# OPERATIONs RESEARCH

# UNIT - 1

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# LINEAR PROGRAMMING PROBLEM

**Definition :**

Linear Programming Problem [LPP] deals with determining optimal allocation of limited resources to meet given objectives . The resources may be in the form of men , raw material , marked demand , money and machines etc .

- Programming means planning . All the relationship between the variables considered in this problems are linear . Hence the name Linear Programming Problem [LPP] .
- The objective is maximizing the profit and minimizing the total cost .

**Types :**

- Maximization problems such as maximize the profit
  Minimization problems such as minimize the cost

**Requirements for employing LPP technique :**

- There must be a well defined objective function
- There must be alternative sources of action to choose
- At least some of the resources must be in the supply , which give rise to constraints must be linear equations or inequalities

Mathematical Formulation of [LPP]

Objective Function :

Minimize or Maximize :

$$Z = c_1x_1 + c_2x_2 + ... + c_nx_n$$

Subject to the Constraints

$a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n \leq \text{(or)} = \text{(or)} \geq b_m$

$a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n \leq \text{(or)} = \text{(or)} \geq b_m$

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$a_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n \leq \text{(or)} = \text{(or)} \geq b_m$

and the non – negativity restrictions $x_1 , x_2 , x_3 , ... x_n \geq 0$

**Formulation of LPP :**

**Step : 1 :** Identify the unknown decision variables to be determined and assign symbols to them.

**Step : 2 :** Identify all the restrictions or constraints (or influence factors) in the problem and express them as linear equations or inequalities of decision variables .

**Step : 3 :** Identify the objective or aim and represent it also as a linear function of decision variables .

**Step : 4 :** Express the complete formulation of LPP as general mathematical model .

**Note :** We consider only those situations where this will help the reader to put proper inequalities in the formulation .

| Less than ≤ | Greater than ≥ | Maximum | Minimum |
|---|---|---|---|
| Available machine<br>Resource man<br>Raw materials<br>Not more than | Atleast<br>Atmost<br>Demand more than | Objective function<br>Profit<br>Sales | Purchase cost<br>waste |

- Usage of man power , time , raw material etc are always less than or equal to the available of man power , time , raw material etc .
- Production is always greater than or equal to the requirement so as to meet the demand .

**Problem :**

1) A firm manufactures two types of products A and B and sells them at a profit of Rs. 2 on type A and Rs. 3 on type B . Each product is processed on two machines $M_1$ and $M_2$ . Type A requires 1 minute of process time on $M_1$ and 2 min on $M_2$ . Type B requires 1 minute on $M_1$ and 1 minute on $M_2$ . Machine $M_1$ is available for not more than 6 hours 40 minutes while machine $M_2$ is available for 10 hours during any working day . Formulate the problem as a LPP so as to maximize the profit .

Solution :

Let $x_1$ be product A and $x_2$ be product B

**Objective function :**

Maximize $Z = 2x_1 + 3x_2$

Subject to constraints

$x_1 + x_2 \leq 400$

$2x_1 + x_2 \leq 600$

Non – negativity

$x_1 , x_2 \geq 0$ .

2) A company makes 3 product x , y , z which pass through three departments drill , lath , assembling the hours available in each department , hours required by each product in each department and profit contribution of each product is given below .

Time required in hours

| Product | Drill | Lathe | Assembly | Profit for unit (Rs) |
|---|---|---|---|---|
| X ($x_1$) | 3 | 3 | 8 | 9 |
| Y ($x_2$) | 6 | 5 | 10 | 15 |
| Z ($x_3$) | 7 | 4 | 12 | 20 |

Hours available 210 , 240 , 260 . Formulate the above as an LPP .

**Solution :**

Let $x_1$ be the product of X , $x_2$ be the product of Y and $x_3$ be the product of Z

Objective function :

Maximize $Z = 9x_1 + 15x_2 + 20x_3$

Subject to constraints

$3x_1 + 6x_2 + 7x_3 \leq 210$

$3x_1 + 5x_2 + 4x_3 \leq 240$

$8x_1 + 10x_2 + 12x_3 \leq 260$

Non – negativity

$x_1 , x_2 , x_3 \geq 0$

3) A person requires atleast 10 , 12 and 12 units of the chemicals A , B and C respectively for his garden . A liquid product contains 1 , 2 and 4 units of A , B and C respectively per jar . A dry product contains 5 , 2 and 1 units of A , B and C per carton . A liquid product sales for Rs. 3 per jar and the dry product sales for Rs. 2 per carton . Formulate this LPP for minimizing the cost .

| Product | Jar ($x_1$) | Carton ($x_2$) | Available |
|---|---|---|---|
| A | 1 | 5 | 10 |
| B | 2 | 2 | 12 |
| C | 4 | 1 | 12 |
| Sales | 3 | 2 | |

**Solution :**

Objective function :

Minimize $Z = 3x_1 + 2x_2$

Subject to conditions :

$x_1 + 5x_2 \geq 10$

$2x_1 + 2x_2 \geq 12$

$4x_1 + x_2 \geq 12$

Non – negativity

$x_1 , x_2 \geq 0$ .

4) A manufacture produces two types of models $M_1$ & $M_2$ . Each $M_1$ requires 4 hours of grinding and 2 hours of polishing . each $M_2$ models requires 2 hours of grinding and 5 hours of polishing . The manufacture has 2 grinders and 3 polishers . Each grinder works for 40 hours a week and Each polisher works for 60 hours a week . Profit on $M_1$ models is Rs. 3 and $M_2$ models is Rs. 4 . Make the maximum profit in a week

| Product | $M_1$ ($x_1$) | $M_2$ ($x_2$) | Hours | Availability |
|---|---|---|---|---|
| Grinder | 4 | 2 | 40 | $2 \times 40 = 80$ |
| Polisher | 2 | 5 | 60 | $3 \times 60 = 180$ |
| Profit | 3 | 4 | | |

**Solution :**

Objective function :

Maximize $Z = 3x_1 + 4x_2$

Subject to constraints

$4x_1 + 2x_2 \leq 80$

$2x_1 + 5x_2 \leq 180$ Non – negativity : $x_1 , x_2 \geq 0$ .

5) An animal feed company must produce 200 kg of a mixed consisting of ingredients of $x_1$ and $x_2$ . The cost of $x_1$ is Rs. 3/kg and $x_2$ is Rs. 5/kg . Not more than 80kg of $x_1$ can be used and atleast 60kg of $x_2$ used . Formulate the LPP .

**Solution :**

Objective Function :

Minimize $Z = 3x_1 + 5x_2$

Subject to constraints

$x_1 + x_2 = 200$

$x_1 \leq 80$

$x_2 \geq 60$

Non – negativity : $x_1 , x_2 \geq 0$ .

6) A firm produces an alloy having the following specifications :

(i)Specific gravity $\leq 0.98$

(ii) Chromium $\geq 8\%$

(iii) Melting point $\geq 450^\circ C$

Raw materials A , B and C having the properties shown in the table can be used to make the alloy .

| Property | Raw materials | | |
|---|---|---|---|
| | A ($x_1$) | B ($x_2$) | C ($x_3$) |
| Specific gravity | 0.92 | 0.97 | 1.04 |
| Chromium | 7 % | 13 % | 16 % |
| Melting point | 440°C | 490°C | 480°C |

Cost of the various raw materials per unit ton are : Rs. 90 for A , Rs. 280 for B and Rs. 40 for C . Find the proportions in which A , B and C be used to obtain an alloy of desired properties while the cost of raw materials is minimum .

**Solution :**

Objective function :

Minimize $Z = 90\, x_1 + 280\, x_2 + 40\, x_3$

Subject to constraints

$0.92\, x_1 + 0.97\, x_2 + 1.04\, x_3 \leq 0.98$

$7\, x_1 + 13\, x_2 + 16\, x_3 \geq 8$

$440\, x_1 + 490\, x_2 + 480\, x_3 \geq 450$

Non – negativity : $x_1 , x_2 \geq 0$

7) A farmer has 1000 acres of land on which he can grow corn , wheat or soya beans . Each acre of corn costs Rs. 100 for preparation requires 7 man – days of work and yields a profit of Rs. 30 . An acre of wheat costs Rs. 120 to prepare , requires 10 man – days of work and yields a profit of Rs. 40 . An acre of soya beans costs Rs. 70 to prepare , requires 8 man – days of work and yields a profit of Rs. 20 . The farmer has Rs. 1,00,000 for preparation and 8000 man – days of work . Formulate this as a LPP .

**Solution :**

| Product | Corn | Wheat | Soya beans |
|---|---|---|---|
| Profit | 30 | 40 | 20 |
| Cost | 100 | 120 | 70 |
| Man - days | 7 | 10 | 8 |

Objective function :

Maximize $Z = 30x_1 + 40x_2 + 20x_3$

Subject to constraints

$100x_1 + 120x_2 + 70x_3 \leq 100000$

$7x_1 + 10x_2 + 8x_3 \leq 8000$

$x_1 + x_2 + x_3 \leq 1000$

Non – negativity : $x_1 , x_2 , x_3 \geq 0$ .

03/02/2021
Monday.

Statistics

1. Operation research:

Operation research is the study of optimizing technique. It is applied decisions theory different teams had do research on military operations. In order to invent techniques who manage with available resources so as to obtain the desired objecting.

Scope (or) uses (or) applications of (O.R):

1) Resources, allocation problems.
2) Inventory control problems.
3) Maintances and replacement problems.
4) Sequencing and scheduling problems.
5) Assignment problems.
6) Transportation problems.
7) Shortest root problems.
8) Like travelling sales person's problems.
9) Marketing management problems.
10) finance management problems.
11) production, planning and control problems.
12) design problems.
13) Queuing problems.

## Graphical methods :-

Linear programming problems involve only two variables can be effectively solved by a Graphical method which provide pictorial representation of the problems and it solution and which gives the basic concepts used you solving general Linear L.P.P which main involve any finite number of variables this method is simple to understand and easy to calculate:

## Graphical Method:

Ex: 1

$z = 3x_1 + 2x_2$, $(-2x_1 + x_2 \le 1$, $x_1 + x_2 \le 3$

and $x_1, x_2 \ge 0$.)

Sol:

① $-2x_1 + x_2 \le 1$

put $x_1 = 0$

$\therefore -2(0) + x_2 = 1$

$= (0, 1)$

put $x_2$

$-2x_1 + 0 = 1$

$x_1 = -1/2$

$= (-0.5, 0)$

② $x_1 + x_2 \le 3$

$x_1 = 2$

$x_1 + x_2 = 3$

$x_1 = 0$

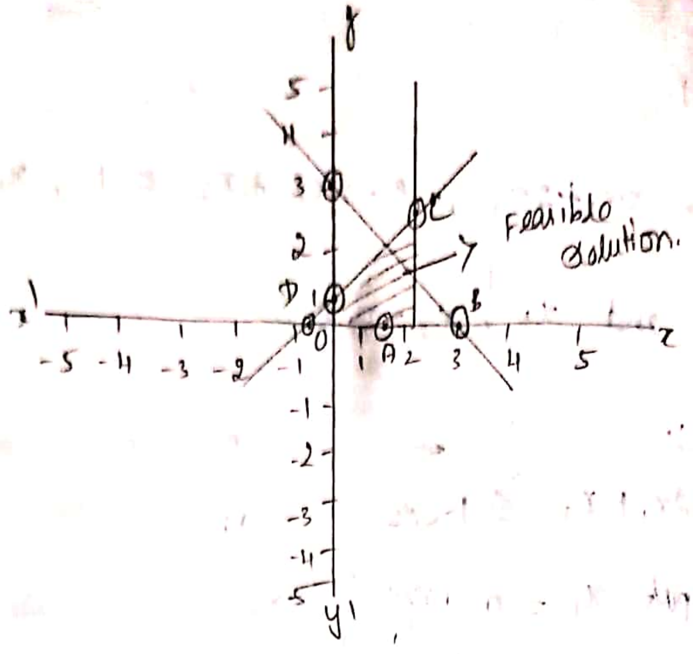
$0 + x_2 = 3$

$= (0, 3)$

$x_2 = 0$

$x_1 + 0 = 3$

$= (3, 0)$




OABCD O(0,0) A(2,0) B(2,1) C$\left(\frac{2}{5},\frac{7}{5}\right)$ D(0,1)

| Vertex | value of | $= 3x_1 + 2x_2$ |
|---|---|---|
| O(0,0) | 0 | $3(0)+2(0) = 0$ |
| A(2,0) | 6 | $3(2)+2(0) = 6$ |
| B(2,1) | 8 | $3(2)+2(1) = 8$ |
| C$\left(\frac{2}{5},\frac{7}{5}\right)$ | 3.6 | $3\left(\frac{2}{5}\right)+2\left(\frac{7}{5}\right) = 1.2+2.4 = 3.6$ |
| D(0,1) | 2 | $3(0)+2(1) = 2$ |

The maximum value of $z = 8$.

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Ex: 2.

Since the profit from type A is Rs. 5 and from type B is Rs. 8 the total profit is $5x_1 + 8x_2$.

∴ The complete formulation of the L.P.P is

Maximum $z = 5x_1 + 8x_2$

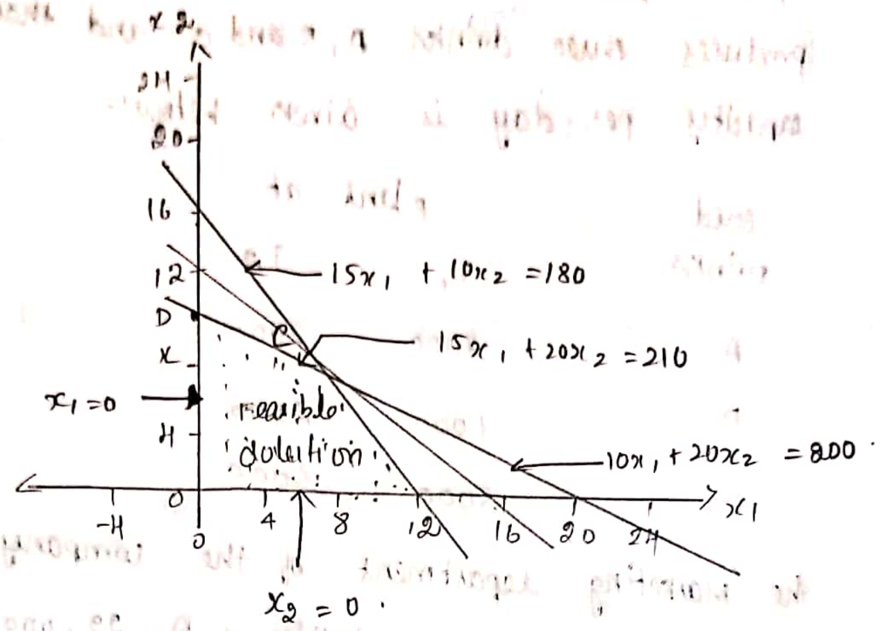
$15x_1 + 10x_2 \le 180$

$10x_1 + 20x_2 \le 200$

$15x_1 + 20x_2 \le 210$

and $x_1, x_2 \ge 0$.

By using Graphical method, OABCD vertices O(0,0), A(12,0), B(10,3), C(2,9) and D(0,10)




The values of z of these vertices are given by.

| Vertex | Value of Z |
|---|---|
| O(0,0) | 0 |
| A(12,0) | 60 |
| B(10,3) | 74 |
| C(2,9) | 82 |
| D(0,10) | 80 |

$(\because Z = 5x_1 + 8x_2)$

Since the problem is of maximization type, the optimum solution is maximum $Z = 82$, $x_1 = 2$, $x_2 = 9$.

Ex: 3

A company making cold drinks has two bottling plants located at towns $T_1$ and $T_2$. Each plant produces three drinks A, B and C and their production capacity per day is given below:

| Cold Drinks | Plant at $T_1$ | Plant at $T_2$ |
|---|---|---|
| A | 6000 | 2000 |
| B | 1000 | 2500 |
| C | 3000 | 3000 |

The marketing department of the company forecasts a demand of 80,000 bottles of A, 22,000 bottles of B & 40,000 bottles of C during the month of June. The operating costs per day of plants at $T_1$ and $T_2$ are ₹6000 and Rs 4000 respectively. Find graphically the number of days for which each plant must be run in June so as to minimize the operating costs while meeting the market demand.

Sol:

Let the plant at $T_1$ and $T_2$ be run for $x_1$ and $x_2$ days respectively.

Since the plants at $T_1$ and $T_2$ run for $x_1$ and $x_2$ days, they will produce

$6000x_1 + 2000x_2$ bottles of A

$1000x_1 + 2500x_2$ bottles of B

$3000x_1 + 3000x_2$ bottles of C.

Since the demand for the cold drinks A, B and C are 80,000, 22,000 and 40,000 respectively and the production is always greater than or equal to the demand, the constraints are.

$6000x_1 + 2000x_2 \geq 80,000 \Rightarrow 6x_1 + 2x_2 \geq 80$

$\Rightarrow 3x_1 + x_2 \geq 40$

$1000x_1 + 2500x_2 \geq 22,000 \Rightarrow x_1 + 2.5x_2 \geq 22$

$3000x_1 + 3000x_2 \geq 40,000 \Rightarrow 3x_1 + 3x_2 \geq 40.$

and $x_1, x_2 \geq 0$.

Since the operating costs per day at $T_1$ is ₹ 6000 and at $T_2$ is ₹ 4000 and $T_1$, $T_2$ run for $x_1$ and $x_2$ days, the total operating costs is ₹ $6000x_1 + 4000x_2$.

Therefore the objectives function is minimize

$z = 6000x_1 + 4000x_2$.

Minimize $z = 6000x_1 + 4000x_2$

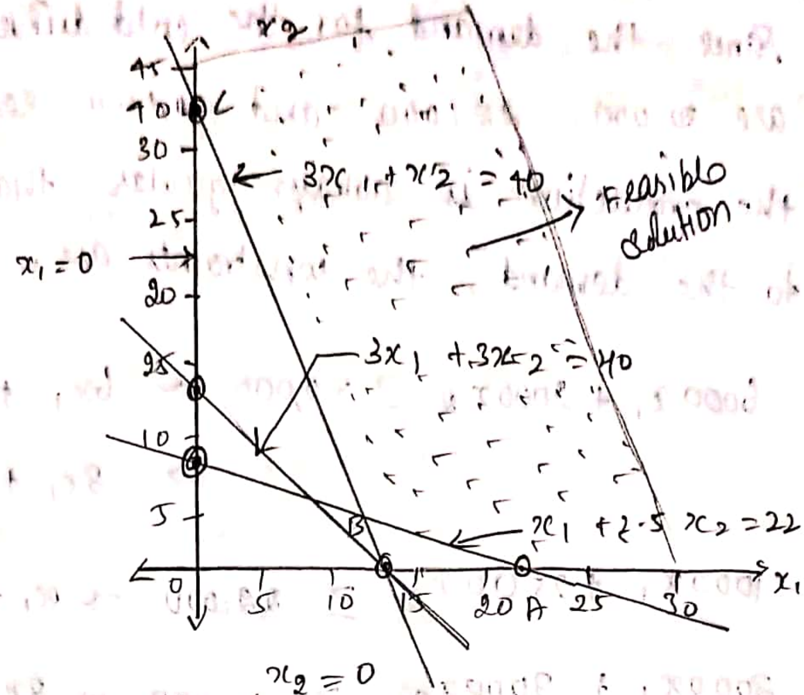
$3x_1 + x_2 \geq 40$

$x_1 + 2.5x_2 \geq 22$

$3x_1 + 3x_2 \geq 40$

and $x_1, x_2 \geq 0$.

By using the Graphical method, A(22, 0), B(12, 4), C(0, 40).




We see that constraint $3x_1 + 3x_2 \geq 40$ does not affect the solution space. So, $3x_1 + 3x_2 \geq 40$ is a redundant constraint.

The values of z at these vertices A (22, 0), B (12, 4) and C (0, 40) are given by.