Chapter 7

UNIT - II 1

Transportation Model

7.1 Introduction

Transportation deals with the transportation of a commodity (single product) from '*m*' sources (origins or supply or *capacity* centres) to '*n*' destinations (sinks or demand or requirement centres). It is assumed that

i) Level of supply at each source and the amount of demand at each destination and

ii) The unit transportation cost of commodity from each source to each destination are known [given].

It is also assumed that the cost of transportation is linear.

The objective is to determine the amount to be shifted from each source to each destination such that the total transportation cost is minimum.

Note : The transportation model also can be modified to account for multiple commodities.

I. Mathematical Formulation of a Transportation Problem :

Let us assume that there are m sources and n destinations.

Let a_i be the supply (capacity) at source i, b_j be the demand at destination j, c_{ij} be the unit transportation cost from source i to destination j and x_{ij} be the number of units shifted from source i to destination j.

Then the transportation problem can be expressed mathematically as

$$\text{Minimize } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}\, x_{ij}$$

subject to the constraints

$$\sum_{j=1}^{n} x_{ij} = a_i, \qquad i = 1, 2, 3 \ldots m.$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \qquad j = 1, 2, 3 \ldots n.$$

and $x_{ij} \geq 0$, for all i and j.

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Note 1: The two sets of constraints will be ***consistent*** if

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

(total supply) *(total demand)*

which is the necessary and sufficient condition for a transportation problem to have a feasible solution. Problems satisfying this condition are called ***balanced transportation problems***.

Note 2: If $\sum a_i \neq \sum b_j$, then the transportation problem is said to be ***unbalanced.***

Note 3: For any transportation problem, the coefficients of all x_{ij} in the constraints are unity.

Note 4: The objective function and the constraints being all linear, the transportation problem is a special class of linear programming problem. Therefore it can be solved by simplex method. But the number of variables being large, there will be too many calculations. So we can look for some other technique which would be simpler than the usual simplex method.

Standard transportation table:

Transportation problem is explicitly represented by the following transportation table.

Destination

		D_1	D_2	D_3	...	D_j	...	D_n	Supply
	S_1	c_{11}	c_{12}	c_{13}		c_{1j}		c_{1n}	a_1
Source	S_2	c_{21}	c_{22}	c_{23}		c_{2j}		c_{2n}	a_2
									$\vdots$
	S_i	c_{i1}	c_{i2}			c_{ij}		c_{in}	$\vdots$
	S_m	c_{m1}	c_{m2}			c_{mj}		c_{mn}	a_m
Demand		b_1	b_2	b_3	...		...	b_n	$\sum a_i = \sum b_j$

The mn squares are called ***cells***. The unit transportation cost c_{ij} from the i^{th} source to the j^{th} destination is displayed in the ***upper left side of the*** ***(i, j)th cell.*** Any feasible solution is shown in the table by entering the value of x_{ij} ***in the centre of the*** $(i,j)^{th}$ cell. The various a's and b's are called ***rim requirements***. The feasibility of a solution can be verified by summing the values of x_{ij} along the rows and down the columns.

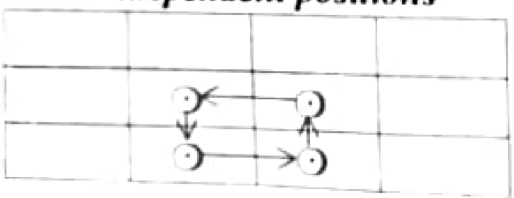
Definition 1: A set of non-negative values x_{ij}, $i = 1,2, \ldots m$; $j = 1,2 \ldots n$. that satisfies the constraints (rim conditions and also the non-negativity restrictions) is called a ***feasible solution*** to the transportation problem.

Note : A balanced transportation problem will always have a feasible solution.

Definition 2: A feasible solution to a $(m \times n)$ transportation problem that contains no more than $m + n - 1$ non-negative allocations is called a ***basic feasible solution*** (BFS) to the transportation problem.

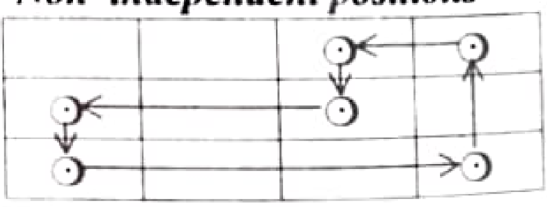
The allocations are said to be in ***independent positions*** if it is impossible to increase or decrease any allocation without either changing the position of the allocation or violating the rim requirements. A simple rule for allocations to be in independent positions is that it is impossible to travel from any allocation, back to itself by a series of horizontal and vertical jumps from one occupied cell to another, without a direct reversal of the route. Example

Non–independent positions



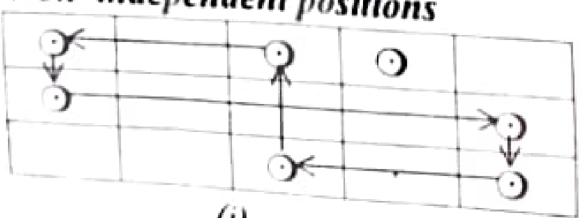
(i)

Non–independent positions



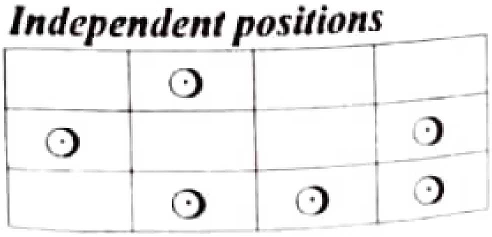
(ii)

Non–independent positions



(i)

Independent positions



(ii)

Definition 3 : A basic feasible solution to a $(m \times n)$ transportation problem is said to be a ***non-degenerate basic feasible solution*** if it contains exactly $m + n - 1$ non-negative allocations in independent positions.

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Definition 4: A basic feasible solution that contains less than $m + n - 1$ non-negative allocations is said to be a ***degenerate basic*** feasible solution.

Definition 5: A feasible solution (not necessarily basic) is said to be an ***optimal solution*** if it minimizes the total transportation cost.

Note : The number of basic variables in an $m \times n$ balanced transportation problem is atmost $m + n - 1$.

Note : The number of non–basic variables in an $m \times n$ balanced transportation problem is atleast $mn - (m + n - 1)$

II. Methods for finding initial basic feasible solution

The transportation problem has a solution if and only if the problem is balanced. Therefore before starting to find the initial basic feasible solution, check whether the given transportation problem is balanced. If not one has to balance the transportation problem first. The way of doing this is discussed in section 7.4 page 7.40 In this section all the given transportation problems are balanced.

Method 1 : North west Corner Rule :

Step 1 : The first assignment is made in the cell occupying the upper left–hand (north–west) corner of the transportation table. The maximum possible amount is allocated there. That is $x_{11} = \min \{a_1, b_1\}$.

Case (i) : If $\min \{a_1, b_1\} = a_1$, then put $x_{11} = a_1$, decrease b_1 by a_1 and move vertically to the 2nd row (*i.e.,*) to the cell (2,1) cross out the first row.

Case (ii) : If $\min \{a_1, b_1\} = b_1$, then put $x_{11} = b_1$, and decrease a_1 by b_1 and move horizontally right (*i.e.,*) to the cell (1,2) cross out the first column

Case (iii) : If $\min \{a_1, b_1\} = a_1 = b_1$ then put $x_{11} = a_1 = b_1$ and move diagonally to the cell (2,2) cross out the first row and the first column.

Step 2: Repeat the procedure until all the rim requirements are satisfied.

Method 2 : Least Cost method (or) Matrix minima method (or) Lowest cost entry method :

Step 1 : Identify the cell with smallest cost and allocate x_{ij} =Min $\{a_i, b_j\}$

Case (i) : If $\min \{a_i, b_j\} = a_i$, then put $x_{ij} = a_i$, cross out the i^{th} row and decrease b_j by a_i. Go to step (2).

Case (ii) : If min $\{a_i, b_j\} = b_j$ then put $x_{ij} = b_j$ cross out the j^{th} column and decrease a_i by b_j Go to step (2).

Case (iii) : If min $\{a_i, b_j\} = a_i = b_j$, then put $x_{ij} = a_i = b_j$, cross out either i^{th} row or j^{th} column but not both, Go to step (2).

Step 2 : Repeat step (1) for the resulting reduced transportation table until all the rim requirements are satisfied.

Method 3: Vogel's approximation method (VAM) (or) Unit cost penalty method : ***[MU. MBA. Nov 96, Apr 95, Apr 97]***

Step 1 : Find the difference (penalty) between the smallest and next smallest costs in each row (column) and write them in brackets against the corresponding row (column).

Step 2 : Identify the row (or) column with largest penalty. If a tie occurs, break the tie arbitrarily. Choose the cell with smallest cost in that selected row or column and allocate as much as possible to this cell and cross out the satisfied row or column and go to step (3).

Step 3 : Again compute the column and row penalties for the reduced transportation table and then go to step (2). Repeat the procedure until all the rim requirements are satisfied.

Example 1 : Determ...

TP. North-West Corner Rule: (1)

1. Calculate the i.b.f.s to the following TPP.

	S_1	S_2	S_3	Available
W_1	5	4	2	~~8~~ 6
W_2	4	7	6	~~12~~ 8
W_3	2	5	8	12
W_4	8	6	7	4.
Requirements	8	10	12	30.

solution:

$\Sigma a_i = \Sigma b_j$, that is Available = Requi[rement]

$\therefore$ The given pb. is Balanced TPP.

	S_1	S_2	S_3	
W_1	$\boxed{6}$ 5	4	2	~~6~~ ~~6~~
W_2	$\boxed{2}$ 4	$\boxed{6}$ 7	6	~~8~~ ~~6~~
W_3	2	$\boxed{4}$ 5	$\boxed{8}$ 8	~~12~~ ~~8~~
W_4	8	6	$\boxed{4}$ 7	4
	~~8~~ ~~2~~	~~10~~ ~~4~~	~~12~~ 4	

$\therefore$ The i b f s is $x_{11}=6$, $x_{21}=2$, $x_{22}=6$, $x_{32}=4$, $x_{33}=8$, $x_{43}=4$.

$\therefore$ Total transportation cost $= \sum_i \sum_j C_{ij} x_{ij}$

$= (6\times5)+(2\times4)+(6\times7)+(4\times5)+(8\times8)+(4\times7)$

$30+8+42+20+64+28 = 192.$

2. Solve the following TPP using N-W CR.

Destination

		D1	D2	D3	D4	Supply
	O1	1	2	1	4	30
Origin	O2	3	3	2	1	50
	O3	4	2	5	9	20
	Demand	20	40	30	10	100

Sn

	D1	D2	D3	D4	Supply
O1	$\boxed{20}$ 1	$\boxed{10}$ 2	1	4	~~30~~ ~~10~~
O2	3	$\boxed{30}$ 3	$\boxed{20}$ 2	1	~~50~~ ~~20~~
O3	4	2	$\boxed{10}$ 5	$\boxed{10}$ 9	~~20~~ ~~10~~
Demand	~~20~~	~~40~~ ~~30~~	~~30~~ ~~10~~	~~10~~	100

∴ the i b f s is $x_{11} = 20$, $x_{12} = 10$, $x_{22} = 30$

$x_{23} = 20$, $x_{33} = 10$, $x_{34} = 10$

Total cost $= \sum_i \sum_j c_{ij} x_{ij}$

$= (20 \times 1) + (10 \times 2) + (30 \times 3) + (20 \times 2) + (10 \times 5) + (10 \times 9)$

$20 + 20 + 90 + 40 + 50 + 90 = 310$.

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Least cost Method. 8 3

1. Obtain e. b. f. s.

	A_1	A_2	A_3	Available.
W_1	5	4	3	6
W_2	4	7	6	8
W_3	2	5	8	12
W_4	8	6	7	4.
Req.	8	10	12	30.

Sn

	A_1	A_2	A_3	
W_1	5	4	$\boxed{6}$ 3	~~6~~
W_2	4	$\boxed{2}$ 7	$\boxed{6}$ 6	~~8~~ ~~2~~
W_3	$\boxed{8}$ 2	$\boxed{4}$ 5	8	~~12~~ ~~4~~
W_4	8	$\boxed{4}$ 6	7	~~4~~
	~~8~~	~~10~~ ~~6~~ ~~4~~	~~12~~ ~~6~~	

$\therefore$ The e b f s is $x_{13} = 6$, $x_{22} = 2$, $x_{23} = 6$, $x_{31} = 8$, $x_{32} = 4$, $x_{42} = 4$

$\therefore$ The Total cost $= \sum_i \sum_j c_{ij} x_{ij}$

$= (6\times3) + (2\times7) + (6\times6) + (8\times2) + (4\times5) + (4\times6)$

$= 18 + 14 + 36 + 16 + 20 + 24 = 128$

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2. obtain Least cost Method.

	S_1	S_2	S_3	S_4	Available
A	5	2	4	3	22
B	1	8	1	6	15
C	4	6	6	5	8
Requirement	7	12	7	19	45

Sn. $\Sigma a_i = \Sigma b_j$, BTPP.

	S_1	S_2	S_3	S_4	
A	5	$\boxed{12}$ 2	4	$\boxed{10}$ 3	~~22~~ ~~10~~
B	$\boxed{7}$ 4	8	$\boxed{7}$ 1	$\boxed{1}$ 6	~~15~~ ~~8~~ 1
C	4	6	6	$\boxed{8}$ 5	~~8~~
	~~7~~	~~12~~	~~7~~	~~19~~ ~~9~~ 1	

$\therefore$ The b.f.s. is

$x_{12} = 12$, $x_{13} = 10$, $x_{21} = 7$, $x_{23} = 7$, $x_{24} = 1$

$x_{34} = 8$! $\therefore$ Total cost $= \sum_i \sum_j c_{ij} x_{ij}$

$= (12\times2) + (10\times3) + (7\times4) + (7\times1) + (1\times6) + (8\times5)$

$= 24 + 30 + 28 + 7 + 6 + 40 = 135$ //

Procedure for the Least cost method or Matrix Minima method.

Step1. Identify the cell smallest cost and allocate $x_{ij} = \min(a_i, b_j)$

case i) If $\min(a_i, b_j) = a_i$ then put $x_{ij} = a_i$ cross cut the i^{th} row and decrease b_j by a_i Go to step 2.

case-ii) If $\min(a_i, b_j) = b_j$, then put $x_{ij} = b_j$ cross out the j^{th} column and decrease a_i by b_j Go to step 2.

case iii) If $\min(a_i b_j) = a_i = b_j$ then put $x_{ij} = a_i = b_j$ cross out the i^{th} row or j^{th} column but both, Go to step 2.

Step2. Repeat step1. for the resulting reduced transportation table until all the rim requirements are satisfied.

Vogel's Approximation Method

Step1. Find the difference (penalty) b/w the smallest and next smallest cost in each row (column) and write them it brackets against the corresponding row (Column)

Step 2: Identify row or column with largest penalty. If a tie occurs, break the tie arbitrarily: choose the cell with smallest cost in that selected row or column and allocate as much as possible to this cell and cross out the satisfied row or column and go to step 3.

Step 3: Again compute the column row penalties for the reduced TP table and then go to step 2. Repeat the ... until all the ...