squares is divided by the Total number of observations. The square root of this sum is known as standard deviation.**
- The standard deviation is the most useful and the most popular measure of dispersion.
- It is always calculated from the **arithmetic mean**, **median** and **mode** is not considered.



Standard Deviation

Standard Deviation shows the variation in data. If the data is close together, the standard deviation will be small. If the data is spread out, the standard deviation will be large.

Standard Deviation is often denoted by the lowercase Greek letter σ sigma, .

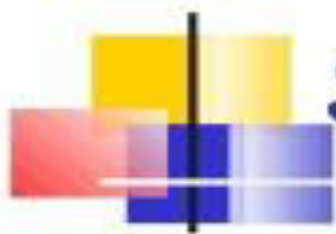


Standard Deviation

Find the **variance**.

- a) Find the **mean** of the data.
- b) Subtract the mean from each value.
- c) Square each deviation of the mean.
- d) Find the sum of the squares.
- e) Divide the total by the number of items.

Take the square root of the variance.



Standard Deviation Formula

The standard deviation formula can be represented using Sigma Notation:

$$\sigma = \sqrt{\frac{\sum (x - \mu)^2}{n}}$$

Notice the standard deviation formula is the square root of the variance.



Find the variance and standard deviation

The math test scores of five students are: 92, 88, 80, 68 and 52.

1) Find the **mean**: $(92+88+80+68+52)/5 = 76$.

2) Find the **deviation from the mean**:

$$92-76=16$$

$$88-76=12$$

$$80-76=4$$

$$68-76=-8$$

$$52-76=-24$$



Find the variance and standard deviation

The math test scores of five students are: 92, 88, 80, 68 and 52.

3) Square the deviation from the

mean:

$$(16)^2 = 256$$

$$(12)^2 = 144$$

$$(4)^2 = 16$$

$$(-8)^2 = 64$$

$$(-24)^2 = 576$$



Find the variance and standard deviation

The math test scores of five students are: 92, 88, 80, 68 and 52.

4) Find the sum of the squares of the deviation from the mean:

$$256 + 144 + 16 + 64 + 576 = 1056$$

5) Divide by the number of data items to find the **variance**:

$$1056 / 5 = 211.2$$



Find the variance and standard deviation

The math test scores of five students are: 92, 88, 80, 68 and 52.

6) Find the square root of the variance: $\sqrt{211.2} = 14.53$

Thus the **standard deviation** of the test scores is **14.53**.

FIND THE VARIANCE AND STANDARD DEVIATION

The math test scores of five students are:

92, 88, 80, 68 and 52.



1) Find the **mean**:

$$(92+88+80+68+52)/5 = 76.$$

2) Find the **deviation from the mean**:

$$92-76=16$$

$$88-76=12$$

$$80-76=4$$

$$68-76= -8$$

$$52-76= -24$$



3) Square the deviation from the mean:

$$(16)^2 = 256$$

$$(12)^2 = 144$$

$$(4)^2 = 16$$

$$(-8)^2 = 64$$

$$(-24)^2 = 576$$



4) Find the sum of the squares of the deviation from the mean:

$$256+144+16+64+576= 1056$$

5) Divide by the number of data items to find the **variance**:

$$1056/5 = 211.2$$



6) Find the square root of the variance:

$$\sqrt{211.2} = 14.53$$

Thus the **standard deviation** of the test scores is **14.53**.

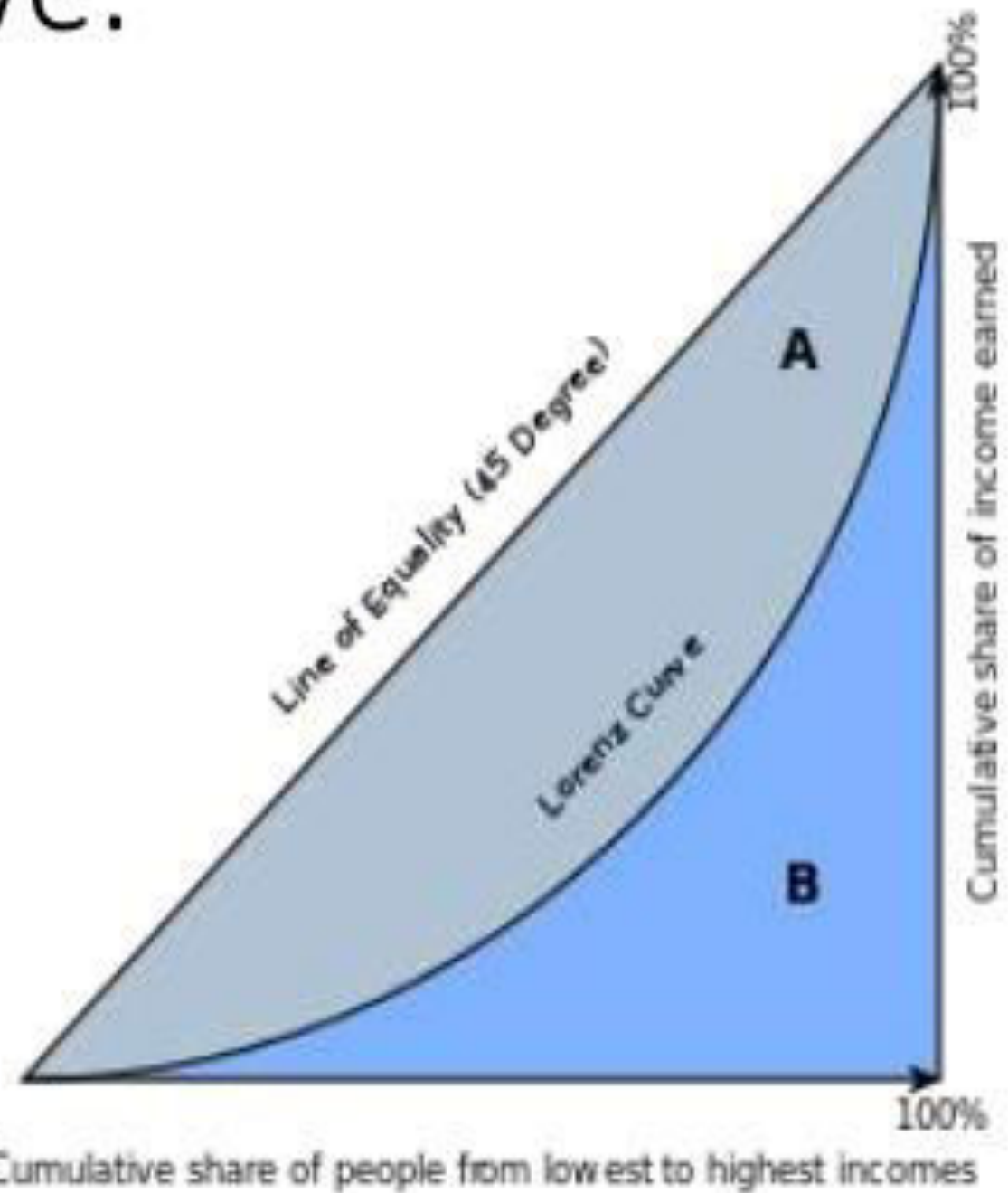


The Lorenz curve.

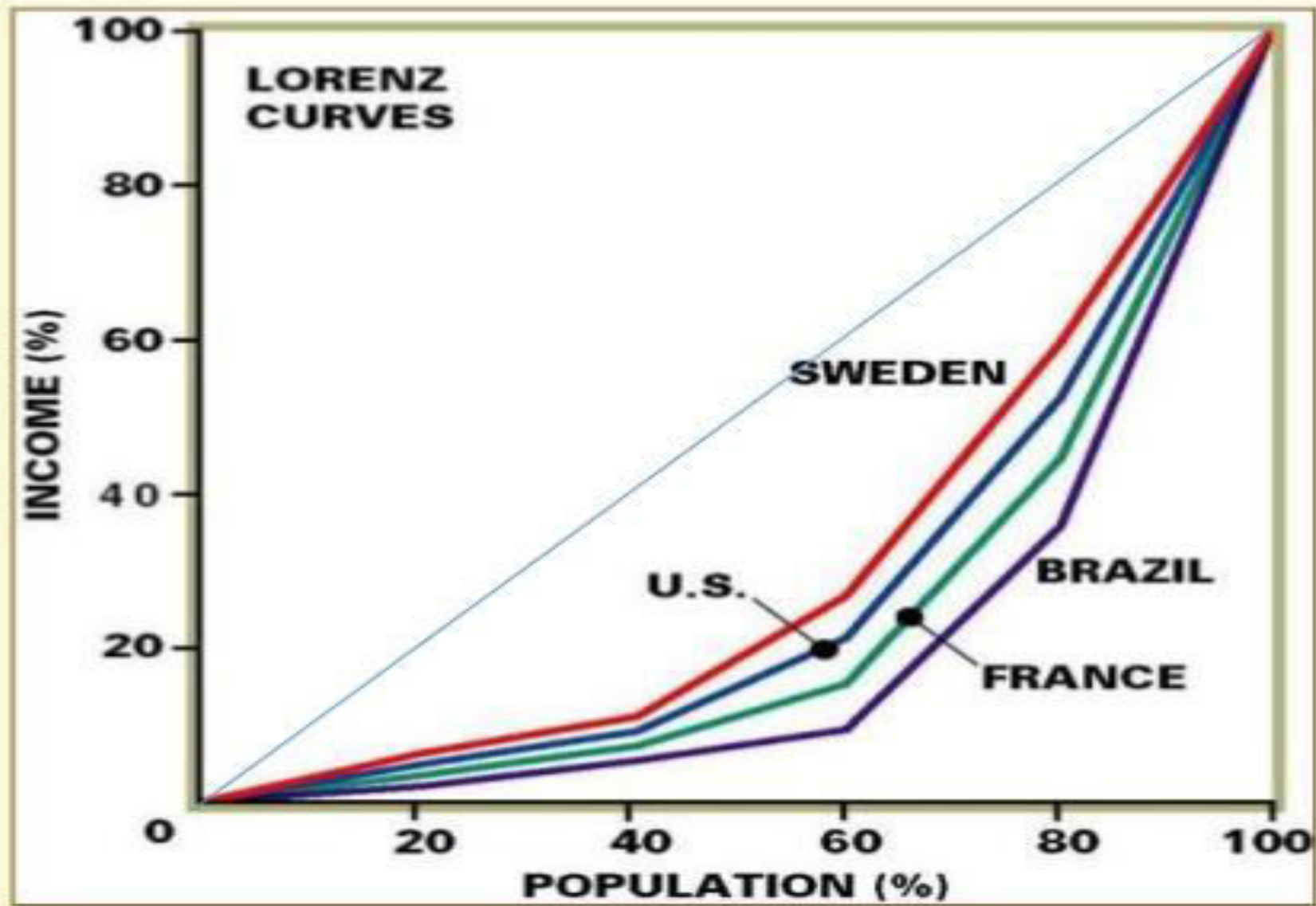
The Lorenz curve represents income distribution. The line of equality shows what an equal distribution would look like, whereas the Lorenz curve shows the actual distribution.

The greater the degree of the bow the greater the
.....

Annotate your diagram to show x and y axis.



Cumulative share of people from lowest to highest incomes



To draw a Lorenz Curve, follow these steps:

1. Gather the data (e.g. census data from two cities)
2. For each set of data, rank the categories and order them by rank in a table.
3. Convert each value in a % of the total.
4. Calculate the running totals (ie cumulative %, by adding the % of one line to the ones before)