29/03/2021
Monday

UNIT- H

Scheduling by PERT and CPM

Introduction:

A project is defined as a combination of interrelated activities all of which must be executed in a certain order to achieve a set goal. A large and complex project involves usually a number of interrelated activities measuring many machines and materials. It is impossible for the management to make and execute an optimum schedule for such a project us by intuition, based on the organisational capabilities and past experience. A systematic scientific approach has become a necessity for projects. So a number of methods applying network scheduling techniques has been developed. programme Evaluation Review technique (PERT) and critical path Method (CPM) are two of the many techniques which are widely used for planning, Scheduling and controlling large complex projects.

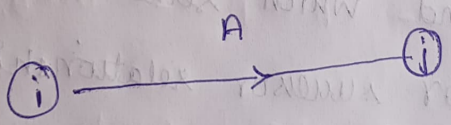
The three main managerial functions for any projects are.

1. planning
2. Scheduling
3. Control

Basic Terminologies:

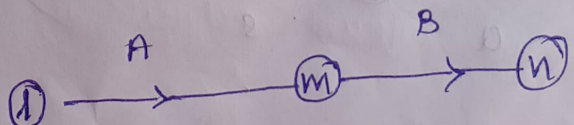
Activity is a task or an item of work to be done in a project. An activity consumes resources like time, money, labour etc..

An activity is represented by an arrow with a node (event) at the beginning and a node (event) at the end indicating the start and termination (finish) of the activity. Nodes are denoted by circles. Since this is a logical diagram length or shape of the arrow has no meaning. The direction indicates the progress of the activity. Initial node and the terminal node are numbered as i & j (j > i) respectively. For example If A is the activity whose initial node is i and the terminal node is j then it is denoted diagrammatically by.

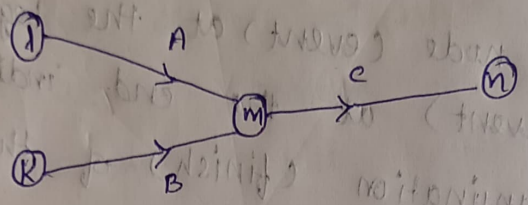


The name of the activity is written over the arrow, not inside the circle. The diagram in which arrow represents an activity is called arrow diagram. The initial and terminal nodes of activities are also called the and head events.

If an activity B can start immediately after an activity A then it is denoted by



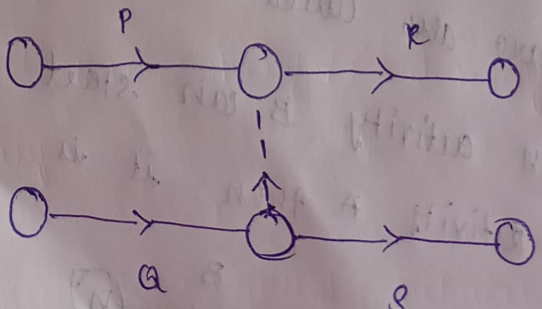
A is called the immediate predecessor of B and B is called the immediate successor of A. If C can start only after completing activity A and B then it is diagrammatically represented as follows:



"A is a predecessor of B" is denoted as "A < B". "B is a successor of A" is denoted by "B > A"

project contains contains two or more activities which have some of their predecessors in common then there is a need for introducing called dummy activity. Dummy activity is an imaginary activity does not consume any resource and which serves the purpose of the predecessor or successor relationship clearly in any activity diagram. The need for a dummy activity is illustrated by the usual example

If P, Q be the predecessors of R and Q be the only predecessor of S.

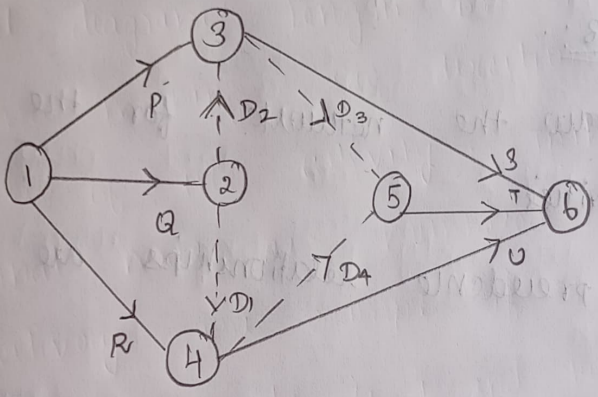


Example : 1

Draw the network for the project whose activities of their precedence relationship are given below.

Activity	P	Q	R	S	T	U
predecessor	–	–	–	P, Q	P, R	Q, R

Solution :-



D, D2 D3 D4 are dummy activities.

Example : 2

Construct the network for the project where activities and their relationships are given below:

Activities ; A, D, E can start simultaneously

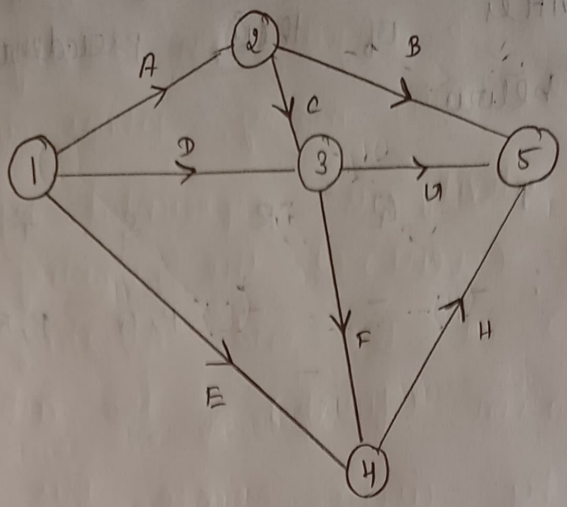
Activities ; B, C > A ; G, F > D, C ; H > E, F

Sol :-

Start activities are A, D, E.

End activities are H, G, B.

The required network is



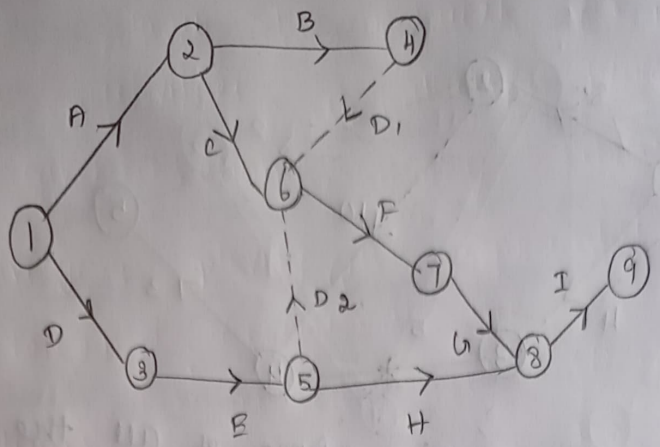
Example 3:

Draw the network for the project who so activities their precedence relationships are as given below:

Activities:	A	B	C	D	E	F	G	H
Immediate predecessor:	-	A	A	-	D	B, C, E	F	E, G, H

Sol: Start Activities: A, D, terminal activities I only. Activities B and C starting with the same node are both the predecessors of the activity F. Also the activity E has to be the predecessor of both F and GH. Therefore dummy activities are necessary.

Thus the required network is



D_1 and D_2 are dummy activities

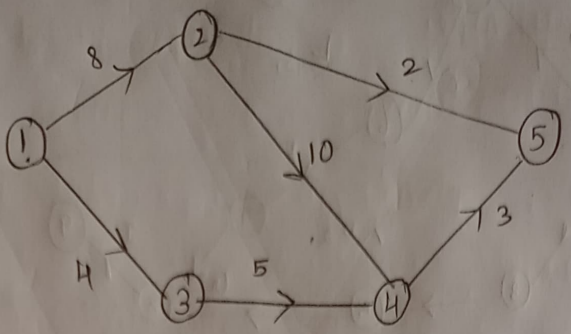
Note: Sometimes while constructing a network you may introduce an dummy activities than necessary. Redundant dummy activities can be found out when one checks whether all the give precedence relationship given in the problem are satisfied exactly. (Nothing more, Less).

Example 1:-

Compute the earliest start, earliest finish latest started latest finish of each activity of the project given below:

Activity	1-2	1-3	2-4	2-5	3-4	4-5
Duration (in days)	8	4	10	2	5	3

First draw the network :-



We take the earliest time of all the start activities as zero.

So earliest starts of 1-2 and 1-3 are zero

To find earliest start of 2-4.

The activity 2-4 can start only after finishing the only preceding activity 1-2 i.e after 8 days.

Earliest start of 2-4 is 8 days. similarly earliest start of 2-5 is and I days.

similarly earliest start of 3-4 is 4 days.

To find the earliest start of 4-5 we first notice that the activity 4-5 is more than one predecessor and also the activity 4-5 can start only the finishing all its preceding activities.

There are two paths leading to the activity 4-5 ; namely 1-2-4 which takes 18 days and 1-3-4 which takes 9 days. Obviously after 18 days by all the activities 1-2, 1-3, 2-4, 3-4 can be finished but n

Wheather than that.

Earliest start of 4-5 is 18 days.

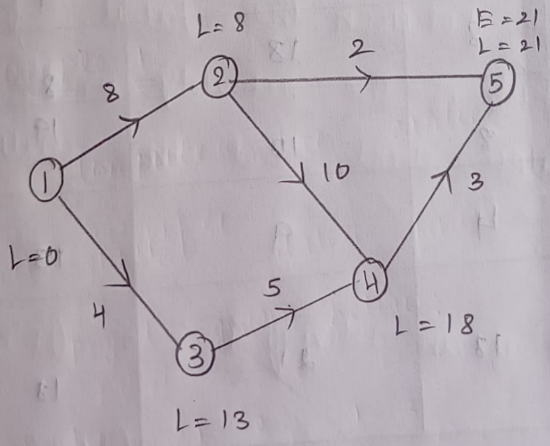
Note: Earliest start of an activity i-j can be denoted as ES, or ES_{ij}. It can also be called the earliest occurrence of the event i.

Earliest finish of any activity i-j is got by adding the duration of the activity denoted by tij to the earliest start of i-j.

Hence the earliest finish of 1-2, 1-3, 2-4, 2-5, 3-4, 4-5 are 14, 18, 10, 9, 21 respectively.

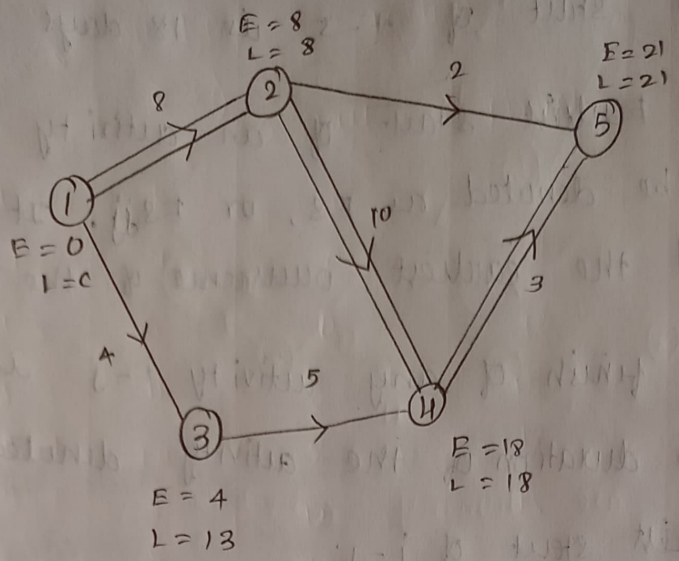
Latest finish

The Latest start of the activities 4-5, 2-5, 2-4, 3-4, 1-3, are 21, 21, 13, 18, 13, 8 respectively.



Latest start time:-

We can tabulate the results and represent these earliest and latest precedence of the events in the network diagram as follows:-



Resource Management Techniques:

		Earliest		Latest	
Activity	Duration days	start ES	Finish EF = ES + tij	start LS = LF − tij	Finish LF
1-2	8	0	8	0	8
1-3	4	0	4	9	13
2-4	10	8	18	8	13
2-5	2	8	10	19	21
3-4	5	4	9	13	18
4-5	3	18	21	18	21

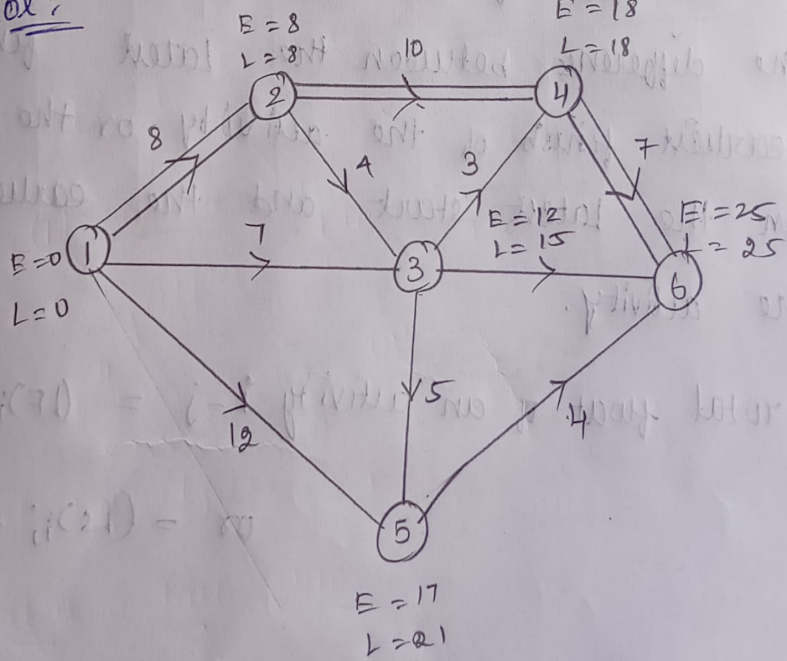
Example 2:

Calculate the earliest finish, earliest start, latest start, latest finish of each activity of the project given below and determine the critical path of the project.

Activity	1-2	1-3	1-5	2-3	2-4
Duration (in weeks)	8	7	12	4	10

Activity	3-4	3-5	3-6	4-6	5-6
Duration (in weeks)	3	5	10	7	4

Sol:-



Activity	Duration (in weeks)	Earliest		Latest	
		Start	Finish	Start	Finish
1-2	8	0	8	0	8
1-3	7	0	7	8	15
1-5	12	0	12	9	21
2-3	4	8	12	11	15
2-4	10	8	18	8	18
3-4	3	12	15	15	18
3-5	5	12	17	16	21
3-6	10	12	22	15	25
4-6	7	18	25	18	25
5-6	4	17	21	21	25

Floats :-

Total float of an activity (T.F) is defined as the difference between the Latest finish and the earliest finish of the activity or the difference between the Latest start and the earliest start of the activity.

Total float of an activity $i-j = (LF)_{ij} - (EF)_{ij}$

or $= (LS)_{ij} - (ES)_{ij}$.

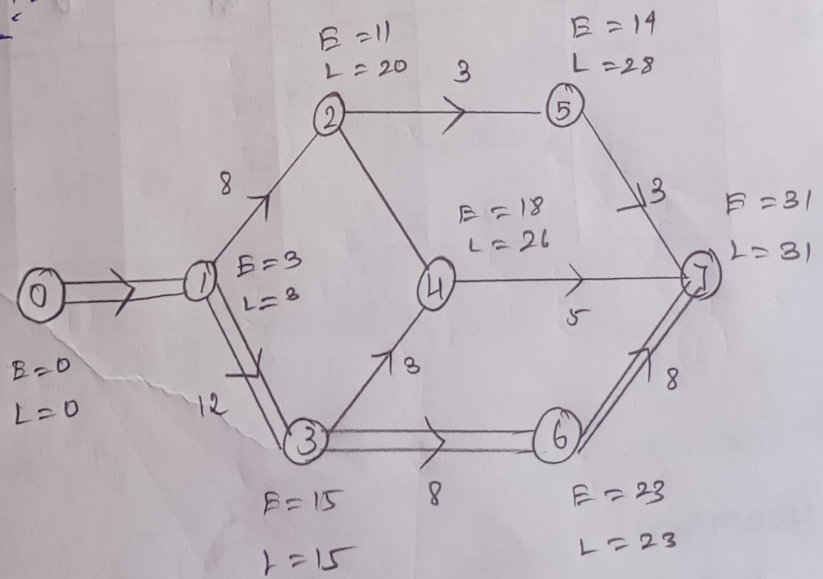
Example 3 :-

Construct the network for the project where the activities are given below and compute the total, free and independent float of each activity and hence determine the critical path and the project duration.

Activity	0-1	1-2	1-3	2-4	2-5
Duration (in weeks)	3	8	12	6	3

Activity	3-4	3-6	4-7	5-7	6-7
Duration (in weeks)	3	8	5	3	8

Sol :-



Activity	Duration (in weeks)	Earliest		Latest		Floats		
		Start	Finish	Start	Finish	TF	FF	IF
0-1	3	0	3	0	3	0	0	0
1-2	8	3	11	12	20	9	0	0
1-3	12	3	15	3	15	0	0	0
2-4	6	11	17	20	26	9	1	-8
2-5	3	11	14	25	28	14	0	-9
3-4	3	15	18	23	26	8	0	0
3-6	8	15	23	15	23	0	0	0
4-7	5	18	23	26	31	8	8	0
5-7	3	14	17	28	31	14	14	0
6-7	8	23	31	23	31	0	0	0