UNIT - III

The Production Function:

The production function expresses a functional relationship between quantities of inputs and outputs it shows how and to what extent output changes with variations in inputs during a specified period of time. In the words of Stigler, The production function is the name given to the relationship between rates of input of productive services and the rate of output of product.

It is the economist's summary of technical knowledge Basically the production function is a technological or engineering concept which can be expressed in the form of a table, graph and equation showing the amount of output obtained from various combinations of inputs used in production, given the state of technology. Algebraically, it may be expressed in the form of an equation as

$Q = F(L, M, N, C, \bar{T})$

where Q stands for the output of a good per unit of time, L for labour, M for management (of organisation), N for land (or natural resources), C for capital and $\bar{T}$ for given technology and F refers to the functional relationship function with many inputs cannot be depicted on a diagram.

Economists, therefore, use a two-input production function. If we take two[4] inputs, labour and capital, the production function assumes the form.

$Q = F(L, C)$

Such a production function is shown in Figure 1.

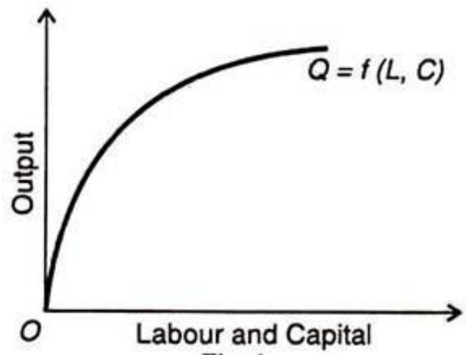

Fig. 1

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The production function as determined by technical conditions of production is of two types: it may be rigid or flexible. The former relates to the short-run and the latter to the long-run. In the short-run, the technical conditions of production are rigid so that the various inputs used to produce a given output are in fixed proportions.

However, in the short-run, it is possible to increase the quantities of one input while keeping the quantities of other inputs constant in order to have more output. This aspect of the production function is known as the Law of Variable Proportions. In the long-run, it is possible for a firm to change all inputs up or down in accordance with its scale. This is known as returns to scale.

The returns to scale are constant when output increases in the same proportion as the increase in the quantities of inputs. The returns to scale are increasing when the increase in output is more than proportional to the increase in inputs. They are decreasing if the increase in output is less than proportional to the increase in inputs.

Let us illustrate the case of constant returns to scale with the help of our production function:

$Q = (L, M, N, C, T)$

Given $\bar{T}$, if the quantities of all inputs L, M, N, C are increased n-fold, the output Q also increases n-fold. Then the production function becomes

$nQ = f(nL, nM, nN, nC)$

This is known as the linear and homogeneous production function, or a homogeneous function of the first degree. If the homogeneous function is of the first degree, the production function is

n^k. Q = f (nL, nM, nN, nC)

If k is equal to 1, it is a case of constant returns to scale, if it is greater than 1, it is a case of increasing returns to scale, and if it is less than 1, it is a case of decreasing returns to scale.

Thus a production function is of two types:

(i) Linear homogeneous of the first degree in which the output would change in exactly the same proportion as the change in inputs. Doubling the inputs would exactly double the output, and vice versa. Such a production function expresses constant returns to scale,

(ii) Non- homogeneous production function of a degree greater or less than one. The former relates to increasing returns to scale and the latter to decreasing returns to scale. One of the important production functions based on empirical hypothesis is the Cobb-Douglas production function.

Originally, it was applied to the whole manufacturing industry in America though it can be applied to the whole economy or to any of its sectors. The Cobb-Douglas production functions is

$Q = A C^a L^{1-a}$ where Q stands for output, L for labour, C for capital employed, A and a are positive constants. In this function, the exponents of L and C added together are equal to 1.

1. The Law of Variable Proportions

If one input is variable and all other inputs are fixed the firm's production function exhibits the law of variable proportions. If the number of units of a variable factor is increased, keeping other factors constant, how output changes is the concern of this law. Suppose land, plant and equipment are the fixed factors, and labour the variable factor.

When the number of labourers is increased successively to have larger output, the proportion between fixed and variable factors is altered and the law of variable proportions sets in.

According to Prof. Left-witch, "The law of variable proportions states that if a variable quantity of one resource is applied to a fixed amount of other input, output per unit of variable input will increase but beyond some point the resulting increases will be less and less, with total output reaching a maximum before it finally begins to decline."

This principle can also be defined thus: When more and more units of the variable factor are used, holding the quantities of a fixed factor constant, a point is reached beyond which the marginal product, then the average and finally the total product will diminish.

The law of variable proportions (or the law of non-proportional returns) is also known as the law of diminishing returns. But, as we shall see below, the law of diminishing returns is only one phase of the more comprehensive law of variable proportions.

Its Assumptions:

(1) It is possible to change the proportions in which the various factors (inputs) are combined.

(2) Only one factor is variable while others are held constant.

(3) All units of the variable factor are homogeneous.

(4) There is no change in technology.

(5) It assumes a short-run situation.

(6) The product is measured in physical units, i.e.. in quintals , tonnes, etc.

(7) The price of the product is given and constant.

Explanation of the Law:

Let us illustrate the law with the help of Table 1, where on the fixed factor (input) land of 4 acres units of the variable factor labour are employed and the resultant output is obtained. The production function is revealed in the first two columns. The average product and marginal product columns are derived from the total product column.

The average product per worker is obtained by dividing column (2) by a corresponding unit in column (1) The marginal product is the addition to total product by employing an extra worker. For instance, 3 workers produce 36 units and 4 produce 48 units. Thus the marginal product is 12 – (48-36) units.

TABLE 1 : Output of Wheat in Physical Units

(1) Land	(2) No. of Workers	(3) Total Product (TP)	(4) Average Product (AP)	(5) Marginal Product (MP)	Stage
4 acres	1	8	8	8	Stage I
4	2	20	10	12	
4	3	36	12	16	
4	4	48	12	12	Stage II
4	5	55	11	7	
4	6	60	10	5	
4	7	60	8.6	0	Stage III
4	8	56	7	-4	

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An analysis of the Table shows that the total, average and marginal products increase a maximum and then start declining. The total product reaches its maximum when 7 units of labour are used and then it declines. The average product continues to rise till the 4th unit while the marginal product reaches its maximum at the 3rd unit of labour, then they also fall.

It should be noted that the point of falling output is not the same for total, average and marginal product. The marginal product starts declining first, the average product following it and the total product is the last to fall. This observation points out that the tendency to diminishing returns is ultimately found in the three productivity concepts.

The law of variable proportions is presented diagrammatically in Figure 2. The TP curve first rises at an increasing rate up to point A where its slope is the highest. From point A upwards, the total product increases at a diminishing rate till it reaches its highest point C and then it starts falling.

Point A where the tangent touches the TP curve is called the inflection point up to which the total product increases at an increasing rate and from where it starts increasing at a diminishing rate.

The marginal product curve (MP) and the average product curve (AP) also rise with TP. The MP curve reaches its maximum point D when the slope of the N curve is the maximum at point A. The maximum point on the AP curve is E where it coincides with the MP curve. This point also coincides with point B on the TP curve from where the total product starts a gradual rise.

When the IP curve reaches its maximum point C, the MP curve becomes zero at point F. When the TP starts declining the MP curve becomes negative i.e. is below X-axis. It is only when the total product declines the average product becomes zero i.e. touches the X-axis. The rising, the falling and the negative phases of the total, marginal and average products are, in fact, the different stages of the law of variable proportions which are discussed below.

Stage-I: Increasing Returns

In stage I, the average product reaches the maximum and equals the marginal product when 4 workers are employed, as shown in Table 1. This stage is portrayed in the figure from the origin to point E where the MP and AP curves meet.

In this stage, the TP curve also increases rapidly. Thus this stage relates to increasing average returns. Here land is too much in relation to the workers employed. It is, therefore, uneconomical to cultivate land in this stage.

The main reason for increasing returns in the first stage is that in the beginning the fixed factor is large in quantity than the variable factor. When more units of the variable factor are applied to a fixed factor, the fixed factor is used more intensively and production increases rapidly.

It can also be explained in another way. In the beginning the fixed factor cannot be put to the maximum use due to the non-applicability of sufficient units of the variable factor. But when units of the variable factor are applied in sufficient quantities, division of labour and specialization lead to per unit increase in production and the law of increasing returns operate.

Another reason for increasing returns is that the fixed factor is indivisible which means that it must be used in a fixed minimum size. When more units of the variable factor are applied on such a fixed factor, production increases more than proportionately. This cause points towards the law of increasing returns.

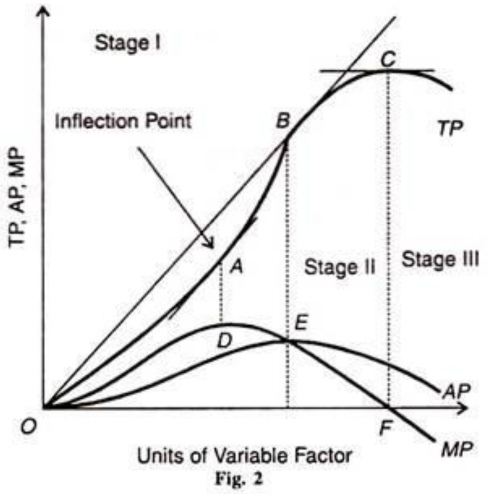

Fig. 2

Stage-II: Law of Diminishing Returns

In between stages I and III is the most important stage of production that of diminishing returns. Stage II starts when the average product is at its maximum to the zero point of the marginal product. At the latter point, the total product is the highest.

Table 1 show this stage when the workers are increased from four to seven to cultivate the given land, in Figure 2 between EB and FC. Here land is scarce and is used intensively. More and more workers are employed in order to have larger output.

Thus the total product increases at a diminishing rate and the average and marginal products decline. Throughout this stage, the marginal product is below the average product. This is the only stage in which production is feasible and profitable.

Hence it is not correct to say that the law of variable proportions is another name for the law of diminishing returns. In fact, the law of diminishing returns is only one phase of the law of variable proportions.

Stage-III: Negative Marginal Returns:

Production cannot take place in Stage III either. For, in this stage, total product starts declining and the marginal product becomes negative. The

Formatted: Font: (Default) Times New Roman, 12 pt, Font color: Blue employment of the 8th worker actually causes a decrease in total output from 60 to 56 units and makes the marginal product minus 4.

In the figure, this stage starts from the dotted line FC where the MP curve is below the X-axis. Here the workers are too many in relation to the available land, making it absolutely impossible to cultivate it. To the right of point F, the variable input is used excessively. Therefore, production will not take place in this stage.

The Law of Diminishing Returns

Benham defines the law of diminishing returns thus: “As the proportion of one factor in a combination of factors is increased, after a point, the average and marginal product of that factor will diminish.”

Marshall applied the operation of this law to agriculture fisheries, mining, forests and the building industry. He defined the law in these words, “An increase in the capital and labour applied in the cultivation of land causes in general a less than proportionate increase in the amount of produce raised, unless it happens to coincide with an improvement in the arts of agriculture.”

It applies to agriculture both in its intensive and extensive forms. The application of additional units of labour and capital to a piece of land causes diminishing returns. Similarly, increasing the proportion of land in relation to doses of labour and capital causes diminishing return. This is because in agriculture close supervision is not possible. Possibilities of division of labour and the use of machines are limited.

Natural calamities like rain, climate, drought, pests, etc. hinder agricultural operations and bring about diminishing returns. Lastly, agriculture is a seasonal industry. So labour and capital cannot be worked to their full capacity. As a result, costs increase in proportion to the product produced. That is why it is also called the law of increasing costs.

This law also applies to river or tank fisheries where the application of additional doses of labour and capital does not bring a proportionate increase to the amount of fish caught. As more and more fish are caught, the quantity of fish decreases because their quantity is limited in a river or tank. In the case of mines and brickfields, the continued application of labour and capital will result in diminishing rate of return.

This is because costs will rise in proportion to the yield from the mines as mining operations are carried deep into the mines. So is the case with forest wealth. In order to get more wood, one has to go deep into the forest which requires clearing of shrubs, paying of ways and handling of wood.

These operations require more and more units or labour and capital, thereby increasing the costs in proportion to the output obtained. Further, the law applies to the construction of buildings.

The construction of a multi-storeyed building or sky-scraper requires additional expenses for providing artificial light and ventilation to the lower storeys and power-lifts to reduce the inconvenience of going to the higher floors. It means increase in costs and diminishing returns.

2. The Law of Returns to Scale

The law of returns to scale describes the relationship between outputs and the scale of inputs in the long- run when all the inputs are increased in the same proportion. According to Roger Miller, the law of returns to scale refers **"to the relationship between changes in output and proportionate changes in all factors of production."**

To meet a long-run change in demand, the firm increases its scale of production by using more space, more machines and labourers in the factory.

Assumptions:

This law assumes that

(1) All factors (inputs) are variable but enterprise is fixed.

(2) A worker works with given tools and implements.

(3) Technological changes are absent.

(4) There is perfect competition.

(5) The product is measured in quantities.

Given these assumptions, when all inputs are increased in unchanged proportions and the scale of production is expanded, the effect on output shows three stages.

Firstly, returns to scale increase because the increase in total output is more than proportional to the increase in all inputs.

Secondly, returns to scale become constant as the increase in total product is in exact proportion to the increase in inputs.

Lastly, returns to scale diminish because the increase in output is less than proportionate to the increase in inputs. This principle of returns to scale is explained with the help of Table 2 and Figure 3.

TABLE 2 : Return to Scale in Physical Units

Unit	Scale of Production	Total Returns	Marginal Returns	
1.	1 Worker + 2 Acres Land	8	8	Increasing Returns
2	2 Workers + 4 Acres Land	17	9	
3.	3 Workers + 6 Acres Land	27	10	
4.	4 Workers + 8 Acres Land	38	11	Constant Returns
5.	5 Workers + 10 Acres Land	49	11	
6.	6 Workers + 12 Acres Land	59	10	Diminishing Returns
7.	7 Workers + 14 Acres Land	68	9	
8.	8 Workers + 16 Acres Land	76	8	

This table reveals that in the beginning with the scale of production of (1 worker + 2 acres of land), total output is 8. To increase output when the scale of production is doubled (2 workers + 4 acres of land), total returns are more than doubled. They become 17.

Now if the scale is trebled (3 workers + 6 acres of land), returns become more than three-fold, i.e., 27. It shows increasing returns to scale. If the scale of production is increased further, total returns will increase in such a way that the marginal returns become constant.

In the case of the 4th and 5th units of the scale of production, marginal returns are 11, i.e., returns to scale are constant. The increase in the scale of production beyond this will lead to diminishing returns. In the case of the 6th, 7th and 8th units, the total returns increase at a lower rate than before so that the marginal returns start diminishing successively to 10, 9 and 8.

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In Figure 3, RS is the returns to scale curve where from R to C returns are increasing, from C to D, they are constant and from D onwards they are diminishing. Why do returns to scale first increase, become constant, and then diminish?

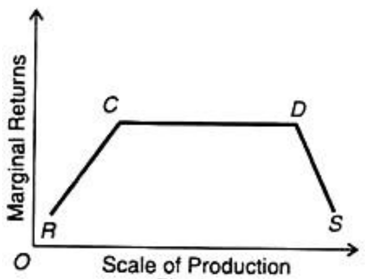

Fig. 3

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(1) Increasing Returns to Scale:

Returns to scale increase because of the indivisibility of the factors of production. Indivisibility m6ans that machines, management, labour, finance, etc., cannot be available in very small sizes. They are available only in certain minimum sizes. When a business unit expands, the returns to scale increase because the indivisible factors are employed to their maximum capacity.

Increasing returns to scale also result from specialisation and division of labour. When the scale of the firm is expanded there is wide scope of specialization and division of labour. Work can be divided into small tasks and workers can be concentrated to narrower range of processes. For this, specialised equipment can be installed. Thus with specialisation, efficiency increases and increasing returns to scale follow.

Further, as the firm expands, it enjoys internal economies of production. It may be able to install better machines, sell its products more easily, borrow money cheaply, procure the services of more efficient manager and workers, etc. All these economies help in increasing the returns to scale more than proportionately.

Not only this, a firm also enjoys increasing returns to scale due to external economies. When the industry itself expands to meet the increased long-run demand for its product, external economies appear which are shared by all the firms in the industry.

When a large number of firms are concentrated at one place, skilled labour, credit and transport facilities are easily available. Subsidiary industries crop up to help the main industry. Trade journals, research and training centres appear which help in increasing the productive efficiency of the firms. Thus these external economies are also the cause of increasing returns to scale.

(2) Constant Returns to Scale

But increasing returns to scale do not continue indefinitely. As the firm is enlarged further, internal and external economies are counterbalanced by internal and external diseconomies. Returns increase in the same proportion so that there are constant returns to scale over a large of output.

Here the curve of returns to scale is horizontal (see CD in Figure 3). It means that the increments of each input are constant at all levels of output. Further, when factors of production are perfectly divisible, substitutable, and homogeneous with perfectly elastic supplies at given prices, returns to scale are constant.

(3) Diminishing Returns to Scale

Constant returns to scale are only a passing phase, for ultimately returns to scale start diminishing. Indivisible factors may become inefficient and less productive. Business may become unwieldy and produce problems of supervision and coordination.

Large management creates difficulties of control and rigidities. To these internal diseconomies are added external diseconomies of scale. These arise from higher factor prices or from diminishing productivities of the factors. As the industry continues to expand, the demand for skilled labour, land, capital, etc. rises.

There being perfect competition, intensive bidding raises wages, rent and interest. Prices of raw materials also go up. Transport and marketing difficulties emerge. All these factors tend to raise costs and the expansion of the firms leads to diminishing returns to scale so that doubling the scale would not lead to doubling the output.

In reality, it is possible to find cases where all factors have tended to increase. Whereas all inputs have increased, enterprise has remained unchanged. In such a situation, changes in output cannot be attributed to a change in scale

alone. It is also due to a shift in factor proportions. Thus, the law of variable proportions is applicable in the real world.

Concept of Costs

1. Accounting costs and Economic costs
2. Outlay costs and Opportunity costs
3. Direct/Traceable costs and Indirect/Untraceable costs
4. Incremental costs and Sunk costs
5. Private costs and Social costs
6. Fixed costs and Variable costs

1. Accounting costs

Accounting costs are those for which the entrepreneur pays direct cash for procuring resources for production. These include costs of the price paid for raw materials and machines, wages paid to workers, electricity charges, the cost incurred in hiring or purchasing a building or plot, etc. Accounting costs are treated as expenses. Chartered accountants record them in financial statements.

2. Economic costs

There are certain costs that accounting costs disregard. These include money which the entrepreneur forgoes but would have earned had he invested his time, efforts and investments in other ventures. For example, the entrepreneur would have earned an income had he sold his services to others instead of working on his own business

Similarly, potential returns on the capital he employed in his business instead of giving it to others, the output generated by his resources which he could have used for others' benefits, etc. are other examples of economic costs.

Economic costs help the entrepreneur calculate supernormal profits, i.e. profits he would earn above the normal profits by investing in ventures other than his.

3. Outlay costs

The actual expenses incurred by the entrepreneur in employing inputs are called outlay costs. These include costs on payment of wages, rent, electricity or fuel charges, raw materials, etc. We have to treat them are general expenses for the business.

4. Opportunity costs

Opportunity costs are incomes from the next best alternative that is foregone when the entrepreneur makes certain choices.

For example, the entrepreneur could have earned a salary had he worked for others instead of spending time on his own business. These costs calculate the missed opportunity and calculate income that we can earn by following some other policy.

5. Direct costs

Direct costs are related to a specific process or product. They are also called traceable costs as we can directly trace them to a particular activity, product or process.

They can vary with changes in the activity or product. Examples of direct costs include manufacturing costs relating to production, customer acquisition costs pertaining to sales, etc.

6. Indirect costs

Indirect costs, or untraceable costs, are those which do not directly relate to a specific activity or component of the business. For example, an increase in charges of electricity or taxes payable on income. Although we cannot trace indirect costs, they are important because they affect overall profitability.

7. Incremental costs

These costs are incurred when the business makes a policy decision. For example, change of product line, acquisition of new customers, upgrade of machinery to increase output are incremental costs.

8. Sunk costs

Suck costs are costs which the entrepreneur has already incurred and he cannot recover them again now. These include money spent on advertising, conducting research, and acquiring machinery.

9. Private costs

These costs are incurred by the business in furtherance of its own objectives. Entrepreneurs spend them for their own private and business interests. For example, costs of manufacturing, production, sale, advertising, etc.

10. Social costs

As the name suggests, it is the society that bears social costs for private interests and expenses of the business. These include social resources for which the firm does not incur expenses, like atmosphere, water resources and environmental pollution.

11. Fixed costs

Fixed costs are those which do not change with the volume of output. The business incurs them regardless of their level of production. Examples of these include payment of rent, taxes, interest on a loan, etc.

12. Variable costs

These costs will vary depending upon the output that the business generates. Less production will cost fewer expenses, and vice versa, the business will pay more when its production is greater. Expenses on the purchase of raw material and payment of wages are examples of variable costs.

Cost in Short Run and Long Run

Cost in Short Run:

Short-Run Total Cost:

A typical short-run total cost curve (STC) is shown in Fig. 14.3. This curve indicates the firm's total cost of production for each level of output when the usage of one or more of the firm's resources remains fixed.

When output is zero, cost is positive because fixed cost has to be incurred regardless of output. Examples of such costs are rent of land, depreciation charges, license fee, interest on loan, etc. They are called unavoidable contractual costs. Such costs remain contractually fixed and so cannot be avoided in the short run.

The total fixed cost (TFC) curve is a horizontal straight line. Total variable is the difference between total cost and fixed cost. The total variable cost curve (TVC) starts from the origin, because such cost varies with the level of output and hence are avoidable. Examples are electricity tariff, wages and compensation of casual workers, cost of raw materials etc.

In Fig. 14.3 the total cost (OC) of producing Q units of output is total fixed cost OF plus total variable cost (FC).

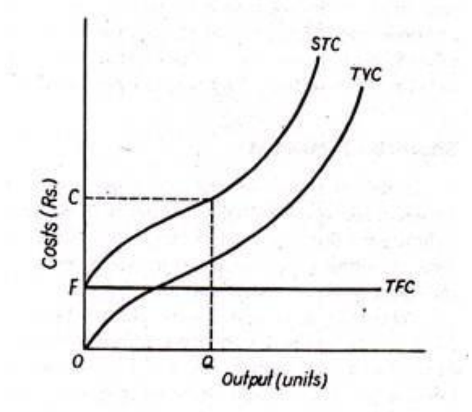

Figure 14.3 Short-run Costs

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Clearly, variable cost and, therefore, total cost must increase with an increase in output. We also see that variable cost first increase at a decreasing rate (the slope of STC decreases) then increase at an increasing rate (the slope of STC increases). This cost structure is accounted for by the law of Variable Proportions.

Average and Marginal Cost:

One can gain a better insight into the firm's cost structure by analysing the behaviour of short-run average and marginal costs. We may first consider average fixed cost (AFC).

Average fixed cost is total fixed cost divided by output,

i.e., AFC = TFC /Q

Since total fixed cost does not vary with output average fixed cost is a constant amount divided by output. Average fixed cost is relatively high at very low output levels. However, with gradual increase in output, AFC continues to fall as output increases, approaching zero as output becomes very large. In Fig. 14.4, we observe that the AFC curve takes the shape of a rectangular hyperbola.

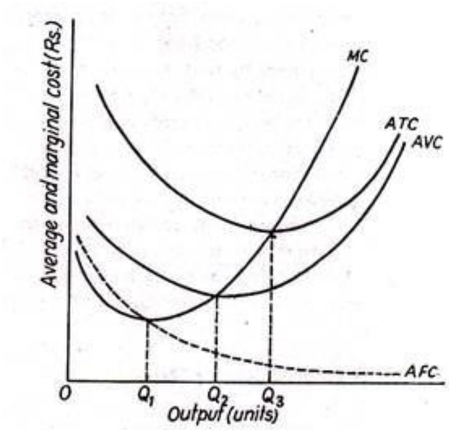

Figure 14.4 Short-run average and marginal cost curves

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We now consider average variable cost (AVC) which is arrived at by dividing total variable cost by output,

i.e., AVC= — TVC/Q

In Fig. 14.4, AVC is a typical average variable cost curve. Average variable cost first falls, reaches a minimum point (at output level Q_2) and subsequently increases.

The next important concept is one of average total cost (ATC).

It is calculated by dividing total cost by output,

i.e., $ATC = \frac{TC}{Q}$

Alternatively, $TC = TFC + TVC$

and $ATC = \frac{TFC}{Q} + \frac{TVC}{Q}$

$= AFC + AVC$

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It is, therefore, the sum of average fixed cost and average variable cost.

The ATC curve, illustrated, is U-shaped in Fig. 14.4 because the AVC cost curve is U-shaped. This is accounted for by the Law of Variable Proportions. It first declines, reaches a minimum (at Q_3 units of output) and subsequently rises. The minimum point on ATC is reached at a larger output than at which AVC attains its minimum. This point can easily be proved.

ATC = AFC + AVC

We know that and that average fixed cost continuously falls over the whole range of output. Thus, ATC declines at first because both AFC and AVC are falling. Even when AVC begins to rise after Q_2, the decrease in AFC continues to drive down ATC as output increases. However, an output of Q_3 is finally reached, at which the increase in AVC overcomes the decrease in AFC, and ATC starts rising.

Since ATC = AFC + AVC, the vertical distance between average total cost and average variable cost measures average fixed cost. Since AFC declines over the entire range of output. AVC becomes closer and closer to ATC as output increases.

We may finally consider short-run marginal cost (SMC). Marginal cost is the change in short-run total cost attributable to an extra unit of output: or

$$SMC = \frac{\Delta STC}{\Delta Q}$$

However, since $STC = TFC + TVC,$

$$SMC = \frac{\Delta TFC}{\Delta Q} + \frac{\Delta TVC}{\Delta Q}$$
$$= 0 + \frac{\Delta TVC}{\Delta Q}$$
$$= \frac{\Delta TVC}{\Delta Q}$$

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Short-run marginal cost refers to the change in cost that results from a change in output when the usage of the variable factor changes. As Fig. 14.4 shows, marginal cost first declines, reaches a minimum at Q_x (note that minimum marginal cost is attained at a level of output less than that at which AVC and ATC attain their minimum) and rises thereafter.

The marginal cost curve intersects AVC and ATC at their respective minimum points. This result follows from the definitions of the cost curves. If marginal cost curve lies below average variable cost curve the implication is clear: each additional unit of output adds less to total cost than the average variable cost.

Thus average variable cost has to fall. So long as MC is above AVC, each additional unit of output adds more to total cost than AVC. Thus, in this case, AVC must rise.

Thus when MC is less than AVC, average variable cost is falling. When MC is greater than AVC, average variable cost is rising. Thus MC must equal AVC at the minimum point of AVC. Exactly the same reasoning would apply to show MC crosses ATC at the minimum point of the latter curve.

a. Short-run Cost Functions:

The total cost function may be expressed as:

TC = k + f(Q) where k is total fixed cost which is a constant, and f(Q) is total variable cost which is a function of output.

ATC = AFC + AVC. Since k is a constant and Q gradually increases, the ratio k/Q falls. Hence the AFC curve is a rectangular hyperbola.

Here

$$MC = \frac{d(TC)}{dQ} = \frac{d}{dQ}(k) + \frac{d}{dQ}[f(Q)] = 0 + f'(Q)$$

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where f'(Q) is the change in TVC and may be called marginal variable cost (MVC). Thus, it is clear that MC refers to MVC and has no relation to fixed cost. Since business decisions are largely governed by marginal cost, and marginal costs have no relation to fixed cost, it logically follows costs do not affect business decisions.

b. Relation between MC and AC:

There is a close relation between MC and AC. When AC is falling, MC is less than AC.

The properties of the average and marginal cost curves and their relationship to each other are as described in Fig. 14.4. From the diagram the following relationships can be discovered.

(1) AFC declines continuously, approaching both axes asymptomatically (as shown by the decreasing distance between ATC and AVC) and is a rectangular hyperbola.

(2) AVC first declines, reaches a minimum at Q_2 and rises thereafter. When AVC is at its minimum, MC equals AVC.

(3) ATC first declines, reaches a minimum at Q_3, and rises thereafter. When ATC is at its minimum, MC equals ATC.

(4) MC first declines, reaches a minimum at Q_1, and rises thereafter. MC equals both AVC and ATC when these curves are at their minimum values.

Table 14.2 : Short-run cost Schedules of a hypothetical firm

(1) Output	(2) Total cost Rs.	(3) Fixed cost Rs.	(4) Variable cost Rs.	(5) Average fixed cost Rs.	(6) Average variable cost Rs.	(7) Average total cost Rs.	(8) Marginal cost (per unit) Rs.
100	6,000	4,000	2,000	40.00	20.00	60.00	20.00
200	7,000	4,000	3,000	20.00	15.00	35.00	10.00
300	7,500	4,000	3,500	13.33	11.67	25.00	5.00
400	9,000	4,000	5,000	10.00	12.50	22.50	15.00
500	11,000	4,000	7,000	8.00	14.00	22.00	20.00
600	14,000	4,000	10,000	6.67	16.67	23.33	30.00
700	18,000	4,000	14,000	5.71	20.00	25.71	40.00
800	24,000	4,000	20,000	5.00	25.00	30.00	60.00
900	34,000	4,000	30,000	4.44	33.33	37.77	100.00
1,000	50,000	4,000	46,000	4.00	46.00	50.00	160.00

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If we compare columns (6) and (8) we see that marginal cost (per unit) is below average variable and average total cost when each is falling and is greater than each when AVC and ATC are rising.

Long-Run

The long run simply refers to a period of time during which all inputs can be varied.

Therefore, a decision has to be made by the owner and/or manager of the firm about the scale of operation, that is, the size of the firm. In order to be able to make this decision the manager must have knowledge about the cost of producing each relevant level of output. We shall now discover how to determine these long-run costs.'

Derivation of Cost Schedules from a Production Function:

For the sake of analysis, we may assume that the firm's level of usage of the inputs does not affect the input (factor) prices. We also assume that the firm's manager has already evaluated the production function for each level of output in the feasible range and has derived an expansion path.

For the sake of analytical simplicity, we may assume that the firm uses only two variable factors, labour and capital, that cost Rs. 5 and Rs. 10 per unit, respectively.

The characteristics of a derived expansion path are shown in Columns 1, 2 and 3 of Table 14.4. In column (1) we see seven output levels and in Columns (2) and (3) we see the optimal combinations of labour and capital respectively for each level of output, at the existing factor prices.

These combinations enable us to locate seven points on the expansion path.

Column (4) shows the total cost of producing each level of output at the lowest possible cost. For example, for producing 300 units of output, the least cost combination of inputs is 20 units of labour and 10 of capital. At existing factor prices, the total cost is Rs. 200. Here, Column (4) is a least-cost schedule for various levels of production.

In Column (5), we show average cost which is obtained by dividing total cost figures of Column (4) by the corresponding output figures of Column (1). Thus, when output is 100, average cost is Rs. 120/100 = Rs. 1.20. All other figures of Column (5) are derived in a similar way.

From column (5) we derive an important characteristic of long-run average cost: average cost first declines, reaches a minimum, then rises, as in the short-run. In Column (6) we show long-run marginal cost figures.

Each such figure is arrived at by dividing change in total cost by change in output. For example, when output increases from Rs. 100 to Rs. 200, the total cost increases from Rs. 120 to Rs. 140. Therefore, marginal cost (per unit) is Rs. 20/100 = Re. 0.20. Similarly, when output increases from 600 to 700 units, MC per unit is 720-560/100 =160/100 =1.60

Column (6) depicts the behaviour of per unit MC: marginal cost first decreases then increases, as in the short run.

We may now show the relationship between the expansion path and long-run cost graphically. In Fig. 14.6 two inputs, K and L, are measured along the two axes. The fixed factor price ratio is represented by the slope of the isocost lines $I_1I'_1$, $l_2l'_2$ and so on. Finally, the known production function gives us the isoquant map, represented by Q_1, Q_2 and so forth.

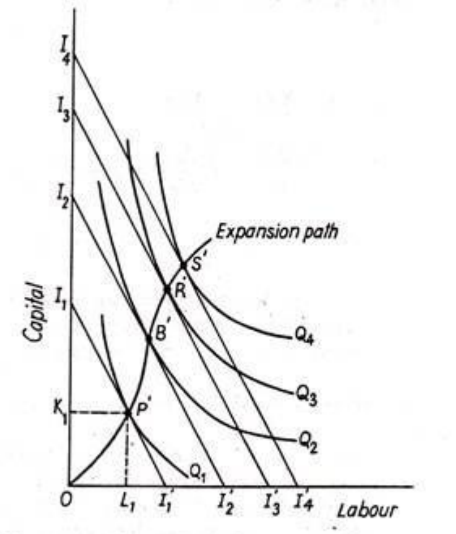

Figure 14.6 The expansion path and long-run cost

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From our earlier discussion of long-run production function we know that, when all inputs are variable (that is, in long-run), the manager will choose the least cost combinations of producing each level of output. In Fig. 14.6, we see that the locus of all such combinations is expansion path OP' B'R'S'.

Given the factor-price ratio and the production function (which is determined by the state of technology), the expansion path shows the combinations of inputs that enables the firm to produce each level of output at the lowest cost.

Table 14.4 : Derivation of long-run cost schedules

(1) Output (Units)	(2) Labour (Units)	(3) Least-cost usage of Capital of labour,	(4) Total cost at Rs. 5 per unit Rs. 10 per unit of capital	(5) Average cost	(6) Marginal cost (per Unit)
100	11	7	Rs. 120	Rs. 1.20	Rs. 1.20
200	12	8	140	Re. 0.70	Re. 0.20
300	20	10	200	0.67	0.60
400	30	15	300	0.75	1.00
500	40	22	420	0.84	Rs. 1.20
600	52	30	560	0.93	1.40
700	60	42	720	Rs. 1.03	1.60

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We may now relate this expansion path to a long-run total cost (LRTC) curve. Fig. 14.7 shows the 'least cost curve' associated with expansion path in Fig. 14.6. This least cost curve is the long-run total cost curve. Points P,B,R and S are associated with points P', B', R' and S' on the expansion path. For example, in Fig. 14.6 the least cost combination of inputs that can produce Q_1 is K_1 units of capital and L_1 units of labour.

Thus, in Fig. 14.7, minimum possible cost of producing Q_1 units of output is TC_1, which is $K_1 + wL_1$, i.e., the price of capital (or the rate of interest) times K_1, plus the price of labour (or the wage rate) times L_1. Every other point on LRTC is derived in a similar way.

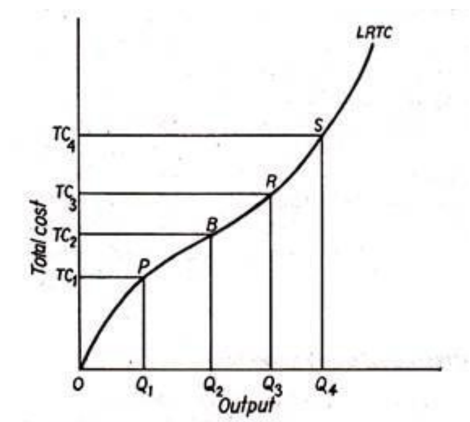

Figure 14.7 Long-run total cost curve

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Since the long run permits capital-labour substitution, the firm may choose different combinations of these two inputs to produce different levels of output. Thus, totally different production processes may be used to produce (say) Q_1 and Q_2 units of output at the lowest attainable cost.

On the basis of this diagram we may suggest a definition of the long run total cost. The time period during which even/thing (except factor prices and the state of technology or art of production) is variable is called the long run and the associated curve that shows the minimum cost of producing each level of output is called the long- run total cost curve.

The shape of the long-run total cost (LRTC) curve depends on two factors: the production function and the existing factor prices. Table 14.4 and Fig. 14.7 reflect two of the commonly assumed characteristics of long-run total costs. First, costs and output are directly related; that is, the LRTC curve has a positive slope. But, since there is no fixed cost in the long run, the long run total cost curve starts from the origin.

Another characteristic of LRTC is that costs first increase at a decreasing rate (until point B in Fig. 14.7), and an increasing rate thereafter. Since the slope of the total cost curve measures marginal cost, the implication is that long-run marginal cost first decreases and then increases. It may be added that all implicit costs of production are included in the LRTC curve.

Long-Run Average and Marginal Costs:

Long-run average cost is arrived at by dividing the total cost of producing a particular output by the number of units produced:

LRTC= LRTC/Q

Long-run marginal cost is the extra total cost of producing an additional unit of output when all inputs are optimally adjusted:

LRTC= Δ LRTC /ΔQ

It, therefore, measures the change in total cost per unit of output as the firm moves along the long run total cost curve (or the expansion path).

Fig. 14.8 illustrates typical long-run average and marginal cost curves. They have essentially the same shape and relation to each other as in the short run. Long-run average cost first declines, reaches a minimum (at Q_2 in Fig. 14.8), then increases. Long-run marginal cost first declines, reaches minimum at a lower output than that associated with minimum average cost (Q_1 in Fig. 14.8), and increases thereafter.

The marginal cost intersects the average cost curve at its lowest point (L in Fig. 14.8) as in the short-run. The reason is also the same. The reason has been aptly summarized by Maurice and Smithson thus: "When marginal cost is less than average cost, each additional unit produced adds less than average cost to total cost; so average cost must decrease.

When marginal cost is greater than average cost, each additional unit of the good produced adds more than average cost to total cost; so average cost must be increasing over this range of output. Thus marginal cost must be equal to average cost when average cost is at its minimum".

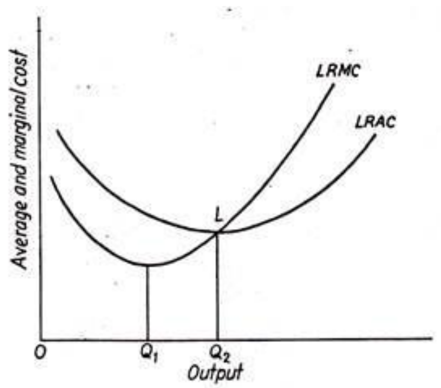

Figure 14.8 Long-run average and marginal cost curves

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Economies and Diseconomies of Scale:

The shape of the long-run average cost depends on certain advantages and disadvantages associated with large scale production. These are known as economies and diseconomies of scale.

1. Economies of Scale:

Various factors may give rise to economies of scale, that is, to decreasing long-run average costs of production.

Greater Specialization of Resources:

With an expansion of a firm's scale of operation, its opportunities for specialization—whether performed by men or by machines—are greatly enhanced. It is because a large-scale firm can often divide the tasks and work to be done more readily than a small-scale firm.

More Efficient Utilization of Equipment:

In some industries, the technology of production is such that a large unit of costly equipment has to be used. The production of automobiles, steel and refined petroleum are obvious examples.

In such industries, companies must be able to afford whatever equipment is necessary and must be able to use it efficiently by spreading the cost per unit over a sufficiently large volume of output. A small-scale firm cannot ordinarily do these things.

Reduced Unit Costs of Inputs:

A large-scale firm can often buy its inputs-such as its raw materials-at a cheaper price per unit and thus gets discounts on bulk purchases. Moreover, for certain types of equipment, the price per unit of capacity is often much less than larger sizes purchased.

Utilization of by-products:

In certain industries, larger-scale firms can make effective use of many by-products that would go waste in a small firm. A typical example is the sugar industry, where by-products like molasses and bagasse are made use of.

Growth of Auxiliary Facilities:

In certain places, an expanding firm often benefits from, or encourages other firms to develop, ancillary facilities, such as warehousing, marketing, and transportation systems, thus saving the growing firm considerable costs. For example, commercial and industrial establishments often benefit from improved transportation and warehousing facilities.

2. Diseconomies of Scale:

With continuous expansion of the scale of operation of a firm, a point may ultimately be reached when diseconomies of scale begin to exercise a more than offsetting effect on the firm's cost curve. As a result, the long-run average cost curve starts to rise.

Decision-Making Role of Management:

As a firm becomes larger, heavier burdens are placed on the management so that eventually this resource input is overworked relative to others and 'diminishing returns' to management set in. In fact, management is an indivisible input which is not capable of continuous variation. With increase in the size of organisation there occurs delay in decision-making.

Competition for Resources:

Rising long-run average costs can occur as a growing firm increasingly bids labour or other resources away from other industries. In the real world, it is very difficult, if not virtually impossible, to determine just when diseconomies of scale are encountered and when they become strong enough to outweigh the economies of scale.